THE PRINCIPLES

OF

STRUCTURAL DESIGN.

PART II.

BY

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PREFACE TO PART II.

THE second part of this treatise has been compiled for the use of officers, Royal Engineers, completing the full course of instruction at the School of Military Engineering.

In the former part of the treatise, among other subjects, the consideration of influences brought to bear on supported beams and cantilevers has been considered. In this, the more complicated questions of fixed and continuous beams, and the effect of rolling loads are first investigated, after which the application of all the foregoing principles to different types of bridges is discussed and exemplified.

In addition to bridges, the subjects of retaining walls, reservoir dams and sea defences are dealt with. This completes the list of those problems in structural engineering which, as a rule, engineers of the army of Great Britain may have to solve.

No apology need be offered for devoting a considerable portion of this work to the construction of bridges, for it will be universally admitted to be of special importance to engineers of the British army.

It may be here pointed out that the bridges which English military engineers have to construct are not those which, as a rule, civil engineers have to consider. They are rather of a light or semi-permanent character, as compared with the railway bridges usually required in civil life; but although the materials are different, the same principles are involved. Since it is rare that precedents can be followed, it is imperative that if the best work is to be done under any given circumstances, the engineers should know all the

varieties of principle so as to select the best type for use in a given case.

Among recent campaigns that of Chitral, where nearly every conceivable type of bridge was constructed, affords evidence of the truth that thorough efficiency in this branch of engineering can only be obtained by an extensive knowledge of principles.

The examples of bridges in this work are taken as far as possible from those designed and built by Royal Engineer officers. It is not intended that these bridges are necessarily the best illustrations of the principles involved, but at least they show how the principle in question can be, and has been, successfully applied. Thus the Hurroo bridge illustrates the principle of the strutted beam, the Barra bridge that of the bow-string girder, the Wangtu bridge that of the cantilever and the Howe truss, Sir John Ardagh's bridge that of the bascule type, Major Bate's portable bridge the braced girder with parallel flanges, the Jumna bridge the suspension principle. It is true that the example of the Fribourg suspension bridge is an exception to the general rule, but that celebrated structure was constructed under conditions so strongly resembling those under which all Royal Engineer officers might ordinarily have to work, that it has been taken as an example

Although it is not possible to do much more in a work of this sort than merely compile information from existing publications yet there are a few points which are here published for the first time, and are little known to English engineers. For instance, in the first chapter is given a series of graphic solutions of problems of fixed beams evolved by Major J. R L. Macdonald, R.E. In another chapter is given Colonel Pennycuick's original and interesting views on masonry dams. Elsewhere in the book there are several new applications of old methods. Everywhere endeavour has been made to substitute graphic for purely mathematical investigation.

As in the former part of the treatise, so in this, mathematical proofs are given as far as possible in appendices to the chapter dealing with the subject concerned. It has not been possible to give such proofs in every case, however, because this would have

involved, in some instances, a greater space than could be afforded: nor has it always been possible to relegate such proofs to the appendices, because to have done so would have destroyed sometimes the point of a demonstration. But the principle has been adhered to in most instances.

The assistance given to the author by the following officers is most gratefully acknowledged:—

Major-General Sir J. C. Ardagh, K.C.I.E., C.B., for permission to quote his description of the equilibrium bridge, and to reproduce the illustrations connected with it.

Colonel Pennycuick, C.S.I., R.E., for examining the chapter dealing with masonry dams.

Major H. D. Love, R.E., for permission to quote from articles written by him on the elastic curve of loaded beams.

Captain J. I. Lang, C.M.G., R.E., for having carefully revised the MSS., and for many valuable suggestions, especially in connection with bridge design.

Captain and Brevet Major J. R. L. Macdonald, R.E., for a paper containing investigations on the subject of fixed and continuous beams, alluded to above.

Captain E. M. Paul, R.E., for having given much assistance in the correction and examination of the text.

The following books have been consulted:—

General Wray, C.M.G., R.E. *Some Applications of Theory to the Practice of Construction.*

Professor Rankine's *Civil Engineering.*

,, ,, *Applied Mechanics.*

Professor Cotterell's *Applied Mechanics.*

Professor De Lanza's *Applied Mechanics.*

Professor Johnson, Turneaure and Bryan's *Theory and Practice of Modern Framed Structures.*

Professor Warren's *Engineering Construction in Iron, Steel and Timber.*

Professor Claxton Fidler's *Bridge Construction.*

Professor Baker's *Masonry Construction.*

Mr. Anglin's *Design of Structures.*

Mr. Patton's *Civil Engineering*.

Mr. FitzMaurice's *Plate Girder Bridges*.

Mr. Stevenson's *Harbours*.

Mr. Shield's *Harbour Construction*.

Mr. Tarn's *Mechanics of Architecture*.

Messrs. Tudsbery Turner & Brightmore's *Principles of Waterworks Engineering*.

M. Pascal's *Traité Pratique des Ponts Metalliques*.

M. Petit's *Aide Mémoire des Ponts et Chaussées*.

The Roorkee Treatise of Civil Engineering.

The Pocket Books of Trautwine and Molesworth.

Proceedings of the Institution of Civil Engineers.

R. E. Professional Papers, etc.

The publication of the plates illustrating the Hurroo, Barra, and Wangtu Bridges is sanctioned by the Secretary of State for India. The War Office has sanctioned the publication of the plate illustrating Major Bate's principle of braced girders.

G. K. S.-M.

Chatham, February, 1898.

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THE PRINCIPLES OF STRUCTURAL DESIGN.

PART II.

CHAPTER I.

Fixed and Continuous Beams.

Fixed Beams.—Definitions.—Advantages.—Major Macdonald's Graphic Methods of Solution.—Various Cases.—Continuous Beams.—Advantages and Disadvantages.—Theorem of Three Moments.—Graphic Solutions. Mr. Claxton Fidler's Method. Appendices.

Fixed Beams

When a beam is supported as in *Fig.* 1, and is not deflected by any load on it, the ends AC, BD will be vertical. If a load of any sort be placed on the beam, the ends will no longer be vertical, but will be inclined towards each other, as in *Fig.* 2.

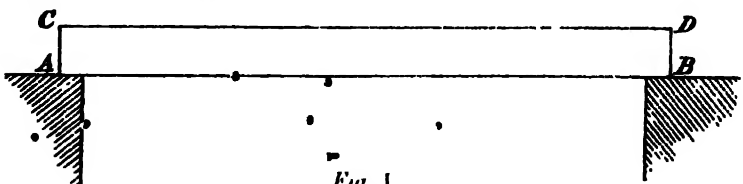


Fig. 1.

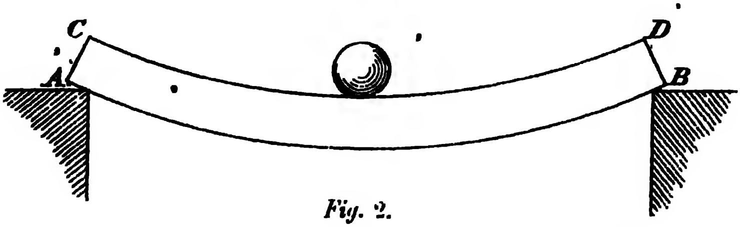


Fig. 2.

B

Definition.

If the beam be supposed to be lengthened at each end, and weighted at those ends, or held down by fastenings, so that the ends still remain vertical, the beam is said to be *fixed*, and the bending will take place as in *Fig.* 3, where the end parts nearest the

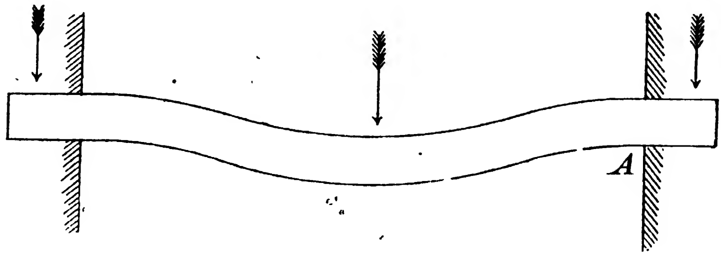


Fig. 3.

fastenings are in "hogg " and the middle part in ging," curvature. The effect of the fixing is to make the tangent to the deflection curve at the fixing horizontal.

If the fastening be insufficient, it is evident that the end sections will assume an inclined position, but not so much inclined as when they are supported only. Such beams are said to be partially, or imperfectly, fixed. It is generally customary to treat such beams as supported only, although there is no doubt that they are stronger to resist transverse stress than supported beams, and, under certain conditions, as strong, practically, as if wholly fixed.

Direct stresses.

The direct stresses in fixed beams are distributed differently from those in supported beams. In the latter there is no direct stress at the points of support, the value of the M_f there being $=0$. In the former the effect of the fastening is to produce a Moment of Flexure at the point of fixing, so that, as in *Fig.* 4, the upper fibres are in tension and the lower in compression.

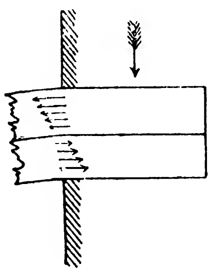


Fig. 4.

Points of contra-flexure.

The points where the curvature changes from hogging to sagging are called "points of *contra or contrary flexure*," or "points of inflexion;" at them there is no direct stress, *i.e.*, the value of the M_f at those points $=0$. We may, in fact, consider the whole beam to be made up, as far as the points of contra-flexure, of two cantilevers, one at each end, and

a supported beam resting on the ends of those cantilevers, as in *Fig.* 5.

It might at first sight be considered possible to work out all problems on fixed beams on the same lines as those for beams strained over two piers, as shown in Chapter IV., Part I. But it will be seen that this is not practicable, as the essential condition of fixing is that the tangent to the deflection curve at the point of fixing must be horizontal---a condition of things that involves consideration of deflection. Therefore the problem cannot be solved by the comparatively simple methods for beams strained over two piers, in which the question of deflection is not considered.

Tangent to deflection curve horizontal at fixing.

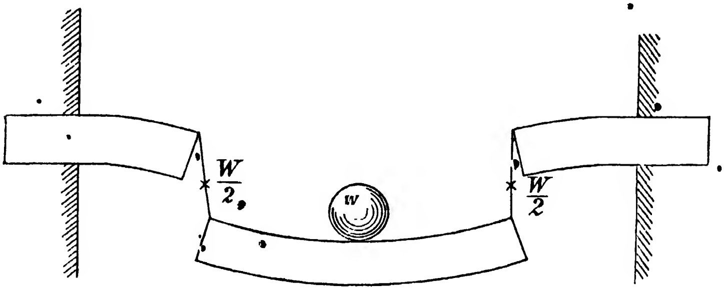


Fig. 5.

As deflection has to be considered, it is evident that the problem will vary according as the beam is of uniform section, or of uniform strength, etc.

The cases which call for solution in ordinary practice are beams of uniform cross section, symmetrical above and below the neutral axis. These alone will here be considered.

Ordinary cases in practice.

The analytical solution of the most usual case, viz., that of a beam fixed at both ends and uniformly loaded, is given in Appendix I. The following results for ordinary cases may all be solved in a similar manner, and are thus tabulated :—

Table I.

Fixed Beams of Uniform Section.

Method of loading	Value of M_{ff}. Fixed.	Value of M_{ff}. If supported only.	Position of M_{ff}. Fixed.	Position of M_{ff}. If supported.	Deflection. Fixed.	Deflection. If supported.	Position of points of contraflexure.
Load W in centre	$\frac{Wl}{8}$	$\frac{Wl}{4}$	Centre and at fixing	Centre	$\frac{1}{192}\frac{Wl^3}{EI}$	$\frac{1}{48}\frac{Wl^3}{EI}$	$\frac{1}{4}l$ from end
,, W uniform	$\frac{Wl}{12}$	$\frac{Wl}{8}$	At fixing	Centre	$\frac{1}{384}\frac{Wl^3}{EI}$	$\frac{5}{384}\frac{Wl^3}{EI}$	·211l from end
,, W in centre* and W uniform	$\frac{5}{24}Wl$	$\frac{3}{8}Wl$	do.	Centre	$\frac{3}{384}\frac{Wl^3}{EI}$	$\frac{13}{384}\frac{Wl^3}{EI}$	·23l from end

Comparison between fixed and supported beams.

From this table we see, for beams of uniform gross section—

(1). That, when the load is in the centre, the effect of fixing the ends is to double the relative strength.

(2). In such a beam there is only $\frac{1}{4}$ of the deflection that there would be if supported only.

(3). That such a beam is equally strong at the fixed ends as in the centre.

(4). That in a uniformly loaded and fixed beam, the fixing has the effect of making the beam stronger than it was when only supported, in the ratio of 3 : 2.

(5). That in a uniformly loaded fixed beam there is only $\frac{1}{5}$ of the deflection that there would be if it were supported only.

(6). That in a uniformly loaded fixed beam the point of greatest stress is at the points of fixing, not in the centre, as in supported beams.

The advantages of fixing, therefore, are very obvious.

Application to rolled steel joist.

If we take, for instance, the wrought-iron rolled joist, 12″ × 6″, shown on *Plate* I., Part I., the Moment of Inertia of which is 360, and Moment of Resistance 296 inch-tons, it would, if supported only, and used in a floor of 16 feet span, allowing the usual weight on the floor (140 pounds to the square foot), require to be spaced at 11·9 feet apart, giving a deflection of 0·24 inches, the load being dis-

* In this case the central load is = the total uniform load.

tributed all along. But with the same unit load and method of loading, the same beam, if fixed, could be placed at 18·1 feet apart, giving a deflection of only ·073″. It would be quite allowable to consider such a girder as fixed, because of the nature of the material, the uniform section throughout, and the symmetry of material about the neutral layer.

Practical difficulties.

Here, however, we are met with a practical difficulty. The advantages of regarding a beam as fixed are so obvious, and lead to such economy in design, that we should at once endeavour to arrange floors, etc., on this basis were it not for two facts—(i.), that the analytical investigations of M_{ft} and deflection are confined to certain particular cases of cross section of beam and method of loading; and (ii.) they depend on absolute perfection in fixing, so that if there be any imperfection in construction the basis of the calculation is at fault.

Hence engineers have been in the habit of calculating beams in floors, etc., as supported only, in default of (*a*) some general rules which could be applied in other cases than the few special ones to which analysis alone applies; and (*b*) of some investigation which would enable the element of uncertainty in fixing to be eliminated.

Major Macdonald's Graphic Solutions.

Graphic methods of solution.

These general rules and investigation, applicable to all ordinary cases, which have hitherto been insoluble by mathematical analysis, have been supplied by the following graphic methods evolved by Capt. and Brevet-Major J. R. L. Macdonald, R.E.*

These rules, which, as far as the writer is aware, have never before been published, give simple and easily applied methods of satisfactorily solving the difficulties above mentioned.

Case 1. One central load.

CASE 1.—*Fixed beam with central load.*

Let AB (*Fig.* 6) be the beam, W the weight. Draw *acb*, the diagram of moments for W, considering AB as supported only. Draw *dce* parallel to *ab*. Join *df* and *ef*.

Then *g* and *h* (or G and H) are the points of contra-flexure.

* Owing to Major Macdonald being now in Central Africa, it has not been possible to communicate further with him regarding the proofs of these graphic problems. Mathematical proofs have, however, been worked out, at considerable length, by Second Lieut. A. C. Robinson, R.E. These proofs are not published here, partly because they would take up too much space, partly because they may not be the same as Major Macdonald's. It is sufficient to note that there is no doubt about the soundness of the graphic methods.

Draw $ighk$. The shaded portion is the required diagram of M_f.

$$R_A = W \times \frac{CH}{GH} = \frac{W}{2}, \text{ and } R_B = W \times \frac{CG}{GH} = \frac{W}{2}.$$

We see that AG = GC and CH = HB, *i.e.*, the position of the points of contra-flexure is half-way between the load and points of fixing.

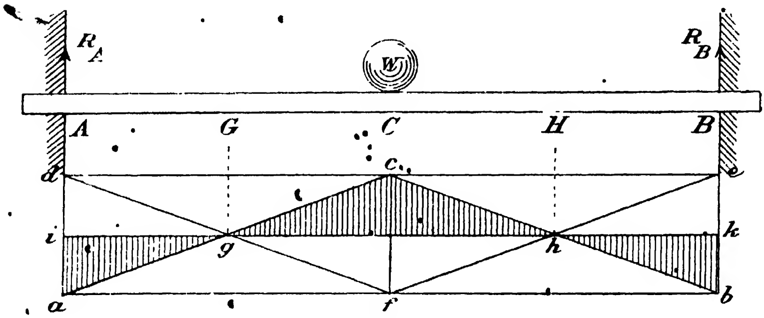


Fig. 6.

Case 2. Uniformly distributed load.

CASE 2.—*Fixed beam with uniformly distributed load.*

Draw acb, the M_f curve for AB (*Fig.* 7), as a supported beam. Then tangents to the parabola at a and b meet at d, when $gd = 2gc$. Draw edf parallel to ab, and draw parabola egf.

Then h and j (or H and J) are the points of contra-flexure.

Draw $khjl$ parallel to ab. The shaded portion is the required M_f diagram.

$$R_A = \text{load on AH} + \tfrac{1}{2}\text{ load on HJ} = \frac{Wl}{2}.$$

AH and BJ may be regarded as cantilevers, each with a uniform load AH and a load at the end equal to $\frac{1}{2}$HJ.

It will be noticed that in the above cases, as compared with those of Chapter IV., Part I., we have altered the datum line from which the M_f ordinates are measured.

Datum line.

In Chapter IV., Part I., this datum line was identical with the beam line, but here it is some other line, as ik (*Fig.* 6) or kl (*Fig.* 7), parallel to the beam line. This *altered datum* will afterwards be freely used in these investigations. It is not always parallel to the beam line, nor is it always quite straight from end to end.

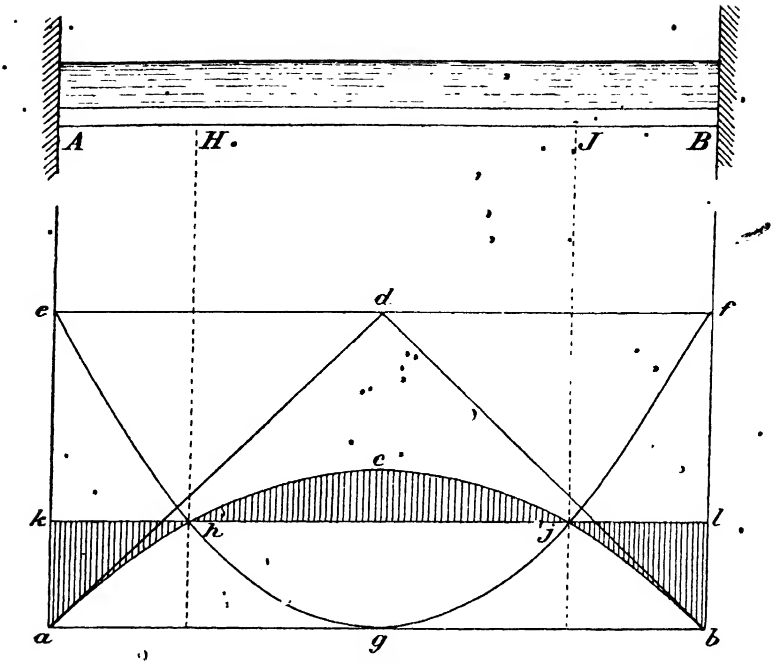


Fig. 7.

Case 3. Single load not at centre.

CASE 3.—*Fixed beam with concentrated load not at the centre.*

The load is at D (*Fig.* 8), while C is the centre of the beam.

Draw the M_f diagram *aeb* as if AB were supported only (see p. 68, Part I.).

Draw *cf* from the centre *c* at right angles to *ab*, and produce *ae* to meet it at *f*. *be* cuts *cf* at *g*.

Draw *gj* and *fh* parallel to *ab*. Join *hd*, *jd*, cutting *ae*, *eb* at *k* and *l*.

Then *k* and *l* (or K and L) are points of contra-flexure.

Join *kl* and produce both ways to *m* and *n*. *mkln* is the datum line.

The shaded portion is the required M_f diagram.

$$R_A = W \times \frac{DL}{KL}, \text{ and } R_B = W \times \frac{KD}{KL}.$$

This is a very important result, inasmuch as the case is not one that can be readily solved analytically.

The deflection can be ascertained by considering KL as a beam supported at K and L, and AK, BL cantilevers loaded with R_A, R_B at their ends, and calculating the combined deflection. This is a matter of no difficulty as regards the cantilevers, but is not so simple for the central part. An expression for the deflection of a supported beam with a load not in the centre will be found in Appendix II. of this Chapter, p. 39.

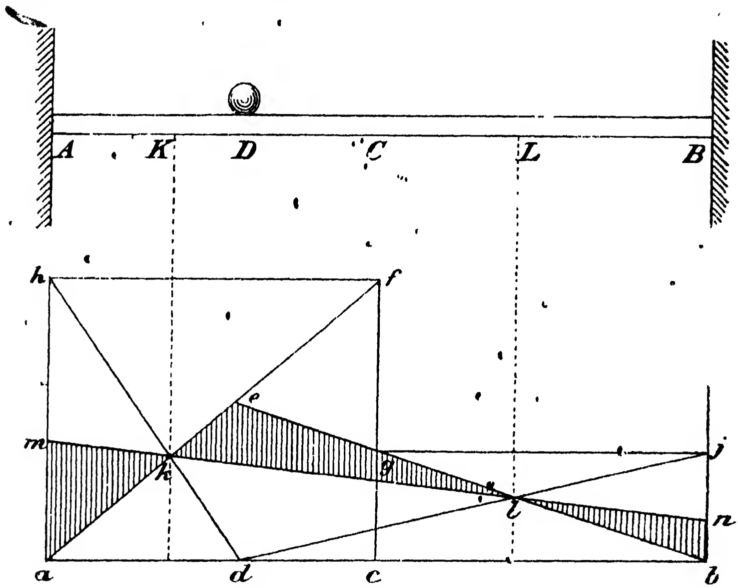


Fig. 8.

Case 4. More loads than one.

CASE 4.—*Fixed beam with more than one load.*

Let C (*Fig.* 9) be the centre of the beam. Draw *akdlbf* (*Fig.* 10) as the M_f diagram for W_1, as in the last case.

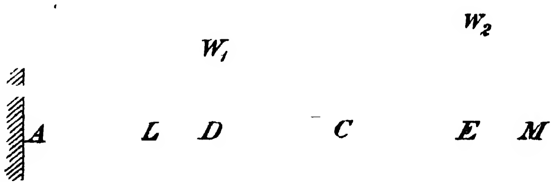


Fig. 9.

Draw $a'k'd'e'b'f'$ similarly for W_2 (*Fig.* 11).

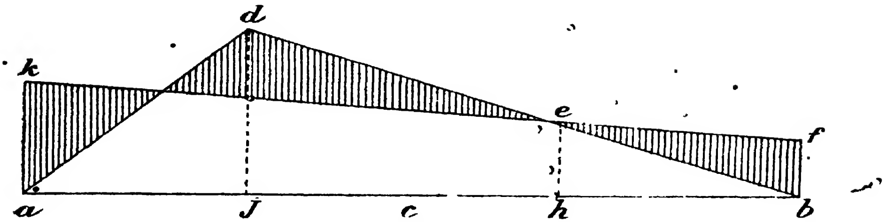


Fig. 10.

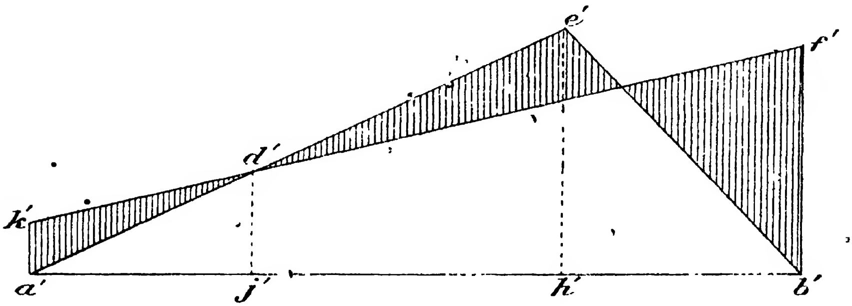


Fig. 11.

Then (*Fig.* 12) make

$$a''k'' = ak + a'k',$$
$$j''d'' = jd + j'd',$$
$$e''h'' = eh + e'h',$$
$$b''f'' = bf + b'f'.$$

Join $k''f''$.

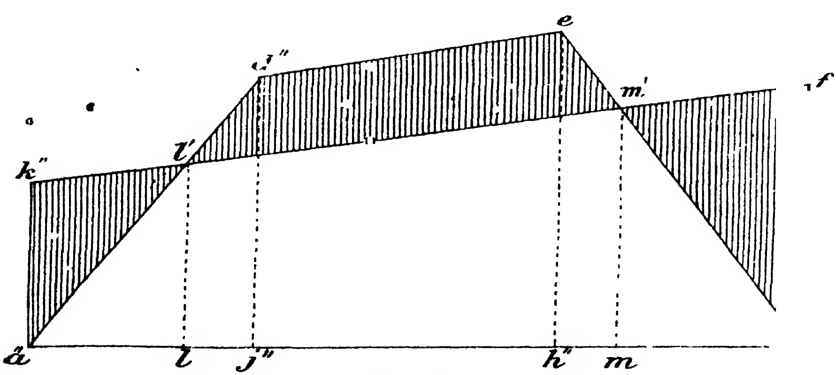


Fig. 12.

The shaded portion in *Fig.* 12 is the diagram required.

The points of contra-flexure are L and M in *Fig.* 9 corresponding to l and m in *Fig.* 12.

Case 5. Planes of fixing different, owing to settlement.

CASE 5.—*Fixed beam, where the planes of fixing at the two ends are not the same owing to settlement, etc.*

Where, owing to settlement of one pier, or from any other cause, the planes of fixing of the two ends are different, the points of contra-flexure, and the bending moments produced, are largely altered.

Let AB be a beam whose fixed ends A and B were originally on the same level as AC, but the end B has settled to d (*Fig.* 13).

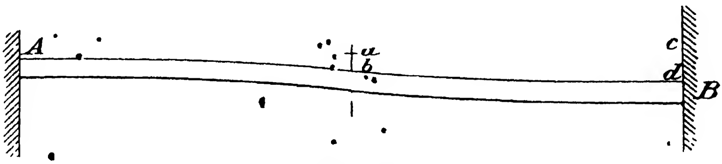


Fig. 13.

The beam has evidently to take the form Abd, with a point of contra-flexure at the centre b. This would form a M_f diagram (*Fig.* 14), irrespective of external loads, of the form $efhj$. The M_f diagram due to external loads would be modified by this; the M_f at A being increased by ef, and at B decreased by hj.

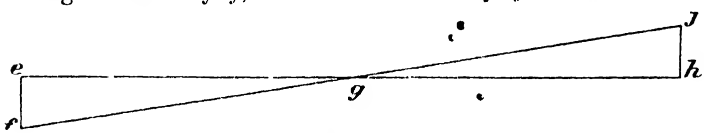


Fig. 14.

This increase or decrease can be found; for it is evident that the settlement cd (*Fig.* 13) = twice the deflection at ab. As there is no M_f at b, we may consider Ab as a cantilever, loaded with an unknown load W at b, which produces a deflection ab. But in the case of a cantilever, the deflection is expressed as follows:—

$$ab = V = \frac{1}{3}\frac{Wl^3}{EI}$$

(see p. 129, Chapter VII., Part I.), where the only unknown is W, since $ab = \frac{1}{2}cd$. Hence W can be found. Then

$$ef = W \times Aa = hj.$$

Thus we can ascertain the unknown bending moments produced by the settlement. In order to find their influence on the bending moments produced by external loading, we have to consider whether the load is on that side of the centre which has settled (the most usual case), or on that side of the centre where no settlement has taken place.

In either case let *Fig.* 15, as in *Fig.* 8, represent the M_f diagram for the fixed beam with load W, and no settlement; kl and qr being the moments at the points of fixing.

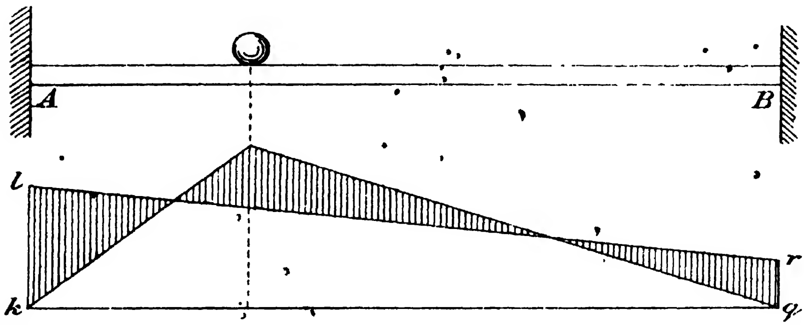


Fig. 15.

Then *Fig.* 16 represents the M_f diagram if, as in *Fig.* 13, B has settled to d and W is on that side of the centre on which no settlement has occurred. Here $k'l' = kl + ef$, and $q'r' = qr - hj$. In this case there is only one point of contra-flexure.

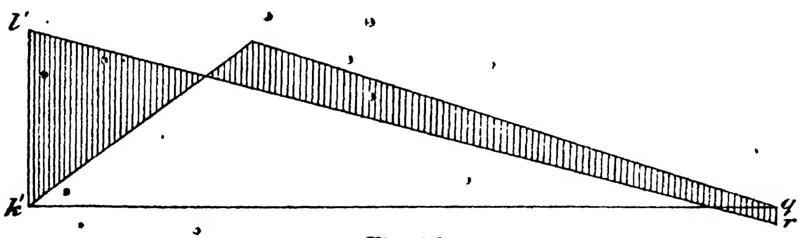


Fig. 16.

Similarly, *Fig.* 17 represents the M_f diagram if A and not B has settled an amount $= cd$. Here $k''l'' = kl - ef$, and $q''r'' = qr + hj$.

The position of the points of contra-flexure here should be studied.

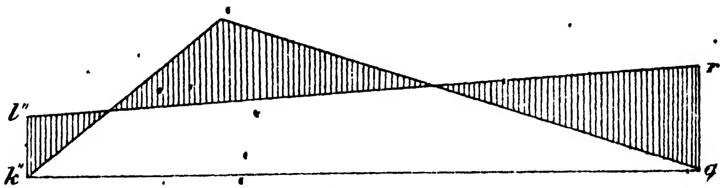


Fig. 17.

Case 6. Beams whose fixing is imperfect.

CASE 6.—*Beams whose fixing is not absolutely perfect.*

This is, of course, a very ordinary case in practice. If it were essential to the consideration of fixed beams that the fixing should be absolutely perfect, it would seldom be possible to base any reliance on the calculations. It would probably be a sound practical rule, while taking all reasonable precautions to ensure perfect fixing, to count upon the beam as being *half fixed* only, *i.e.*, to assume the negative M_f at the *points of fixing* to be only half what it would be if the beam were perfectly fixed.

We have already seen that in the M_f diagrams of fixed beams given above, the curves or polygons of the moments are the same as those that would be evolved if the beams were supported only, but the datum line from which the moments are measured is different. In *supported* beams this datum line corresponds to the beam line, but in *wholly fixed* beams it is at some definite distance from the beam line. In *partially fixed* beams it will occupy some intermediate position. For practical purposes it is suggested that this intermediate position may be half-way between the beam line and the datum line for wholly fixed beams.

Thus for beams loaded centrally, as in *Fig.* 18, the diagram in

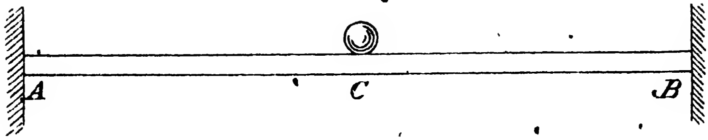


Fig. 18.

Fig. 19 gives the M_f when wholly fixed. Now draw *Fig.* 20 making $a'e' = \frac{1}{2}ae$, and $b'f = \frac{1}{2}bf$, and $c'd' = cd$.

The M_{ff} in AB if supported only is cd (*Fig.* 19).
,, ,, wholly fixed ,, $gd = \frac{1}{2}cd$ (*Fig.* 19).
half ,, $g'd' = \frac{3}{4}cd$ (*Fig.* 20).

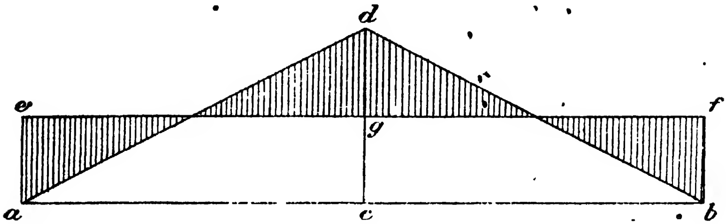


Fig. 19.

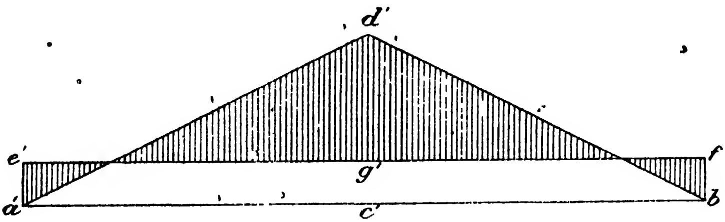


Fig. 20.

For a uniformly loaded beam, wholly fixed (*Fig.* 21), the M_{ff} is at the points of fixing and $= \frac{1}{12}Wl$. The M_f at the centre is $= \frac{1}{24}Wl$.

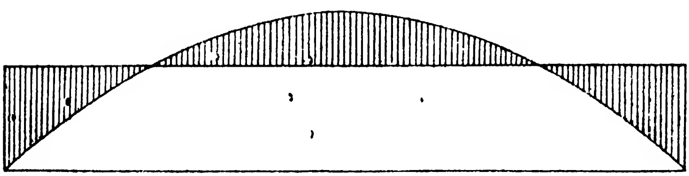


Fig. 21.

Uniformly loaded beams imperfectly fixed.

If the beam be half fixed, the M_{ff} will now be at the centre $= \frac{1}{12}Wl$, and the M_f at the points of fixing will be $= \frac{1}{24}Wl$ (*Fig.* 22). **Thus a uniformly loaded beam of uniform cross section is equally strong whether half fixed or wholly fixed.** Paradoxical as it may seem, such a beam is really strongest when it

is *not* wholly fixed, but when the M_{ff} at the points of fixing and at the centre are each $=\frac{1}{16}Wl$.

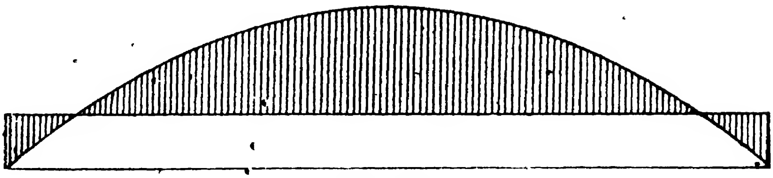


Fig. 22.

It may here be pointed out that in calculating the strength of the fixed planking of the roadway of a bridge, such as a military pontoon bridge where the ends of the chesses are fixed by ribands and rack-lashings, as in *Fig.* 23, we may practically consider loads,

Fig. 23.

such as wheels of wagons, guns etc., which may deviate slightly from the centre, as symmetrical, in view of the investigation shown in the cases above.

For, in the first place, it is clear that, so far as the M_{ff} is concerned (but not the position of the points of contra-flexure), the M_{ff} due to the weights on two wheels, as shown in dotted lines in *Fig.* 24, may

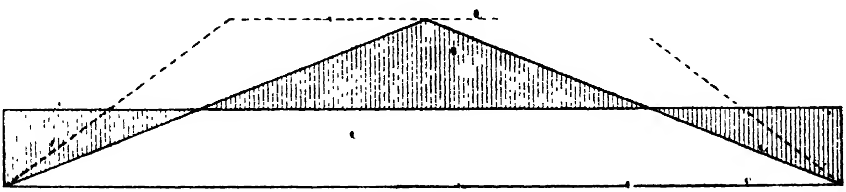


Fig. 24.

be replaced by one central weight giving the same value of M_{ff}. This substitution will simplify the demonstration, for any influences

that would modify the M_{ff} in the one case would equally modify it in the other.

Suppose, therefore, the load on the two wheels replaced by one central load. This has a M_f diagram as shown shaded in *Fig.* 24. If that load be placed a little to one side, it would, *if both supports are still on the same level,* tend to produce a diagram as in *Fig.* 25.

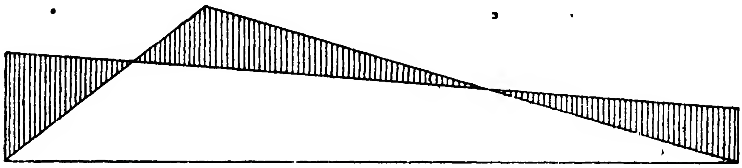


Fig. 25.

But the excess of weight on one longitudinal causes an excess of deflection on that beam, which, as we have seen in Case 5 and *Fig.* 17, tends to counteract the unsymmetrical nature of the M_f' diagram, and bring it back to the symmetrical diagram shown in *Fig.* 24.

CASE 7.—*Beam fixed at one end, supported at the other, with plane of fixing, and support in the same plane (general case).* Case 7. Beam fixed at one end, supported at the other.

Let AB be the beam (*Fig.* 26). First consider the beam as fixed, and draw the M_f diagram (*Fig.* 27). Now if A be fixed and B supported, we know that the M_f at $B=0$. In *Fig.* 28 make $a'e' = ae + \frac{1}{2}bf$, and join $e'b'$, $a'd'b'$ being the same as adb. The shaded portion is the required M_f diagram.

Similarly if B be fixed and A supported. Draw $a''d''b''$ (*Fig.* 29) as in *Fig.* 27. Make $b''f'' = bf + \frac{1}{2}ae$ (*Fig.* 27), and join $a''f''$. Then the shaded part in *Fig.* 29 is the M_f diagram.

The above procedure is a general solution for all cases, whether the loads be concentrated or distributed. It affords another proof of the value of graphic solutions, since only a few such cases are readily soluble by mathematical analysis.

CASE 8.—*Beam fixed at one end, supported at the other, but with settlement at one side or other.* Case 8. Beam fixed at one end, supported at the other, and one end settled.

If *Fig.* 28 shows the diagram when there is no settlement at either side, it is clear that if the *fixed* end settle a negative M_f will be produced, and the diagram will be something like *Fig.* 30. If the settlement at A has been x inches, it is equivalent to a cantilever *ab*

with an unknown load W, which produces a deflection $= x$: $x = \frac{Wl^3}{3EI}$; hence W is calculable. Then the M_f at a due to V W $\times$ ab, and ac is reduced by W $\times$ ab. It will be seen, by compai *Figs.* 28 and 30, that, although the M_f has been reduced at the p of fixing, it has been increased under the load.

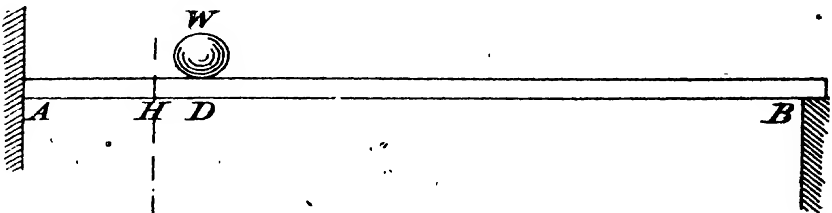


Fig. 26.

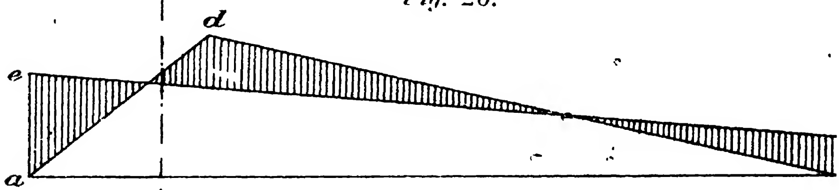


Fig. 27.

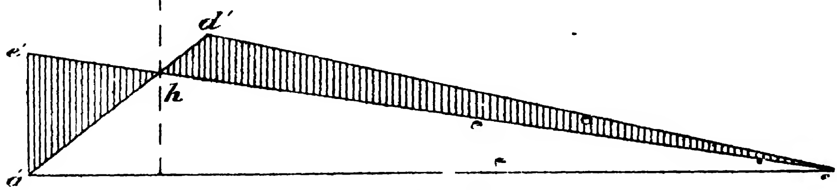


Fig. 28

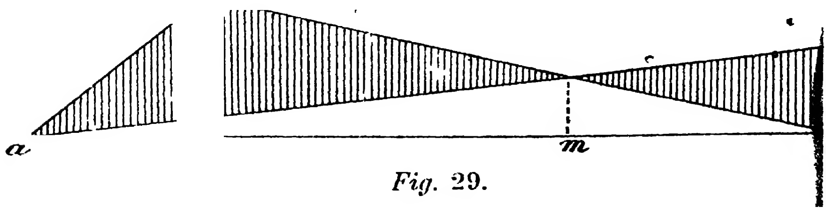


Fig. 29.

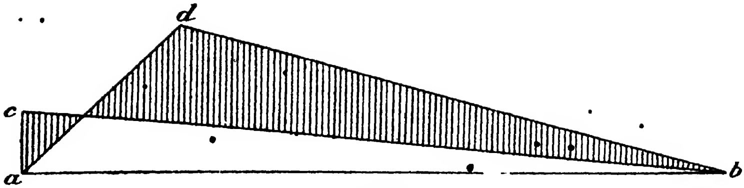


Fig. 30.

Similarly, if the point of support *b* be lowered, the M_f at *a* will be increased, and that under the load decreased, the diagram assuming the form in *Fig.* 31.

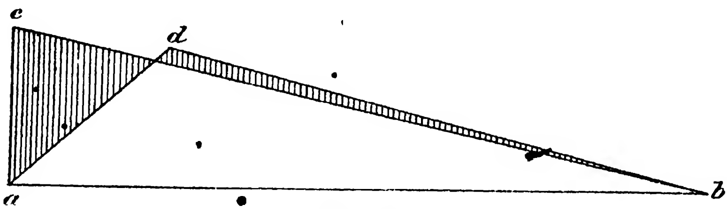


Fig. 31.

As an example of the economy of calculating beams as fixed, when, for practical reasons, it is possible to do so, we may take the following :—

EXAMPLE. Framed floo with fixed beams.

EXAMPLE 1.—*A framed floor is required for a room* 16′ × 30′, *and the girders proposed for use are rolled steel joists* 12″ × 6″ (*as in* Plate I., *Part I.*). *It is proposed to use wooden binders and bridging joists, to fix the ends of the girders, and it may safely be assumed that the walls can bear the weight of such girders, binders, or joists as may be brought upon them. Timber for binders and joists, red fir* (pinus sylvestris).

Weights on floors.

As regards weights on floors, Mr. Hurst gives in his *Handbook* the following rules :—

Ordinary dwelling house floors, including the weight of the floor-boards	$1\frac{1}{4}$ cwt. = 140 lbs. per sq. foot.
Public buildings, lecture rooms, etc.	$1\frac{1}{2}$,, = 168 ,, ,,
Warehouses and factories ...	$2\frac{1}{2}$ to 4 cwt. = 280 to 448 lbs. per sq. foot.

Messrs. Dorman, Long & Co., of Middlesborough, in their valuable *Handbook of Steel Sections*, give the following:—

Dwellings or office buildings	150 lbs.
Public halls or schools	180 „
Warehouses	200 to 400 lbs.
Heavy machinery	300 to 500 „

The factors of safety, however, recommended by this firm for their steel (see p. 57, Part I.) are less than those usually adopted in practice by the War Department. Hence Mr. Hurst's rules are taken in this example.

The safe Moment of Resistance of the $12'' \times 6$ rolled joist, if of steel, is 443 inch-tons, if of wrought iron, 296 inch-tons (see p. 98, Part I.).

Let the assumed distribution of floor timbers be as shown in *Fig.* 1. *Plate* I.

The calculations of the various parts are based on the following considerations:—That the floor joists, binders, and girders are fixed, or at least half fixed, and that they are uniformly loaded, but the ceiling joists are only supported, as their fixing under the binders is necessarily most imperfect.

Floor joists.

Floor Joists.—Span 5·3′, weight per square foot 140 lbs., distance apart of joists 1 foot.

Hence W, total weight on each joist between binders—

$$= 5{\cdot}3 \times 1 \times 140 = 742 \text{ lbs.}$$

And M_{ff} for a fixed beam uniformly loaded is

$$\frac{Wl}{12} = \frac{742 \times 64}{12} = 3{,}957 \text{ inch lbs.}$$

The joists being rectangular—

$$M_r = \frac{rbd^2}{6}, \text{ and } r = \frac{6000}{5} \text{ (average)} = 1{,}200.$$

Hence $\quad 3{,}957 = \frac{1200}{6}\, bd^2$, or $bd^2 = 19{\cdot}8$.

Make $\quad 2\frac{1}{4} \times 3''$.

Ceiling joists.

Ceiling Joists.—These joists may be 1 foot apart centre to centre, which is the usual distance in practice. The weight of the ceiling

is 10 lbs. per square foot. From the deflection formula (*d*), page 139, Part I.—

$$bd^3 = \frac{L^3 S w}{133}, \text{ where } L = 5\cdot3,\ S = 1, \text{ and } w = 10 \text{ lbs.}$$

Hence $bd^3 = 11\cdot1$. $2\frac{1}{2}'' \times 1''$ would do, but leaves very little room for nailing. Make $2\frac{1}{2}'' \times 1\frac{1}{2}''$.

Binders.—Span 15 feet. Weight on each binder = floor weight on $\frac{1}{6}$ whole area + weight of floor joists on $\frac{1}{6}$ area + weight of ceiling and joists ditto. Binders.

Floor weight	$\frac{1}{6} \times 16' \times 30' \times 140$	... ...	= 11,200 lbs.
,, joists	$\frac{1}{6} \times 30 \times 16 \times 2\frac{1}{4}'' \times 3'' \times 36$	..	= 135 ,,
Ceiling ,,	$\frac{1}{6} \times 30 \times 15 \times 2\frac{1}{2}'' \times 1\frac{1}{2}'' \times 36$	...	= 70 ,,
Ceiling	$\frac{1}{6} \times 16 \times 30 \times 10$	..	= 800 ,,
	Total	...	12,205 lbs.
To this must be added the weight of the binder itself, say	$1 \times 15 \times 12'' \times 12'' \times 36$		540 ,,
		Hence W =	12,745 lbs.

or 5·7 tons.

The M_{ff} in inch-lbs. will be

$$\frac{Wl}{12} = \frac{12745 \times 15 \times 12}{12} = 191{,}175,$$

and

$$M_r = \frac{rbd^2}{6} = \frac{1200}{6} bd^2.$$

Hence $\qquad bd^2 = 191{,}175 \div 200 = 955.$

Make $\qquad 10'' \times 10''.$

The weight of the binder will then be $15 \times 10'' \times 10'' \times 36 = 375$, and not 540 lbs., as approximately estimated above. We see that the true value of W will then be 12,680 lbs., not 12,745, say 5·6 tons.

Girder.—From *Fig.* 2, *Plate* I., we see that the distribution of weights is symmetrical as regards the centre. The M_{ff} will be either at the fixing or under either weight. *Fig.* 3 shows the M_f diagram for one weight taken by itself. If the beam had been supported only, the M_{ff} under either load, taken separately, would be $5\cdot6 \times \frac{2}{3} \times 64$ inches $= 240$ inch-tons, or *de* (*Fig.* 3). Drawing the M_f diagram for a fixed beam as in Case 3, *i.e.*, making the line *a′b′* pass through the points *o*, *p*, we see that $aa' = 160$ inch-tons, and $bb' = 85$ inch-tons. As the loads Girder.

at D and C are equal, and symmetrically situated with regard to the centre, it is evident that the M_f diagram for the other weight would be similar, only reversed, *i.e.*, *bb'* would be 160 and *aa'* 85. Hence, combining the two as in *Fig.* 4, we have aa'' (*Fig.* 4) $= aa' + bb'$ (*Fig.* 3), and $e'd' = ed + cf$. From this we see that the M_f is at the point of fixing, and is 245 inch-tons.

Hence the rolled joist is amply strong enough, even if made of wrought iron only, as its M_r, if of wrought iron, is 296 inch-tons. A lighter section of *steel* would be strong enough.

Details of floor are shown in *Figs.* 5 and 6, *Plate* I.

Deflection.

It will be observed that the effect of fixing is to increase the stiffness in a far greater ratio than the strength in beams of uniform section. For instance, a beam with a single central load is, when fixed, twice as strong as when supported. But it is four times as stiff, the deflection at the centre being $\frac{1}{192}\frac{Wl^3}{EI}$ instead of $\frac{1}{48}\frac{Wl^3}{EI}$. In the case of a uniformly loaded beam, the fixing increases the strength by two-thirds, but the stiffness by 5.

It may hence be inferred that if the requirements of strength are satisfied in a fixed beam, the requirements for stiffness are probably more than satisfied, and might be neglected in calculation. For instance, the binders in the last example would have a deflection

$$= \frac{1}{384}\frac{Wl^3}{EI} = \frac{1}{384}\frac{12680 \times (15 \times 12)^3 \times 12}{1440000 \times 10000} = \cdot 16 \text{ inches,}$$

whereas the permissible deflection of $\frac{1}{40}''$ per foot run would be $\frac{15}{40}'' = 0\cdot375''$.

Even if the fixing is imperfect, this would allow an ample margin.

It will thus be seen that, whereas in supported beams the requirements for stiffness generally are greater than those for strength, and therefore govern the scantling adopted, *in fixed beams the opposite occurs.*

Economy of fixing.

In the example thus worked out, the scantlings which would required if the beam were supported, compared with those required on the basis of the beam being fixed :—

	Fixed.	Supported.*
Rolled steel joists, No.	1	2
Binders 32′ long, scantling ...	$2 \times 10'' \times 10''$	$2 \times 5'' \times 10''$
Floor joists 18′ ,, ,, ...	$30 \times 2\frac{1}{4}'' \times 3''$	$30 \times 2'' \times 6''$

There would be a saving in the fixed floor of 9 cwts. of steel

girders,* and about 6 cubic feet of timber. This would amount to about £3 at English market rates. There would also be saving in the brickwork of the walls 3″ in height all round. *Plate* I. shows the floor as designed.

The economy of fixing in timber would be so slight as not to make it worth while adopting in ordinary floors. In framed floors, however, the chief advantage of fixing lies in the economy of the girders, an economy which increases with the span.

It need hardly be said that cast-iron beams, if designed in the usual way with one flange three times the area of the other, are unsuited for use as fixed beams, because the smaller flange, which is designed for compression only, would in certain parts of the fixed beam be brought into tension. Fixing not applicable to the ordinary cast-iron beams.

It is not necessary to say much about the shearing stresses in fixed beams. They would be little different from those in supported beams. In any case, they would be found by precisely the same rules. It would only be necessary to find the reactions at the points of fixing, and apply the rules found in Chapter VI., Part I., for shearing

Continuous Beams.

In many cases in actual practice beams are not fixed at their points of support, but are continuous over several spans. The joists of a floor, the common rafters in a roof, the bressumer in a verandah, etc., are common instances of beams which are continuous, though they are usually calculated as supported only, for the sake of simplicity. CONTINUOUS BEAMS.

It is proposed here not to enter into the full investigation of the problem of finding the bending moments in such beams, but to give a few graphic methods whereby such investigation may be arrived at with a reasonable approximation to accuracy. The mathematical analysis of such beams has formed the subject of many treatises in several languages, but however interesting the problem may be from a purely academic point of view, there are certain objections to such analysis, which cause it to be regarded with distrust from a practical point of view.

One objection is that the theory is based on the supposition that all the points of support are on the same level. Hence, if any settlement occurs, there will be, as we have seen in the case of fixed Objections to theory of continuous beams.

* Even if the 12″ × 6″ girder is used. A slightly lighter section might have been adopted, with still greater saving.

beams, corresponding alterations in the points of contra-flexure, and in the bending moments.

Another objection is that the theory is based upon a consideration of uniform values of the elasticity, which in practice can never be obtained.

Again, in built-up work, such as occurs in bridges and roofs, variations of workmanship upset calculations for uniformity of stress.

Finally, if the structure is subject (as in the case of bridges) to varying loads, the positions of the points of contra-flexure and the resulting moments will vary with the various methods and positions of loading.

Hence we must regard such investigations as approximate only. This, however, is no reason why the subject should be wholly disregarded, for the economic advantages of continuous girders under certain circumstances give them unquestionable claims to practical consideration.

Difficulty of investigation.

The main difficulty of the investigation lies in the fact that the vertical reactions cannot be found by the law of the lever. The internal stresses must, therefore, be determined by the consideration of the elastic deflection of the beam.

Beam continuous over two spans.

In the case of a beam continuous over two equal spans and uniformly loaded, the reactions at the central pier and the supports can be ascertained from the laws of deflection. The procedure would be first to calculate the central deflection which would occur if the girder were merely supported at both ends, then to consider the reaction at the central pier, which will evidently be equal to the single load which would cause at the centre the calculated deflection. For instance, if the beam be of uniform cross section and uniformly loaded with a total load = W, the deflection at the centre will be $\frac{5}{384}\frac{Wl^3}{EI}$ (p. 189, Part I.). Under a single central load the same deflection would be $\frac{1}{48}\frac{W'l^3}{EI}$. Hence $\frac{5}{384}W = \frac{1}{48}W'$ or $W' = \frac{5}{8}W$. We see, therefore, that in such a case $\frac{5}{8}W$ is the reaction at the centre, and $\frac{3}{16}W$ at each of the supports. This has already been taken into consideration in investigating the stresses on a king-post roof (p. 250, Part I.).

Now if we suppose the same beam to be absolutely rigid,* we see

* From Mr. Claxton Fidler's *Treatise on Bridge Construction.*

that the smallest settlement of the central pier would bring the entire load on the abutments. Similarly, a settlement of the abutments would bring the load on the piers. In the former case the M_f diagram would be *abc* (*Fig.* 32), and in the latter case *ab′c*.

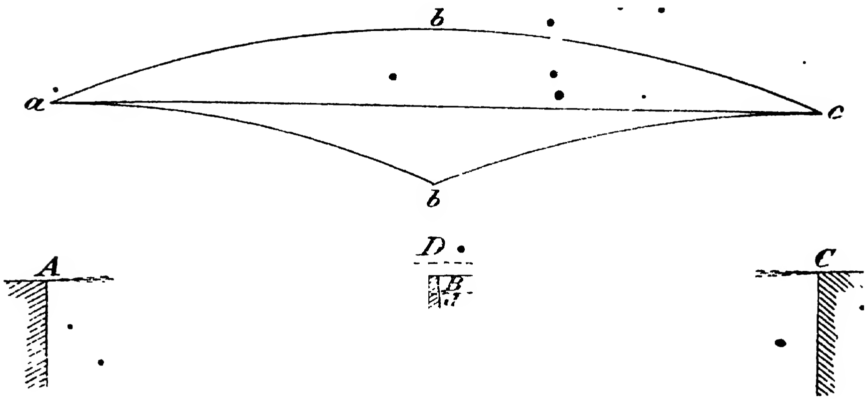


Fig. 32.

The range of stress *bb′* would be enormous, inasmuch the one case the stresses at the centre would be of the opposite character to what they would be in the other. It is, however, evident that this difference of stress could only occur in practice (since the actual materials we have to deal with are elastic) when the difference in level was an appreciable amount, viz., for a uniform load W all along the girder (if of equal section throughout)—

$$Bd = \frac{5}{384}\frac{Wl^3}{EI},$$

and

$$BD = \frac{Wl^3}{EI}.*$$

The solution must, in any case, depend on the elastic deflection of

* For a cantilever uniformly loaded—

$$V = \frac{W'l'^3}{EI}.$$

Here $W' = \frac{W}{2}$ and $l' = \frac{l}{2}$.

Hence $V = \frac{1}{12}\frac{Wl^3}{EI}$

the girder; and the reactions produced by the piers, in order to produce the best effect, should be such as to give the deflection curve such a form as to bring the three points A, B, C on the same level.

Clapeyron's Theorem of Three Moments.

Analytically, the relation between the bending moments at any three consecutive piers may be expressed by what is known as Clapeyron's Theorem of Three Moments, which is as follows :—

Let M_0, M_1, M_2 and M_3 denote the Moments of Flexure at four consecutive piers; l_1, l_2, l_3 the lengths of the three spans; w_1, w_2, w_3 the weights per lineal foot, then

$$M_0 l_1 + 2M_1 (l_1 + l_2) + M_2 l_2 + \tfrac{1}{4}(w_2 l_2^3 + w_1 l_1^3) = 0 \text{ ...... (i.)}$$

for uniformly distributed loads.

For concentrated loads the theorem becomes

$$\frac{M_1}{3}(l_1 + l_2) + \frac{M_0 l_1}{6} + \frac{M_2 l_2}{6} + \Sigma \frac{W_1 x_1}{6 l_1}(l_1^2 - x_1^2) + \Sigma \frac{W x}{6 l_2}(l_2^2 - x^2) = 0 \text{...(ii.)},$$

where W and W_1 are the loads in the two spans, and x_1 and x the distances from those loads to the piers at the extremities of the section under consideration. If there be several weights in one bay, take the algebraic sum of the moments, *i.e.*, $\Sigma W x$. For proof of Clapeyron's Theorem see Appendix II. to this chapter.

Major Macdonald's Graphic Solutions.

Major Macdonald's graphic solutions of continuous beams.

The application of Clapeyron's theorem to examples may now be considered and compared with Major Macdonald's graphic system of investigation. This system is somewhat as follows :—Each separate span of the continuous beam is first treated as if fixed, and the M_f diagram drawn on that supposition, as explained above. The end spans are then treated as beams fixed at one end, supported at the other, and half the sum of the resulting moments at either side of the central pier is taken as the moment for the continuous beam. Thus, in *Fig.* 33, if AB, BC, CD represent three adjacent spans of a continuous beam, the principles involved in the investigation are as follows:—

(1). If at B we have two bending moments B*e* due to load on AB, and B*f* due to load on BC, and B be set free (or considered as unfixed), then the resulting M_f at B is a mean between B*f* and B*e*, or is = B*h*.

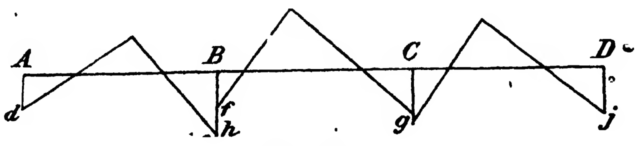


Fig. 33.

(2). But as B*f* has been increased to B*h*, C*g* will be decreased by $\frac{1}{2}fh$, and

is increased by $\frac{1}{2}fh$, and as Be has been decreased by eh, Ad will be increased by $\frac{1}{2}eh$.

(3). By first releasing the points from left to right in succession, and then from right to left in succession, and taking the means of the results so obtained, the second differences are cut out.

The above assumptions give results which closely approximate to those obtained by the Theorem of Three Moments.

Take for example the beam shown in *Fig.* 34, where $AB=40'$, $BC=30'$, $AD=25'$, $BE=15'$, $W=24$ tons, $W'=20$ tons. Let M_a, M_b and M_c be the bending moments at A, B and C respectively.

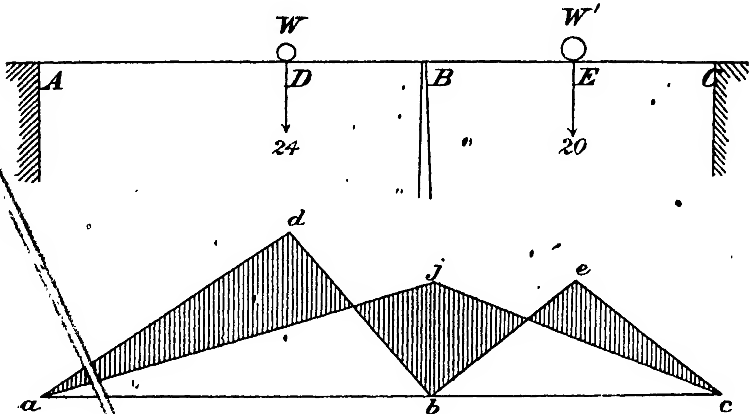


Fig. 34.

EXAMPLE. Beam continuous over two spans.

Then, by the Theorem of Three Moments (ii.)—

$$\frac{M_b\,(AC)}{3}+\frac{M_a\,(AB)}{6}+\frac{M_c\,(BC)}{6}=\frac{W\,(AD)}{6AB}(AB^2-AD^2)+\frac{W'(EC)}{6BC}(BC^2-EC^2).$$

But M_a and M_c are each $=0$.

Therefore, substituting values—

$$\frac{M_b\times 70}{3}=\frac{24\times 25}{6\times 40}(40^2-25^2)+\frac{20\times 15}{6\times 30}(30^2-15^2).$$

Whence $$M_b=152{\cdot}7, \text{ say } 153 \text{ foot-tons.}$$

If we draw bj to scale $=153$, and adb, bec for the spans AB, BC as if supported only, and join aj, jc, then ajc is the distorted datum line, and the shaded part of *Fig.* 34 gives the diagram of moments.

To solve the same problem by Major Macdonald's graphic method. Consider AB, BC as fixed (*Fig.* 35), and draw the diagram of moments for each, viz., $afdbg$ and $bhecj$. Then by measurement

$$af=85,\ bg=140,\ bh=75,\ cj=75 \text{ foot-tons.}$$

If we suppose A and C let free, and thus supported while B remains fixed; we have the moments as follows :—

A	B		C
0	$(140 + \frac{85}{2})$	$(75 + \frac{75}{2})$	0
0	182·5	112·5	0

Now set free B

A	B	C
0	$\frac{182·5+112·5}{2}$	0
0	147·5	

Make bk 147·5 (*Fig.* 35) and join ak and ck. The shaded portion is the required M_f diagram. akc is the distorted datum line.

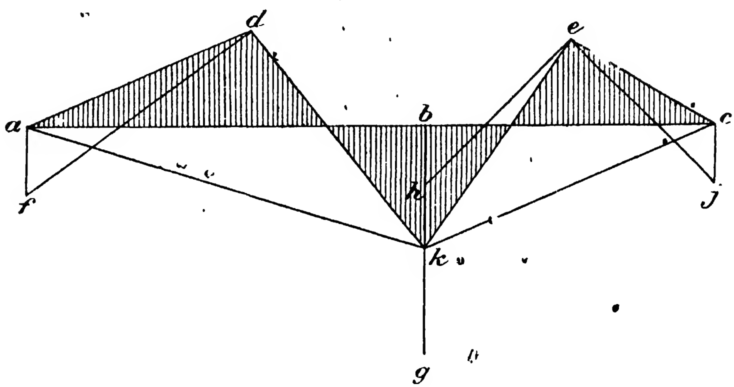


Fig. 35.

The calculated value of bk is 152·7, the graphical value is 147·5. The error is about 3·4 per cent.

EXAMPLE. Beam continuous over three spans.

Another example may be given to illustrate the application of the principle to beams with several spans :—

Let AE (*Fig.* 36) = 30′, EB = 10′, AB = 40′, BF = 20′, FC = 10′, BC = 30′, CG = 20′, GD = 10′, CD = 30′, W′ = 20′ tons, W″ = 30′ tons, W‴ = 24′ tons.

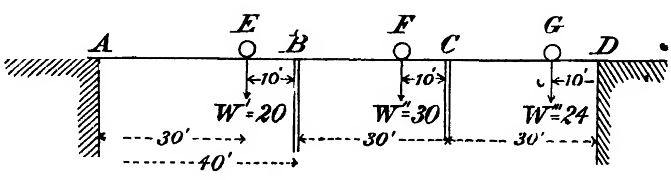


Fig. 36.

Let M_a, M_b, M_c, M_d be the pier moments.

Then, by Theorem of Three Moments, Equation (ii.) —

$$\frac{M_b\,(AB+BC)}{3} + \frac{M_a \times AB}{6} + \frac{M_c \times BC}{6}$$
$$= \frac{W' \times AE}{6 \times AB}(AB^2 - AE^2) + \frac{W'' \times CF}{6 \times BC}(BC^2 - CF^2).$$

But $$M_a = 0\,;$$

$$\frac{M_b \times 70}{3} + \frac{M_c \times 30}{6} = \frac{20 \times 30}{6 \times 40}(1600 - 900) + \frac{30 \times 10}{6 \times 30}(900 - 100),$$

or $$140\,M_b + 30\,M_c = 15 \times 700 + 10 \times 800 = 18500 \quad . \quad \text{(i.)}.$$

Similarly—

$$\frac{M_c \times (BC+CD)}{3} + \frac{M_b \times BC}{6} + \frac{M_d \times CD}{6}$$
$$= \frac{W'' \times BF}{6 \times BC}(BC^2 - BF^2) + \frac{W''' \times DG}{6 \times CD}(CD^2 - DG^2)$$

and $$M_d = 0\,;$$

$$\therefore \quad \frac{M_c \times 60}{3} + \frac{M_b \times 30}{6} = \frac{30 \times 20}{6 \times 30}(900 - 400) + \frac{24 \times 10}{6 \times 30}(900 - 100)$$

$$120\,M_c + 30\,M_b = 20 \times 500 + 8 \times 800 = 16400\,;$$

$$\therefore \quad M_c = \frac{1640}{12} - \frac{M_b}{4}$$

Substitute this value in (i.)—

$$140\,M_b + 30\left(\frac{1640}{12} - \frac{M_b}{4}\right) = 18500,$$
$$M_b = 108{\cdot}7\,;$$

and since $$120\,M_c + 30\,M_b = 16400,$$
$$M_c = 109{\cdot}5.$$

If we draw *aeb*, *bfc*, *cgd* for AB, BC, CD as supported beams, and make *bh* on the same scale $= M_b = 108{\cdot}7$, and $cj = M_c = 109{\cdot}5$, the shaded portion is the M_f diagram (*Fig.* 37).

The above is the solution by the Theorem of Three Moments.

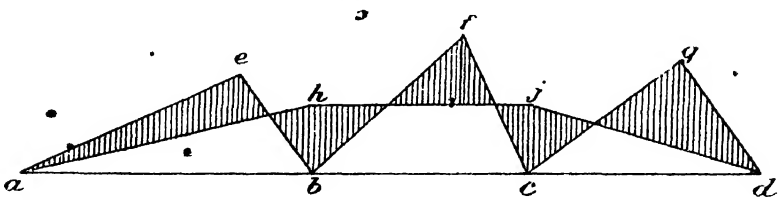


Fig. 37.

To apply Major Macdonald's graphic method, consider AB, BC and CD

fixed beams, and draw the M_f diagrams as in *Fig.* 38. In practice, it is well to draw these to a large scale.

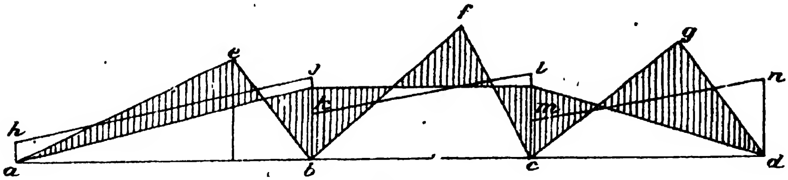


Fig. 38.

Th 37, $bj = 113$, $bk = 69$, $cl = 129$, $cm = 55$, $dn = 106$.

A	B	B	....	C	C	..... D
37	113	69	... .	129	55	 106

Consider A and D supported, the others *fixed*. Then we have

0	.. .. $113 + \frac{37}{2}$	69	... .	129	$55 + \frac{106}{2}$	... 0
$= 0$	.... 131·5	69	.. . ...	129	108	0

Now if we relieve B *and* C, each re-acts on the other, and the problem is indeterminate. But if we release the points in succession from left to right, and then from right to left, and finally take the mean of the results, we get very close approximation to the results obtained by the Theorem of Three Moments, thus:—Work from left to right and release B.

A	B		C		D
0	$\frac{131{\cdot}5 + 69}{2}$	....	$\left(129 - \frac{100{\cdot}2 - 69}{2}\right)$	108	0
$= 0$	$= 100{\cdot}2$	.. ..	113·4	 108	

Now release C.

0	$\left(100{\cdot}2 + \frac{113{\cdot}4 - 110{\cdot}7}{2}\right)$	$\frac{113{\cdot}4 + 108}{2}$	0
$= 0$	101·5	.... 110·7 ...	. . 0...(A).

Now work from right to left, release C.

0	131·5	$\left(69 + \frac{129 - 118{\cdot}5}{2}\right)$	$\left(\frac{129 + 108}{2}\right)$	0
$= 0$	131·5	74·2	118·5	

Now release B.

0	$\frac{1}{2}(131{\cdot}5 + 74{\cdot}2)$	$8{\cdot}5 - \frac{129 - 102{\cdot}8}{2}$	
$= 0$	102·8	104·4	... .0...(B).

The mean of A and B gives $M_b = 102{\cdot}15$ and $M_c = 107{\cdot}5$, as against $M_b = 108$ and $M_c = 109$, by the Theorem of Three Moments.

This graphic method has been applied to a great number of different loads and spans, and has always been found to give results which approximate so closely to the calculated values as to be practically identical.

Mr. Claxton Fidler's Graphical Method.

Another graphical method evolved by Mr. Claxton Fidler is especially useful in the case of beams uniformly loaded. Mr. Claxton Fidler's graphic solution.

In *Fig.* 39, let AB, BC, CD represent a beam continuous over three spans, and loaded uniformly.

First draw the M_f diagram for each span as if it were supported only, and independent of the others. These will be parabolas which will vary with the span and the unit load.

Take points G, H, O, L at one-third of each span from the points of support. Draw GK at right angles to AB, cutting the line joining the apex of the parabola a' to B at K, draw HL, OM, LN cutting similar lines at L, M, and N. Join KL, MN, and divide these lines at P and Q in the inverse proportion of the adjacent spans, *e.g.*, make KP : PL : : BC : AB. If the spans are equal, as in *Fig.* 39, P and Q will be vertically above B and C.

Then P and Q are *pivot points.*

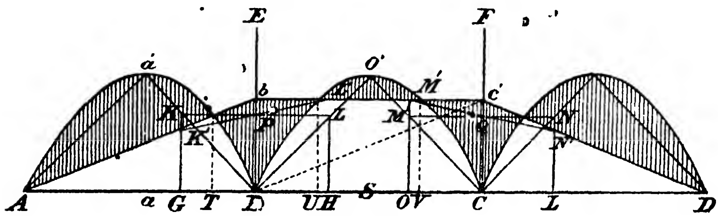


Fig. 39.

The distorted datum line, from which we wish to measure the M_f ordinates, must pass at such distances above the pairs of points on each side of the pier, that KK′ and LL′ are in the inverse ratio of the spans AB and BC. The line must start at A and terminate at D. There is only one position of the line which will satisfy these conditions, and that is easily found by trial and error. In the figure the spans and weights are equal, and the problem is simple. But it is almost equally simple where the spans and weights are unequal (see *Plate* II. as an example). "The moveable base line can be adjusted in a few minutes by trial and error, so as to comply with the governing condition" that it must pass over one point and under the other. "The engineer who is accustomed to laying down the gradients upon a railway section will have no difficulty in effecting this familiar adjustment, for he has only to suppose that B is a

break of gradient, and he has to adjust his grades so as to pass under the fixed point K and over L by equal depths of cutting and embankment. Perhaps the quickest methods will be to draw the lines KL and MN, connecting each pair of fixed points and intersecting the pier verticals at P and Q. Then the point b must be raised until a straight-edge laid across K'L' cuts through the point P, and the same at each pier of the bridge."*

The shaded portion of the figure (29) represents the M_f diagram.

Principles involved in the graphic method.

Although space will not admit of the somewhat lengthy and difficult, but very ingenious, proof of the above graphic method, it may be pointed out that the steps involved are as follows:—

(1). The actual deflected curve that a beam of any form assumes when under a load is derived from the M_f diagram in precisely the same way that the M_f curve is derived from the load (see Appendix II.).

(2). The position that such a curve occupies in space can, however, only be determined by reference to fixed points.

(3). In the case of girders (of uniform section) continuous over several spans of known length and with known weights, we can construct M_f diagrams for each span as if the girder for each span were supported only and independent. This will indicate the form of the deflection curve.

(4). In the case of such girders, we have certain fixed points, viz., the piers and abutments. We know also that the tangent to the deflection curve at the piers will be common to the curve in two adjacent spans, measuring the inclination upwards in one span and downwards in the next.

(5). If M_a, M_b, and M_c represent the unknown pier moments at the abutment and at piers B and C, and if M_1 and M_2 represent the central moments of the spans AB, BC (*Fig.* 39), and if E be the Modulus of elasticity and d the depth of the girder, it may be proved that the deflection below the tangent at B in the span AB is expressed by

$$\left(\tfrac{2}{3}M_b - \tfrac{2}{3}M_1\right)\frac{AB^2}{Ed} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\alpha),$$

and that the deflection below the tangent at B in the span BC is expressed by

$$\left(\tfrac{2}{3}M_b + \tfrac{1}{3}M_c - \tfrac{2}{3}M_2\right)\frac{BC^2}{Ed} \quad \ldots\ldots\ldots\ldots\ldots \quad (\beta).$$

* *A Practical Treatise on Bridge Construction*, by T. Claxton Fidler, M.Inst.C.E., p. 141.

(6). If by any geometric device we can obtain a value for the expressions within the brackets in (α) and (β), we can ascertain the position of the true datum line. This has actually been done in the foregoing graphic construction, for in *Fig.* 39

$$KK' = \tfrac{2}{3}\,aa' - \tfrac{2}{3}Bb, \text{ and } LL' = \tfrac{2}{3}Bb + \tfrac{1}{3}Cc - \tfrac{2}{3}So'.*$$

By this method, therefore, we may fix the points of contra-flexure T, U, V, X, and, therefore, obtain the M_f diagram for the whole beam.

As regards the shearing stresses, we see at once that when the points of contra-flexure have been ascertained, the shearing stresses can be easily calculated. For the shearing stress at the side of any pier will be (in the case of uniformly distributed loads) the weight on the cantilever represented by the distance from the pier to the point of contraflexure at that side, plus half the weight supported on the portion between the points of contra-flexure. Thus in *Fig.* 39 the shearing stress at $A = \frac{1}{2}w \times AT$, shearing stress at $B = \frac{1}{2}w \times AT + w \times BT$ in the span AB, and $w \times BU + \frac{1}{2}\,w \times UV$ in the span BC, and so on. — Shearing stresses.

As an illustration, *Plate* II. shows the graphic representation of moments in a continuous girder bridge with irregular spans when loaded all over. The total width is 135′, and the spans 30′, 60′ and 45′. The weight per foot run is 2 tons, *i.e.*, temporary load 1 ton and permanent load 1 ton. It will be seen that there is very considerable economy by adopting the continuous form in this case, for the M_{ff}, instead of being 900 foot-tons, is only 590 foot-tons. The pier moments calculated by the Theorem of Three Moments are 476 and 597. By the graphic method they are 470 and 590.

The same bridge loaded only on the central span gives a M_f diagram shown in dotted lines. There is a very considerable difference in the values of the M_f, and still greater difference in the position of the points of contra-flexure. We see from this that the effect of rolling loads on continuous girder bridges is to alter the positions of the points of contra-flexure, the values of the pier moments, and both the values and the nature of the shearing stresses. Where the dead weight of the bridge is great in comparison with the rolling load, such changes are of little consequence. But where the opposite conditions occur, as is the case in railway bridges of short span (say under 100′), the changes produced are very important, and are difficult to meet — Disadvantages of continuity

* For if we join BC′ cutting HL′ in h—

$$LL' = L'h + hH - LH.$$

practically. Hence, in *such bridges, continuous girders are not considered economical for spans less than* 150 *feet.*

"The uncertainty* arising from the shifting of the points of contra-flexure may, however, be met by fixing them in the most advantageous positions for the general economy of the structure. This can be done by connecting the ends of the different segments together at the points of contra-flexure decided on in such a way that the continuity of the direct stresses in the upper and lower flanges is broken, as is done in the case of the Forth Bridge, etc., or, in other words, by constructing each span with cantilevers for part of the distance supporting a beam of reduced span."*

A great disadvantage of continuous girders lies in the fact that differences of level in the piers or abutments causes also shifting of the points of contra-flexure, and although minor differences may be accounted for by applications of the graphic methods of Mr. Fidler, as described above, the uncertainty that arises from this cause has had the effect of diminishing the confidence of engineers in their use in bridges.

Case where continuous beams may be advantageously used.

There is no doubt, however, that in simple construction the principle of the continuous girder may be applied with advantage. Numerous instances will at once suggest themselves—common rafters, bressumers, etc. In such cases there is not likely to be much shifting of the points of contra-flexure, and the advantage of calculating the beams as continuous may in some cases be very considerable.

As a guide to the calculation of pier moments, etc., in equal spans, equally loaded uniformly, the following table is given, dealing with beams continuous up to five spans, each span being of length $= l$, and the weight on each span being $W = wl$.

* General Wray, *Applications of Theory to the Practice of Construction*, 4th edition, p. 107.

Table II.

W = distributed load on *one* span. l = length of *one* span.

No. of Spans	Reactions at Supports (abutments or piers)						Moments at Supports						Shearing Stresses at Supports on either side.					
Supports.	1st.	2nd.	3rd	4th	5th	6th.	1st	2nd	3rd	4th.	5th.	6th	1st.	2nd.	3rd	4th.	5th	6th.
2	$\frac{3}{8}W$	$\frac{10}{8}W$	$\frac{3}{8}W$				0	$\frac{Wl}{8}$	0				0 | $\frac{3}{8}W$	$\frac{5}{8}$ | $\frac{5}{8}W$	$\frac{3}{8}W$ | 0			
3	$\frac{4}{10}W$	$\frac{11}{10}W$	$\frac{11}{10}W$	$\frac{4}{10}W$			0	$\frac{Wl}{10}$	$\frac{Wl}{10}$	0			0 | $\frac{4}{10}W$	$\frac{6}{10}$ | $\frac{5}{10}W$	$\frac{5}{10}$ | $\frac{6}{10}W$	$\frac{4}{10}W$ | 0		
4	$\frac{11}{28}W$	$\frac{32}{28}W$	$\frac{26}{28}W$	$\frac{32}{28}W$	$\frac{11}{28}W$		0	$\frac{3Wl}{28}$	$\frac{Wl}{14}$	$\frac{3Wl}{28}$	0		0 | $\frac{11}{28}W$	$\frac{17}{28}$ | $\frac{15}{28}W$	$\frac{13}{28}$ | $\frac{13}{28}W$	$\frac{15}{28}$ | $\frac{17}{28}W$	$\frac{11}{28}W$ | 0	
5	$\frac{15}{38}W$	$\frac{43}{38}W$	$\frac{37}{38}W$	$\frac{37}{38}W$	$\frac{43}{38}W$	$\frac{15}{38}W$	0	$\frac{4Wl}{38}$	$\frac{3Wl}{38}$	$\frac{3Wl}{38}$	$\frac{4Wl}{38}$	0	0 | $\frac{15}{38}W$	$\frac{23}{38}$ | $\frac{20}{38}W$	$\frac{18}{38}$ | $\frac{19}{38}W$	$\frac{19}{38}$ | $\frac{18}{38}W$	$\frac{20}{38}$ | $\frac{23}{38}W$	$\frac{15}{38}W$ | 0

In general, it may be said that the moments at the supports next to the ends are always the greatest, and are there about $= \frac{Wl}{10}$, that they are least at the third supports from the ends, where they are about $\frac{Wl}{13}$, and that near the centre they are nearly uniform at about $\frac{Wl}{12}$.

APPENDIX I.

Analytical solution of M_f and deflection in a fixed beam uniformly loaded.

As an example, we give here the process of finding analytically the values of the bending moments and deflection in a beam of uniform and symmetrical section fixed at both ends and uniformly loaded.*

Let AB (*Fig.* 40) represent the beam; w = unit of load, l = length, P = force fixing the ends. Let M_f = Moment of Flexure at any point distant x from A.

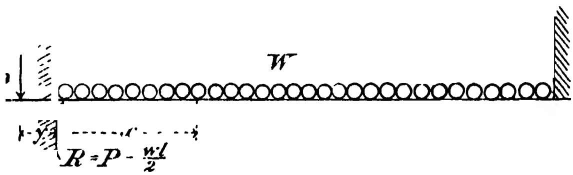


Fig. 40.

Let $$M_{fA} = Py = M_f \text{ at A}.$$
Then at any point x

$$M_{fx} = +P(y+x) + wx\frac{x}{2} - Rx$$

$$= P(y+x) + \frac{wx^2}{2} - Px - \frac{wlx}{2} = M_f + \frac{wx^2}{2} - \frac{wlx}{2} \quad ..(1).$$

But $$M_{fx} = \frac{d^2v}{dx^2} EI.$$

Hence, integrating between limits 0 and x—

$$EI\frac{dv}{dx} = M_{fA}x + \frac{wx^3}{6} - \frac{wlx^2}{4} + C \quad \ldots \quad ..(2).$$

When $x = 0$, *i.e.*, at the point of fixing, the tangent to the deflection curve is, from the nature of the fixing, horizontal; therefore $\frac{dv}{dx} = 0$. Hence $C = 0$.

* General Wray's *Applications of Theory to Practice of Construction*, 4th Edition, p. 83.

When $x=\frac{l}{2}$, *i.e.*, in the centre of the beam, $\frac{dv}{dx}=0$, or v is a maximum. This is evident from the symmetry of the load.

In (2), by substituting $\frac{l}{2}$ for x, and making $\frac{dv}{dx}$ and $C=0$ –

$$M_{fA}\frac{l}{2}+\frac{wl^3}{48}-\frac{wl^3}{16}=0, \text{ or } M_{fA}=\frac{wl^2}{12}.$$

Substituting this value of M_{fA} in (1) –

$$M_f=\frac{wl^2}{12}+\frac{wx^2}{2}-\frac{wlx}{2} \qquad .(3).$$

When $x=\frac{l}{2}$, M_f has its maximum negative value, *i.e.*, at the centre of the beam

$$M_f=\frac{wl^2}{12}+\frac{wl^2}{8}-\frac{wl^2}{4}=-\frac{wl^2}{24}=-\frac{Wl}{24} \qquad ...(4).$$

At the fixed ends, $x=0$.

$$M_f=\frac{wl^2}{12}, \text{ or } M_{ff}=\frac{Wl}{12} \qquad ..(5).$$

When $M_f=0$ we have the points of contra-flexure, *i.e.* —

$$\frac{wl^2}{12}+\frac{wx^2}{2}-\frac{wlx}{2}=0.$$

Whence $$x=(\tfrac{1}{2}-\sqrt{\tfrac{1}{12}})\,l=\cdot 211\,l,$$

or the points of contra-flexure are ·211l from the point of fixing.

From (1) and (3)—

$$M_f=EI\frac{d^2v}{dx^2}=\frac{wl^2}{12}+\frac{wx^2}{2}-\frac{wlx}{2}.$$

Integrating between the limits 0 and x –

$$\frac{dv}{dx}=\frac{w}{EI}\left(\frac{l^2x}{12}+\frac{x^3}{6}-\frac{lx^2}{4}\right), \text{ and } v=\frac{w}{EI}\left(\frac{l^2x^2}{24}+\frac{x^4}{24}-\frac{lx^3}{12}\right).$$

Putting $x=\frac{l}{2}$, to find the maximum deflection at the centre—

$$v=\frac{w}{EI}\left(\frac{l^4}{24\times 4}+\frac{l^4}{24\times 16}-\frac{l^4}{12\times 8}\right)=\frac{Wl^3}{EI}\times\frac{1}{384} \qquad .... (6).$$

From (4) we see that the M_f at the centre is $-\frac{Wl}{24}$, from (5) we see that the M_{ff} (at the point of fixing) is $\frac{Wl}{12}$, and from (6) we see that the maximum deflection $=\frac{1}{384}\frac{Wl^3}{EI}$, as in Table I.

APPENDIX II.

Relation between curves of load, shearing stress, M_f, slopes, and elastic curve

As the relation between the M_f curve and the actual elastic curve produced by the deflection of a loaded beam has been alluded to in the text, it is perhaps fitting that this matter should here be more fully investigated, as it is not only interesting, but of practical value.

The curves represented by (1) the load on a beam; (2), the shearing stress produced by that load; (3), the Moments of Flexure or Resistance; (4), the slopes or angles formed with the beam line by the deflected beam at each point; and (5) the actual elastic curve of the beam, are related to each other in regular sequence. Each is derived from the one before it by a process of integration.

This subject has been pointed out by several writers. The application of it in a graphic form is the subject of an article in the *R.E. Journal* for August, 1897, by Major H. D. Love, R.E., from which extracts are given below.

In Chapter VI., Part I., p. 112, it was demonstrated that if S be the shearing stress at any point of a loaded beam, and M_f the Moment of Flexure, then

$$S = \frac{dM_f}{dx},$$

or M_f = the integral of the shearing stress. Again, it was shown on p. 127 that the fundamental equation for computing the slope and deflection of a loaded beam is

$$\frac{M_f}{EI} = \frac{d^2 v}{dx^2}.$$

By one integration of this we find the slope whose tangent is $\frac{dv}{dx}$, and by a second integration we find the value of the deflection v at distance x from the origin. One of the co-ordinate axes is a tangent to the curve at the origin.

From these facts we see that the five curves of load, shearing, bending moment, slope, and deflection form a continuous series.

To take the simplest example, viz., for a beam of uniform cross section supported at both ends and loaded in the centre.

The curve of load (*Fig.* 41) is simply one ordinate, enclosing no area, the

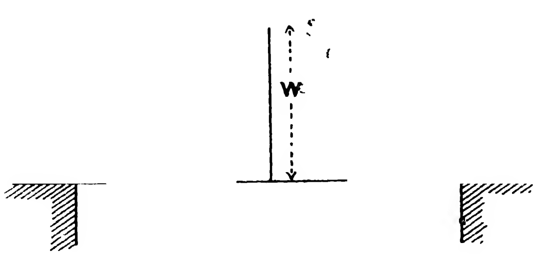


Fig. 41.

shearing stress diagram is therefore a curve with ordinates of constant value when each $= \frac{1}{2}$W (*Fig.* 42).

The M_f curve (*Fig.* 43) is derived from this curve by integration. Thus for any point a between A and C the corresponding ordinate $a'b'$ represents the area of the shaded rectangle in *Fig.* 42, and the M_{ff} C'D' = area of the rectangle AD, *i.e.*

$$\tfrac{1}{2}W \times \tfrac{1}{2}l = \frac{Wl}{4}.$$

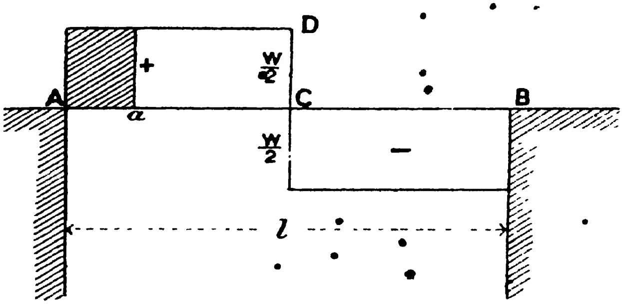


Fig. 42.

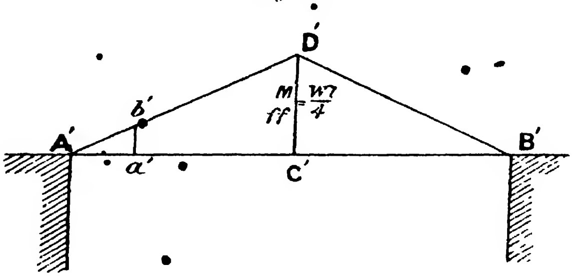


Fig. 43.

To find the curve of slopes (*Fig.* 44) from the M_f curve we take the central point of the beam as origin, since the slope is = 0 at this point.

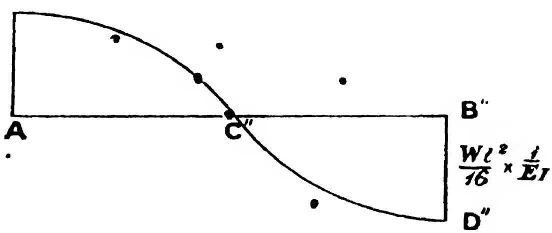


Fig. 44.

The graphic integral of the triangle B'C'D' (*Fig.* 43) is the parabola B''C''D'' (*Fig.* 44), the maximum ordinate of which is the area of the triangle B'C'D', *i.e.* —

$$\tfrac{1}{2}M_{ff} \times \tfrac{1}{2}l = \tfrac{1}{2} \cdot \frac{Wl}{4} \cdot \frac{l}{2} = \frac{Wl^2}{16},$$

multiplied by a constant which varies obviously with the material and shape of the cross section. This may be proved to be $\frac{1}{EI}$.

For the deflection curve B″ is taken as the origin, since the deflection there is $=0$, and the maximum deflection at the centre will be represented by the area of the semi-parabola, *i.e.* —

$$V = \tfrac{2}{3}\frac{Wl^2}{16} \times \frac{l}{2} \times \frac{1}{EI} = \tfrac{1}{48}\frac{Wl^3}{EI}$$

(see Table XIX., p. 129, Part I.). This curve is shown in *Fig.* 45.

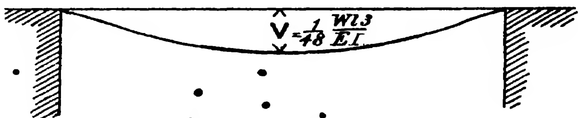


Fig. 45.

If the section of the beam is not uniform, the ordinates of the M_f curve must be reduced in the proportion $I:I_0$, where I_0 is the Moment of Inertia at the datum section.

EXAMPLE 2.—*Find the central deflection of a uniform beam loaded symmetrically with the equal loads* (*see Case* 8, *p.* 72, *and Example* 11, *p.* 105, *Part I.*).

Fig. 46 is M_f curve, and *Fig.* 47 the curve of slopes, for half the beam.

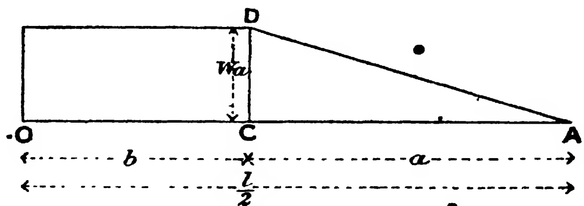


Fig. 46.

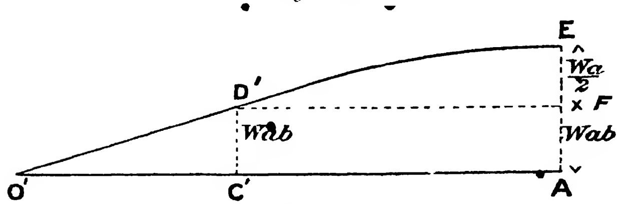


Fig. 47.

The triangle O′C′D′ (*Fig.* 47) is the graphic integral of the rectangular portion OCD (*Fig.* 46) of the M_f curve, and the parabola D′EF that of the

triangular portion in *Fig.* 46. The whole figure A′O′D′E (*Fig.* 47) is the graphic integral of the M_f curve. The area of *Fig.* 47

$$=\tfrac{2}{3}\frac{Wa^2}{2}\times a+Wa^2b+Wab\,.\,\frac{b}{2}=\frac{Wa}{6}\,(2a^2+6ab+3b^2).$$

Substituting $\frac{1}{2}l-a$ for b—

$$V=\frac{Wa}{24EI}(3l^2-4a^2).$$

It may further be proved that the intersection of any two tangents to the M_f curve must lie in the vertical through the c.g. of the intervening portion of the load. For instance, in *Fig.* 48 let AB represent a cantilever uniformly loaded (as on p. 64, Part I.). The diagram of moments is shown on the lower part of the figure. The tangent at d intersects the line ab vertically below the centre of gravity of the load. This property connects the curves of load and bending moments. Hence a similar property must hold good for the M_f curve and the deflection curve, *i.e.*, any two tangents to the deflection curve must intersect vertically below the c.g. of the corresponding area of the M_f curve.

This may be practically applied to such examples as the following:—

EXAMPLE 3.—*A supported beam of uniform section carries a single load at any point. Find the deflection at the load.*

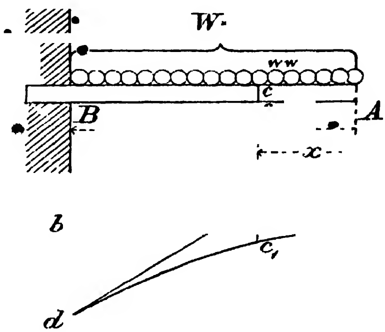


Fig. 48.

In *Fig.* 49 AOBC is the M_f curve; $AO=a$, $BO=b$; OO′ is the unknown deflection, AT′, BT″, T′T″ are the tangents to the deflection curve at A, B,

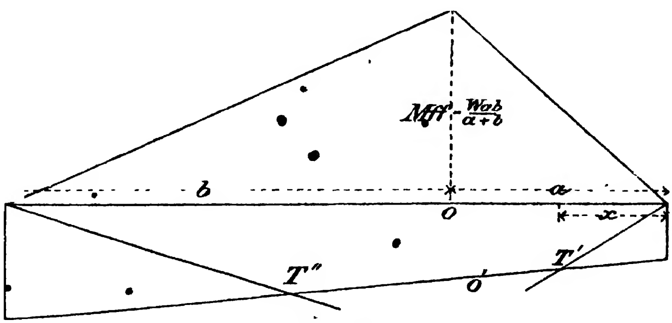


Fig. 49.

and O respectively, y_a and y_b are the distances below the supports of the tangent T′T″, and x the horizontal distance of T′ from A.

The area AOC = S measures the tangent at A, viz., $\frac{y_a}{x}$, and T′ is vertically under the c.g. of the area S.

Hence $$y_a = \frac{S x}{EI} = \tfrac{1}{2} M_{ff} \times a \times \tfrac{2}{3} a \times \frac{1}{EI} = \frac{M_{ff} a^2}{3EI}.$$

Similarly $$y_b = \frac{M_{ff} b^2}{3EI}.$$

Hence

$$V = OO' = \frac{a y_a + b y_b}{a + b} = \frac{1}{3EI\,(a+b)}\{M_f a b^2 + M_f a^2 b\} = \frac{M_f a b}{3EI} = \frac{W a^2 b^2}{3EI\,(a+b)}.$$

Example 4.—*A uniform beam is fixed at one end and supported at the other. Find the* M_f *at the fixed end—(1), for central load; and (2) for uniform load.*

(1). Central load (*Fig.* 50). Let ABC be the M_f curve for the beam if supported only, ABD the curve for fixation. Then the actual M_f curve is the shaded area, which we may call S. Let the distance of its c.g. from B be $\bar{x}$.

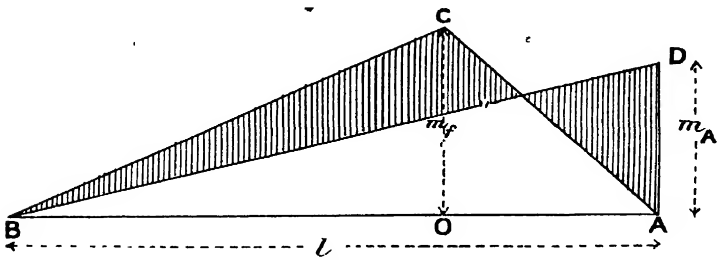


Fig. 50.

Then $$S\bar{x} = \tfrac{1}{2} M_f l \times \tfrac{l}{2} - \tfrac{1}{2} M_A l \times \tfrac{2}{3} l.$$

But the tangent at A, being horizontal, must intersect the tangent at B in the point B; $\therefore \bar{x} = 0$.

$$\therefore \qquad M_A = \tfrac{3}{4} M_f = \tfrac{3}{4} \times \frac{Wl}{4} = \tfrac{3}{16} Wl.$$

(2). Uniform load (*Fig.* 51).

$$S\bar{x} = \tfrac{2}{3} M_f l \cdot \frac{l}{2} - \tfrac{1}{2} M_A l \cdot \tfrac{2}{3} l = 0.$$

$$\therefore \qquad M_A = M_f = \frac{Wl}{8}.$$

The Theorem of Three Moments may be derived from the above property of the intersection of the tangents.*

* Cotterill's *Applied Mechanics*, p. 336. Notation altered to suit foregoing.

Let ACO′, BDO′ (*Fig.* 52) be the M_f curves due to the load on two spans AO′, BO′ of a beam AOB, continuous over three supports A, O, B, and let the centre support be below the other two by the small quantity V.

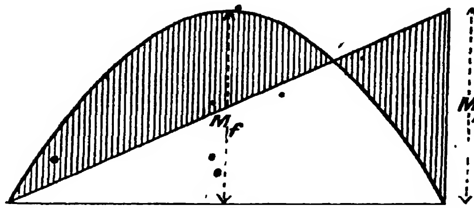


Fig. 51.

The bending moments at A, O, and B are represented by AE, O′L and BF.

The curves ACO′ and BDO′ representing the M_f curves for each span when supported only, the actual M_f diagram will be, as before, represented by the intercept between the line ELF and those curves.

The curve AOB is the deflection curve, AT, BT′ are the tangents at A and B, and TOT′ is the tangent at O, intersecting AT, BT in the points TT′.

Let a represent the area of curve ACO′, and l_1 be the span AO′; similarly, let b represent the area O′DB and l_2 the length O′B. If the load on these lengths be w per unit of length, the curves are parabolas and the area

$$a = \tfrac{2}{3} l_1 \times \frac{wl_1^2}{8} = \frac{wl_1^3}{12}, \text{ and } b = \frac{wl_2^3}{12}.$$

Now let i_A be the angle between the tangents at O and A, and S the difference of the two areas, the curve ACO′ and the trapezium EO′, then, as before—

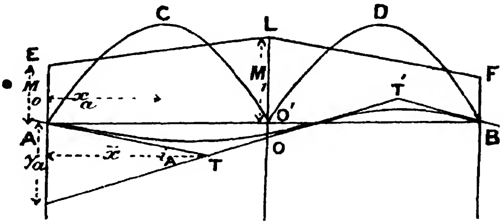


Fig. 52.

$$i_A = \frac{S}{EI}$$

But $\quad S = a - \dfrac{M_0 + M_1}{2} l_1.$

Let the horizontal distance of the c.g. of S from A be $\bar{x}$, then, as before, the horizontal distance of T from A is also $\bar{x}$. Hence if y_a be the vertical distance of the tangent from A, as before—

$$y_a = \frac{S\bar{x}}{EI}.$$

To find $\bar{x}$, let x_a be the horizontal distance of the centre of gravity of ACO′ from A, then

$$S\bar{x} = ax_a - M_0 l_1 \times \frac{l_1}{2} - \frac{M_1 - M_0}{2} l_1 \cdot \tfrac{2}{3} l_1 = ax_a - \tfrac{1}{6} M_0 l_1^2 - \tfrac{1}{3} M_1 l_1^2,$$

whence we find the value of y_a, and y_b, and the corresponding distance of B from the same tangent, is written by a change of letters.

If V be the central deflection—

$$\frac{y_a - V}{l_1} = \frac{-y_b + V}{l_2},$$

or

$$\frac{y_a}{l_1} + \frac{y_b}{l_2} = +V\left(\frac{1}{l_1} + \frac{1}{l_2}\right);$$

hence, dividing the values of y_a and y_b by l_1 and l_2 respectively, and adding, we have, when $V=0$, *i.e.*, when all supports are level—

$$a\frac{x_a}{l_1} + b\frac{x_b}{l_2} - \tfrac{1}{3}M_1(l_1 + l_2) - \tfrac{1}{6}M_0 l_1 - \tfrac{1}{6}M_2 l_2 = 0,$$

which is the general form of the Equation of Three Moments, and from it either of the forms on p. 24 may be derived.

For instance, if the load be uniform, and

$$a = \frac{wl_1^3}{12}, \quad b = \frac{wl_2^3}{12}, \quad x_a = \tfrac{1}{2}l_1, \text{ and } x_b = \tfrac{1}{2}l_2$$

we have

$$\frac{wl_1^3}{12} \times \tfrac{1}{2} + \frac{wl_2^3}{12} \times \tfrac{1}{2} - \frac{M_1}{3}(l_1 + l_2) - \tfrac{1}{6}M_0 l_1 - \tfrac{1}{6}M_2 l_2 = 0,$$

i.e.—

$$M_0 l_1 + 2M_1(l_1 + l_2) + M_2 l_2 + \tfrac{1}{4}(wl_2^3 + wl_1^3) = 0,$$

as on p. 24, Equation (ii.).

CHAPTER II.

EFFECT OF ROLLING LOADS ON BEAMS.

Conventional Methods of Considering Loads.—Dynamic Effect.—Unit Loads in English Railways.—Static Effects in Various Cases.—Examples.—Lateral Effect.—Centrifugal Forces.—Wind Pressure.—Impact in Rolling Loads.—Appendix.

BEFORE considering the details of the design of bridges, it is first necessary to devote some attention to the effect of rolling loads,* and of live loads generally. EFFECT OF ROLLING LOADS.

In a highway bridge the greatest loads are produced either by a dense crowd of human beings over the whole bridge, taken at a live load of 70 to 80, or dead load of 140 to 160 lbs. (many authorities consider that 80 lbs. is quite sufficient, especially in country districts), per square foot, distributed all over, or by a steam roller or traction engine. The weights of the latter are usually, in England, 9 tons on the rear and $3\frac{1}{2}$ tons on the forward axle, the axles being $10\frac{1}{2}$ feet apart. This weight has, however, been recently exceeded, and it is less than the weights allowed in America (15 tons total; 6 tons on forward and 9 tons on rear axle, 11 feet apart). Loads on highway bridges.

If a traction engine were crossing a bridge, other weights, such as the few foot passengers that might be on the bridge at the time, would not add appreciably to the load. Traction engines.

In railways it is common to consider the distributed load, which would be equivalent to the most unfavourable position of the actual wheel loads of the heaviest engines and trains passing over the bridge. No doubt the rolling load is distributed on the main girders by means of the rails, sleepers, cross girders, etc. Railways—usual method of considering load.

Another method is to consider the position of the wheel loads of two of the heaviest engines in use, which produce the maximum stress in each particular member of the structure. This method has

* This chapter is based partly on information given in Anglin's *Design of Structures*, partly on *The Theory and Practice of Framed Structures*, by Johnson, Turneaure and Bryan.

the disadvantage that, although exact for any particular engines, it would not apply to others, and there are always several classes in use on any line.

Another method adopted in America consists in considering the train load uniformly distributed over the structure, with an excess allowed for a certain length at the head of the train for the engine and tender.

Dynamic Effect.

Variation caused by dynamic effect.

Apart from the fact that the permanent way tends to spread the weight uniformly, it must be remembered that the distribution of load which obtains when an engine is at rest does not continue when it is running at high speed. Any irregularities in the permanent way cause variations. The action of the mechanism, the oscillation caused by the plunging and rapidly-moving mass, the difference of weights according to whether the engine is running in backward or forward gear, all tend to produce varying pressures at different axles, and to render nugatory calculations based upon the distribution of weights when the engine is stationary.

In Chapter VII., Part I., the method of calculating the maxima stresses in the *flanges* of bridges from the consideration of the observed deflection under passing loads is mentioned. This, however, is only approximately applicable, for not only do the web members modify the result, but the reiterated incidence of the wheels upon the cross girders of a bridge brings excesses on the track at regular intervals which may correspond to the periods of vibration just in the same way as troops marching in step across a field bridge cause both increased vertical and lateral oscillation.*

Besides, it must be borne in mind that some of the web members in a bridge may be struts. These may be just on the point of failure by lateral flexure, or may have actually failed by combined bending and crushing, and yet no indication of this is seen in the deflection as a whole. Hence deflection is not, *per se*, a reliable indication of the strength of the whole structure.

The deflectic of railway bridges under live loads are about 20 per cent. greater than with the same load standing on the bridge. The amount depends, of course, on the speed and on the smoothness of the road, for it will be readily understood that a badly-laid track causes impact and sudden blows, which are far

* See Appendix to this chapter.

more dangerous to the structure than any other form of load. Again, as energy = mass × square of velocity, the speed of passing loads has a very marked effect on the stresses produced.

Equivalent uniform loads

It is possible, practically, to work out bridge stresses on the basis of equivalent loads producing uniform distribution, without producing excessive errors. It will be evident, however, that the unit for any given class of rolling stock must be greater in the case of a short span than in the case of a long span. For instance, an express engine on the Midland Railway weighs 78 tons, and is 52′ 3″ long (*Fig.* 53). On a 50′ bridge this weight, equally distributed, would give 1·55 tons per foot run. On a 10′ bridge, however, the greatest weight that could be brought on it would be when the driving and trailing wheels, which are 10′ apart, are over the span. The weights on these wheels amount to 29 tons, thus producing, if uniformly distributed, a stress of 3 tons per foot.

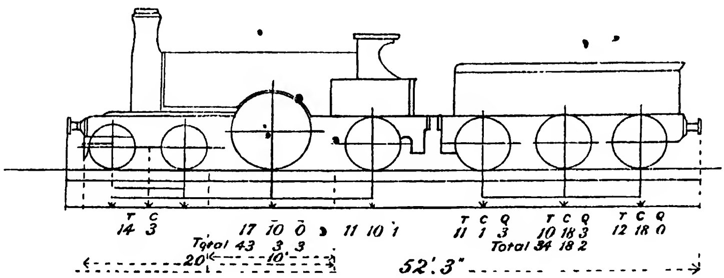


Fig. 53.

Unit load for English railways.

Sir B. Baker gives the following unit live loads for various spans in English railways :—

For a 10′ span	...	3	tons per foot run.
,, 20′ ,,	...	2·4	,, ,, ,, ,,
,, 30′ ,,	...	2·10	,, ,, ,, ,,
,, 60′ ,,	...	1·50	,, ,, ,, ,,
,, 100′ ,,	...	1·375	,, ,, ,, ,,
,, 150′ ,,	...	1·25	,, ,, ,, ,,
,, 200′ ,,	...	1·125	,, ,, ,, ,,
,, 300 ,,	...	1·000	,, ,, ,, ,,

These are in accord with English practice generally. On the Indian broad gauge (5′ 6″) lines these weights would have to be

slightly increased, and on the mètre gauge sensibly diminished. The heaviest types of engines now used abroad weigh as much as 100 tons, with a total wheel base of 54¼ feet. The greatest load on a 10′ span would be 51 tons.

Although the uniform rolling load may be taken, as just shown, for the main girders of a bridge, it would not be correct to use the same quantity for determining the stresses in the cross girders, longitudinals, etc. Each of these must be considered separately with reference to the weights of the heaviest engines that may come upon them. Mr. Claxton Fidler gives the following rule for the spacing of cross girders:—Let b be the distance apart in feet, Q = weight on driving axle, and q = greatest intensity of engine load per foot of wheel base. Then $b = \frac{Q}{q}$. "Thus, if the engine wheels were spaced at uniform distances of 6 feet, with a load of 15 tons on each axle, the cross girders might be spaced at 6 feet centres, and their maximum load would then be 15 tons, an amount which evidently could not be reduced by any closer spacing, and would be only slightly increased if a wider spacing were adopted. For constructive reasons a wider spacing would generally be still more economical."*

Static Effect in Various Cases.

Static effect. It will be well now to examine the static transverse moments on simple beams caused by loads passing across. The notation here adopted is M_f = bending moment produced at any section by the load in any position. M_{ff} = maximum bending moment produced *at a given section* during the passage of the load. M_{fff} = the maximum bending moment produced in the beam by the passing load.

Case 1. Single load. CASE 1.—*Beam supported at both ends, with a single load rolling across* (Fig. 54).

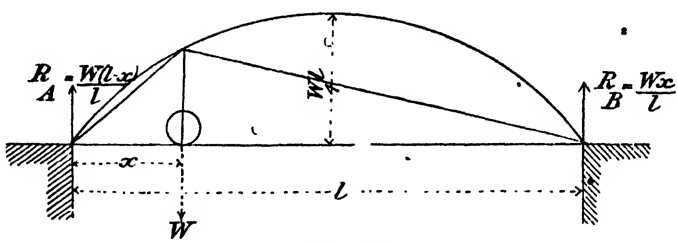


Fig. 54.

* *Treatise on Bridge Construction*, p. 268.

The reactions are

$$R_A = -\frac{W(l-x)}{l},$$

and
$$R_B = -\frac{Wx}{l}.$$

For any position of the load distant x from A the M_{ff} will be directly under it and $= \frac{W}{l}(lx - x^2)$, which is the equation to a parabola.

At the centre, where $x = \frac{l}{2}$, $M_{fff} = \frac{Wl}{4}$

The curve of moments will, therefore, be represented by a parabola passing through the points of support, and the central ordinate $= \frac{Wl}{4}$.

As regards *shearing*, when the load is at either abutment, the reaction, and, therefore, the positive shearing stress at that abutment, will be $= W$, and at the other $= 0$. When the load is at the centre, the reaction and both positive and negative shearing forces $= \frac{1}{2}W$. Hence the diagram of shearing stress will be as shown on *Fig.* 55, the ordinate at any point from the beam line, above or below, representing the maximum shear at that point, positive or negative.

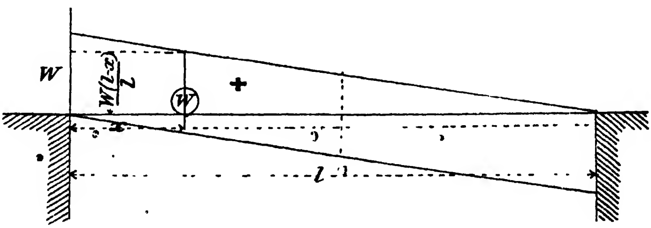


Fig. 55.

CASE 2.—*Beam supported both ends, load uniform, but of length less than the span.* Case 2. Load uniform of length less than span.

Let a (*Fig.* 56) = length of load, b = distance of any section AA′ from one abutment, x = distance of the c.g. of the load from AA′ and w = unit of weight of the load.

Then at A_1

$$M_f = -R_A b + w\left(\frac{a}{2} - x\right)\frac{1}{2}\left(\frac{a}{2} - x\right)$$

$$= -\frac{wab}{l}(l - b - x) + \frac{w}{2}\left(\frac{a}{2} - x\right)^2$$

$$= -\frac{wab}{l}(l - b) + \frac{wa^2}{8} - \frac{w}{2}\left(ax - \frac{2abx}{l} - x^2\right) \qquad .(\alpha).$$

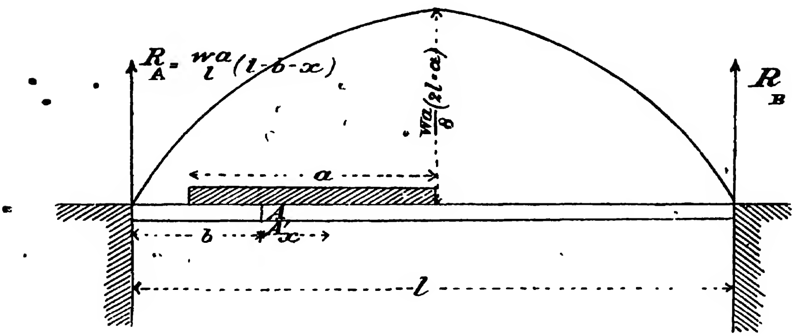


Fig. 56.

Differentiate for a maximum—

$$\frac{dM_f}{dx} = a - \frac{2ab}{l} - 2x = 0\ ;$$

$$x = \frac{1}{2}\left(a - \frac{2ab}{l}\right) = \frac{a}{2l}(l - 2b). \qquad .(\beta).$$

This gives the distance of the centre of the load from the given section when M_f becomes M_{ff}. Substituting this value of x in (α)—

$$M_{ff} = \frac{wab}{2l^2}(l - b)(2l - a) \ldots \qquad .(\gamma).$$

If $$b = \frac{l}{2},$$

$$M_{fff} = \frac{wa}{8}(2l - a) \ \ldots\ldots\ldots\ \ldots\ldots\ldots\ldots(\delta).$$

When $a = l$, *i.e.*, when the load covers the old beam, $M_{fff} = \frac{wl^2}{8}$,

which is the familiar expression for the M_{ff} in a beam supported at both ends and uniformly loaded (Case 9, Chapter IV., Part I.).

Equations (γ) and (δ) may be represented graphically by drawing an ordinate from the centre, of length $=$ (δ), and describing a parabola through it and the points of support (*Fig.* 56).

As regards shearing stress for a moving uniform load, the maximum positive shear at any point N (*Fig.* 57) occurs when all possible loads are added to the right, and when there are no loads to the left; for adding a load to the right increases the left reaction, and, Shearing stress.

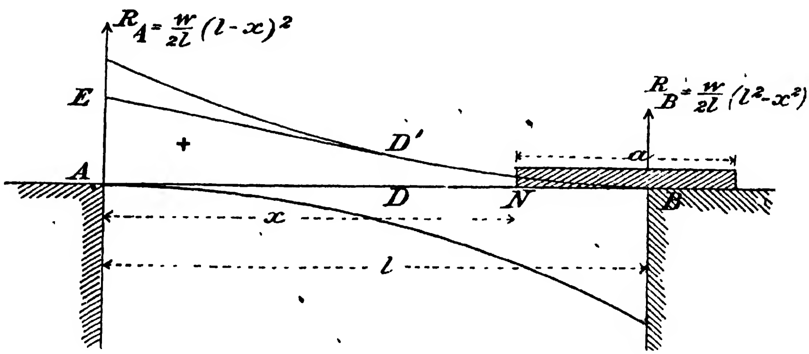


Fig. 57.

therefore, the positive shear. On the other hand, when the load has passed to the left, still covering the right portion, the increment to the left causes a negative shear to be set up. Hence the maximum shear at any point is when the head of the load has advanced to that point. The value of the positive shear at N

$$= R_A - w\,(l-x)\,\frac{(l-x)}{2l} = \frac{w}{2l}\,(l-x)^2.$$

When the whole load has just come on the beam $l - x = a$, and the shearing stress $= \frac{\quad}{2l}$ This is the equation to a parabola with vertex at right end, and maximum ordinate $= \frac{wa^2}{2l}$ at a distance a from the vertex. Beyond that distance, *i.e.*, for any point at any position x_1, measured from A, the shearing stress

$$= \frac{wa}{l}\left(l - x_1 - \frac{a}{2}\right) = wa - \frac{wax_1}{l} - \frac{wa^2}{2l},$$

i.e., the curve becomes a straight line tangential to the parabola (*Fig.* 57). If x_1 be measured from the point B, the positive shear

$$=\frac{wax_1}{l}-\frac{wa^2}{2l}$$

When the load has advanced over the whole beam, the positive shearing stresses produced by the load in its passage are represented by a parabolic curve, found by laying off at each abutment an ordinate $=\frac{wl}{2}$. Where the length of the load a is less than l, set off $BD=a$, and draw DD′ to the curve. From D′ draw D′E tangential to the curve. ED′B gives the diagram of positive shearing stress, and a similar inverted diagram gives the negative shearing stress (*Fig.* 57).

Combined moving and dead load. Shearing stress.

Where the beam is subjected to both a dead and a live load, the one uniform throughout (as in the case of the dead weight of a bridge), the other moving from one side to the other (as in the case of passing trains), the maximum positive and negative shears are found by combining the shears due to each system of loading. In *Fig.* 58 EF represents the shears due to the dead load (Case 10, p. 114, Part I.), and COB, AO′D represent the shears due to the rolling load. The maximum positive shears are found graphically by adding to the ordinates of EF those of CB, giving the curve C′F. This curve crosses the axis at G, the dead load negative shears to the right of this point being greater than the live load positive shears. Similarly for the negative shears. Between H and G both kinds of shear are possible. The dotted line C′D′ represents the shear that takes place when the whole rolling load is stationary on the bridge. It will be observed that the ordinates from AB to C′O and D′O′ do not differ much from those to the dotted line C′D′ except for a short distance in the centre. In *Fig.* 58 the live load has been drawn of greater magnitude than the dead load. The difference between the ordinates to the curve C′O and to the dotted line would be still less if the live load were diminished. Hence we conclude that in a bridge of considerable span, such as that constructed for the passage of troops, where the dead load is relatively great, we may usually calculate the shearing stress on the assumption that the whole bridge is covered with a uniform load.

As illustrating the foregoing we may take the following example:—

EXAMPLE 5. *An engine 20′ long, with a weight of 3 tons per foot run, comes on a 50′ bridge. Find* (a) *the $M_{\rm ff}$ at 15′ from one abutment*

and the position of the load; (b), *the M_{ff} at the centre and the position of the load;* and (c) *the shearing stress at* 15′ *from the centre.*

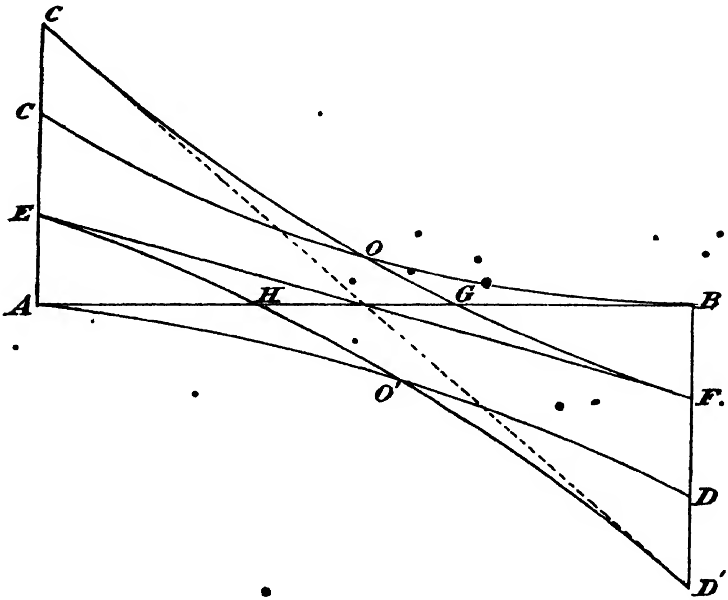


Fig. 58.

(a). The M_{ff} at 15′ is found from equation (γ). Substituting values—

$$M_{ff} = \frac{3 \text{ tons} \times 20' \times 15'}{2 \times (50')^2} (50' - 15')(100' - 20')$$

$$= 504 \text{ foot-tons.}$$

The position of the centre of the load is found from (β).

$$x = \frac{20}{2 \times 50} \{50 - (2 \times 15)\} = 4,$$

i.e., the distance of the centre of the load from the left abutment, when the M_f at 15′ is a maximum, is 15′ + 4′ = 19 feet.

(b). At the centre, from (δ)—

$$M_{fff} = \frac{3 \text{ tons} \times 20'}{8} \{(2 \times 50) - 20\} = 600 \text{ foot-tons},$$

and

$$x = \frac{20\{50 - (2 \times 25)\}}{2 \times 50} = 0,$$

i.e., the centre of the load is directly over the centre of the beam.

(3). As 15′ from the centre is 10′ from one abutment, the maximum shearing stress will be when $l = 50$, $a = 20$, and $x = 40$, and is

$$\frac{wa^2}{2l} + \frac{wax}{l} = -\frac{3 \times 20 \times 20}{100} + \frac{3 \times 20 \times 40}{50} = 36 \text{ tons.}$$

Case 3. Two or more loads at a fixed distance apart.

Case 3. —*Beams supported, exposed to rolling load of two equal or unequal weights at a fixed distance apart. This would be the case with the wheels of a truck, or a gun and limber, etc. The* M_{ff} *will evidently occur either under one of the loads or between them, at some point, for any position.*

Let W_1, W_2 represent the loads, a the distance between them. Let *Fig.* 59 represent the loads in any position. Take a point F in

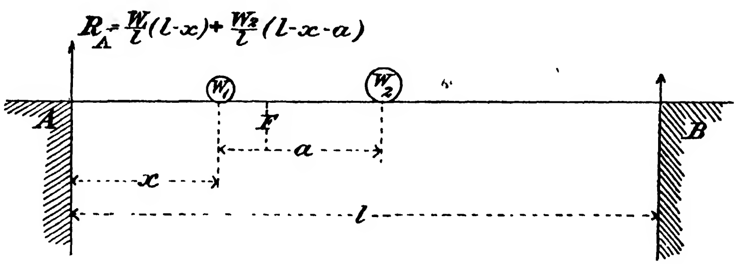


Fig. 59.

the beam such that AF : FB :: W_1 : W_2, and consider the M_f on any section between A and F. Let x = the distance from A of such a section. When W_1 comes on this section—

$$R_A = W_1\left(\frac{l-x}{l}\right) + W_2\left(\frac{l-x-a}{l}\right),$$

and at x

$$M_f = \frac{x}{l}\{(W_1 + W_2)(l - x) - W_2 a\} \quad \ldots\ldots \ldots (a).$$

Similarly for any section x_1 between F and B, measured from A, when W_2 comes on this section—

$$M_f = R'_A x_1 - W_1 a = \frac{x_1}{l}\{(W_1+W_2)(l-x_1)\} - \frac{W_1 a}{l}(l-x_1) \quad \ldots(\beta).$$

Differentiate (α) for a maximum—

$$\frac{dM_f}{dx} = \frac{W_1+W_2}{l}(l-2x) - \frac{W_2 a}{l} = 0.$$

Hence $$x = \frac{l}{2} - \frac{W_2 a}{2(W_1+W_2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(\gamma).$$

Similarly differentiating (β) for a maximum—

$$\frac{dM_f}{dx} = \frac{W_1+W_2}{l}(l-2x_1) + \frac{W_1 a}{l} = 0,$$

and $$x_1 = \frac{l}{2} + \frac{W_1 a}{2(W_1+W_2)} \quad \ldots\ldots\ldots\ldots\ldots(\delta).$$

Hence the distances of the positions of maxima bending moments may be measured from the centre, and are, for the left portion, $\frac{W_2 a}{2(W_1+W_2)}$, and for the right portion $\frac{W_1 a}{2(W_1+W_2)}$. But the expressions $\frac{W_2 a}{W_1+W_2}$ and $\frac{W_1 a}{W_1+W_2}$ represent the distances of W_1 and W^2 respectively from their common centre of gravity. Hence we see that the M_{fff} will be when the point midway between the common centre of gravity and the heavier of the two loads is over the centre of the beam, and is under the heavier load.

When the loads are equal, as in the case of a railway truck—

$$x = \frac{l}{2} - \frac{Wa}{2(W+W)} = \frac{l}{2} - \frac{a}{4}.$$

Hence the position of the M_{fff} will be on either side of the centre, and at a distance $= \frac{1}{4}a$ from the centre.

Graphically, the diagram of moments may be represented by first finding the position of the maxima bending moments as in (γ) and (δ), and then drawing ordinates, the values of which are obtained by substituting in (α) and (β) the values of x and x_1 found in (γ) and (δ). In any actual case the substitution of these values presents no difficulty, although algebraically it gives a somewhat lengthy equation.

To illustrate the foregoing we may take a few examples:—

Examples of two loads at fixed distances.

EXAMPLE 6. *A bridge 30′ span is traversed by a truck with wheels 10′ apart, and loaded with 5 tons on each wheel. Find the value and position of the maximum Moment of Flexure.*

Here $$W_1 = W_2 = 5 \text{ tons.}$$

$$a = 10'.$$

Then equation (γ) becomes $x = 15 \frac{5 \times 10}{2 \times (5+5)} = 12{\cdot}5$ feet. This gives the position of M_{fff}. Substituting this value in (α)—

$$M_{fff} = \frac{12{\cdot}5}{30}\{(5+5)\ 17{\cdot}5 - (5 \times 10)\} = 52{\cdot}08 \text{ foot-tons.}$$

EXAMPLE 7. *A truck with 3 tons on one wheel, 7 tons on the other, and axles 8 feet apart, is crossing the same bridge as in the former example (span 30′). Find the value and position of the M_{fff}, and draw the curve of moments. Here*

$$W_1 = 7,\ W_2 = 3,\ a = 8,\ l = 30$$

From (γ) and (δ) the positions of the M_{ff} are

$$x = 15 - \frac{3 \times 8}{2\,(3+7)} = 13{\cdot}8 \text{ feet,}$$

and

$$x_1 = 15 + \frac{7 \times 8}{2\,(3+7)} = 17{\cdot}8 \text{ feet.}$$

The points of M_{ff} are on either side of the centre

$$15 - 13{\cdot}8 = 1{\cdot}2',$$

and

$$17{\cdot}8 - 15 = 2{\cdot}8'.$$

Substituting the values of x and x_1 in (α) and (β), in the left half of the beam—

$$M_{ff} = \frac{x}{l}\{(W_1 + W_2)(l - x) - W_2 a\}$$

$$= \frac{13{\cdot}8}{30}\{10 \times 16{\cdot}2 - (3 \times 8)\} = 63{\cdot}48 \text{ foot-tons,}$$

and in the right half

$$M_{ff} = \frac{x_1}{l}(W_1 + W_2)(l - x_1) - \frac{W_1 a}{l}(l - x_1)$$

$$= \frac{17{\cdot}8}{30}(10 \times 12{\cdot}2) - \frac{7 \times 8}{30}\ 12{\cdot}2 = 49{\cdot}3 \text{ foot-tons.}$$

The M_{fff} is, therefore, in the left half of the beam, at a distance 1·2 feet from the centre, with value = 63·48 foot-tons.

Graphically, this would be as represented in *Fig.* 60, the curves being parabolas passing through one end of the beam line and the maximum ordinate at each of the two positions of M_{ff}. In this

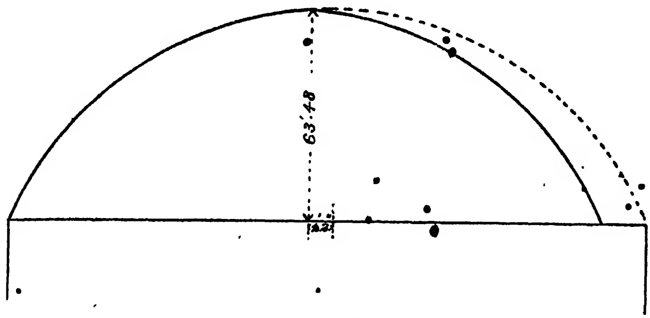


Fig. 60.

example the heavier of the two loads is in rear when the bridge is crossed from left to right, and *vice versâ*. As the truck may be reversed, the bridge must be designed to take the greatest loads coming in either direction; it would be necessary to consider the curve of moments as shown in dotted lines, *i.e.*, on the right a repetition of the left or maximum curve.

Shearing Stress.

For two loads W_1 and W_2 the maximum positive shearing stress at any point is (*Fig.* 61) when W_1 is just to the right of that point. The shear due to W_1 at x is $S_1 = \frac{W_1}{l}(l - x)$, being the reaction produced by W_1 at A. The stresses produced by W_1 are represented graphically by the line CB, where AC = W_1. Shearing stress for two loads at a fixed distance.

The shear due to W_2 when W_1 is at x is

$$S_2 = \frac{W_2}{l}\{l - (x + a)\}.$$

When $x = l - a$ this = 0, and when $x = -a$ it is = W_2. Hence by drawing AG = $-a$, and BF also = a, (AF = $l - a$), and drawing the ordinate GE = W_2, we get the line EF = the graphic representation of the shear due to W_2. The total shear due to both weights is the

sum of those produced by each singly. Then by making CI = AL, and taking a point K vertically above F, we get the positive shearing stress diagram IKB. The negative shearing stress diagram will be similar, but inverted. It would not be an *equal* diagram unless the loads were equal, but it would be found by a precisely similar process.

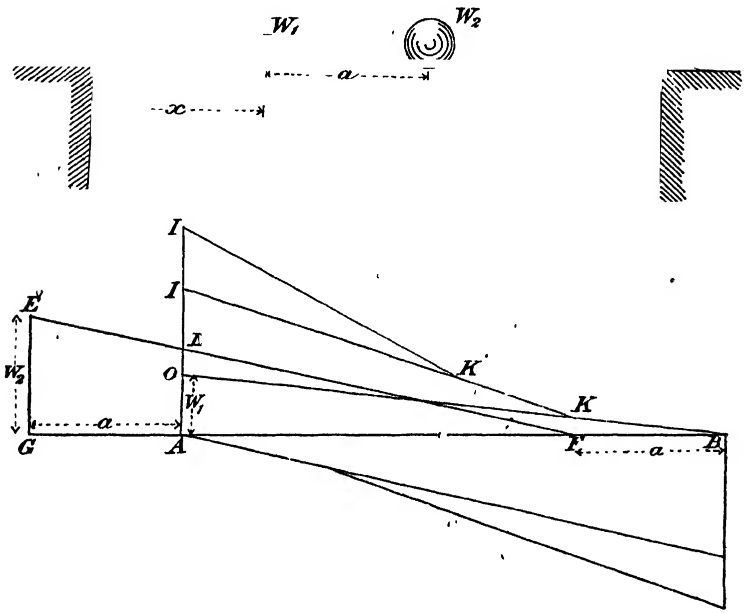


Fig. 61.

For three loads the diagram becomes I′K′B, and if the loads are equal the curve approaches a parabola in the limit when the load is uniform all over.

*Lateral Effect.**

Effect of centrifugal force.

We have still to consider the lateral pressure arising from the centrifugal force due to loads moving in a curve and from wind pressure. There are also indeterminate stresses due to oscillations in the passing load. This total lateral pressure must be resisted by horizontal bracing.

* Chiefly from *Modern Framed Structures*, by Professors Johnson, Turneaure, and Bryan.

As regards centrifugal force, the amount F of a weight P moving in a radius r is $= \frac{v^2}{gr} \times P$.

If the radius r is expressed in the degrees* D of a curve with chords of 100′, $r = \frac{5730}{D5280}$, where 5730 = radius in feet of a 1° curve, and since $g = 32{\cdot}2$ feet per second $= \frac{32{\cdot}2 \times 60 \times 60}{5280}$ 79,100 miles per hour, we have

$$F = \frac{v^2 D \times 5280}{5730 \times 79100} \times P = {\cdot}0000117 v^2 DP = kP,$$

where v is in miles per hour, D = degree of curve, P and F are in tons.

Fig. 62 represents a transverse section of a bridge. The c.g. of the load is at G, with an eccentricity e, partly caused by curvature of the track, and partly due to the horizontal displacement caused by superelevation of the rails, or one rail in a single line. The line DD′ is the centre of a cross girder or beam. AA′ and CC′ are lateral struts.

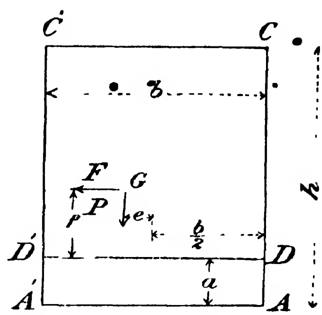


Fig. 62.

The loads P and F, caused by the dead weight and the centrifugal force, are transmitted to the cross girder, and thence to the verticals at DD′. *Fig.* 63 shows the cross girder with reactions. The horizontal reactions are each $= \frac{F}{2}$, assuming them to be equal. The portion of these forces taken by the lower laterals will be $F \times \frac{h-a}{h}$, and the portion at the upper laterals $F \times \frac{a}{h}$, one-half of each being applied at each side. The M_f at D and D′ are $= \frac{F}{2} \frac{a(h-a)}{h}$.

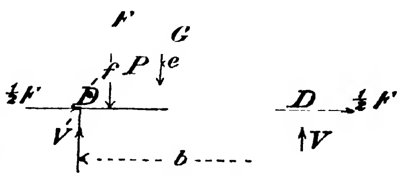


Fig. 63.

* This refers to the American practice of laying out curves, a practice, however, which is universally adopted in India.

If the moving load is taken as uniform, F and P are the same for all parts, but if wheel loads are taken, F and P vary. Since F is a constant function of P for any one problem, the maximum moments and shears in the lateral systems, due to centrifugal force, will be a constant function of the maximum moments and shears in a vertical truss due to the actual wheel loads. Hence for the lower laterals, to get the maximum moments and shears, multiply those found in the vertical truss by $k\left(\frac{h-a}{h}\right)$, and for the upper laterals multiply by $k\frac{a}{h}$. The maximum value of F = maximum vertical floor beam load × k.

Usually one of the lateral systems lies practically in the plane of the floor beams or cross girders, in which case the whole of F is carried by this one lateral system, and the verticals receive no bending moment.

The vertical main trusses supply the reactions V and V′. Taking moments about D′ we have

$$V = \frac{P\left(\frac{b}{2} - e\right) - Ff}{b} = P\left(\frac{1}{2} - \frac{e + kf}{b}\right),$$

and since the sum of the vertical components = 0—

$$V' = P\left(\frac{1}{2} + \frac{e + kf}{b}\right).$$

From these two equations we see that the inner truss receives its maximum load for a minimum value of k, *i.e.*, when the line is straight; and that the outer truss receives its maximum load for a maximum value of k.

If there are longitudinals at DD′, each will receive a lateral moment and shear equal to k times the maximum vertical moment and shear due to half the actual wheel loads. This moment is taken by the upper flange and adds to its stress, and the shear necessitates extra rivets to fasten the longitudinal to the cross girder.

Wind Pressure.

Lateral wind pressure on bridges.

The subject of wind pressure on structures has been considered in Part I., pp. 233 *et seq.*, in its general bearings, and specially in

connection with the design of roofs. It remains to consider this matter as influencing the design of bridges.*

The exposed area of the span of a bridge consists of the exposed vertical surface of the windward girder, of the floor system, and of the leeward girder (or at least so much of it as is not completely and closely sheltered by the train). In addition to this, there is the surface of a train on the bridge, reckoned at 10′ high and with its bottom 2·5′ above the rails. The wind pressure on this may be taken at 30 lbs. per square foot.

On ordinary heavy railway bridges the exposed areas per linear foot may be roughly estimated at 10 square feet for the train, 1 square foot for the ends of the sleepers and sides of the guard rails, 4 square feet for the longitudinal floor girders, and 5 square feet for each truss or main girder, or a total of 10 to 14 square feet for the dead wind load on the two trusses and floor.

In ordinary double track railway bridges, with vertical wind bracing, the *weight* of this bracing for both trusses may be roughly estimated at

$$\left(\frac{6Nl}{170}+\frac{1136}{p}\right)\frac{b}{15} \text{ lbs. per lineal foot of track,}$$

where l = length in feet, N = number of panels, p = panel length in feet, b = breadth of the bridge in feet.

As the wind may blow from either direction, some of the members of the bracing may be called upon to bear alternate tensile or compressive stress. Counter-bracing may be necessary to prevent this, as will be presently shown. Where there are members thus subject to alternate stress, they should be designed to resist either class of stress, and the factor of safety should be large, on account of the deterioration produced by reversal of stress.

Vertical lateral bracing is often introduced between the vertical posts of the main girders at panel points, in order to prevent the independent lateral vibration and swaying of vertical trusses, also to stiffen the long vertical posts, and assist in carrying some of the wind stresses to the leeward girder. Vertical wind bracing.

The modifications due to diagonal directions of the wind, *i.e.*, when blowing in directions neither horizontal, nor exactly normal to the axis of the bridge, must be considered. The *uplifting pressure* is

* The treatment of the subject here given is largely taken from a paper by Captain W. H. Bixby, U.S. Engineers, published in 1895, and also from *Minutes of Proceedings, Institution of Civil Engineers*, Vol. LXIX.

generally met by so arranging the flooring that it may be torn up from the floor girders before the upward lift of the wind becomes great enough to throw serious strain upon the main girders.

Downward pressure is of comparatively rare occurrence and little force, and need not be considered.

The greatest strains will usually come on a bridge where the wind is blowing horizontally and normal to the line of the bridge. But as a considerable deviation from the normal must take place before the pressure is sensibly diminished (see *Fig.* 212, p. 258, Part I.), a diagonal wind may deprive the leeward girders of the advantage of shelter which they would have had from the windward girders if the wind had been truly normal. Thus a diagonal wind may cause greater strains than a normal wind. Therefore, if the strain on a bridge be computed from that of a normal wind, the effective area of all vertical trusses must include at least all surfaces that may be reached by diagonal winds of about 45° angle. This of itself will usually prevent the necessity of considering the effect of any possible shelter by any verticals or diagonals in braced girders.

Sudden gusts. As regards gusts, and the sudden pressures thereby brought on the bridge, it seems reasonable to assume that these will not extend at one time over more than 600′ to 1,000′ length of any bridge, and such pressure should be treated as any other live load. As the maximum pressure of gusts during storms of maximum intensity will occur so rarely, and the strain of the metal will therefore be so slight, it seems reasonable in such cases to allow the stress to reach nearly the elastic limit of the material.

Provisions for wind pressure. Consequently it would seem that wind pressures would be amply provided for by allowing for (1) a dead load wind pressure = the average steady pressure of high winds over the entire effective area of the bridge with the train on it; (2), a live load wind pressure of 50 lbs. per square foot over from 600′ to 1,000′ in length of the bridge, and 30 lbs. per square foot on the remainder. If the shape or form which the members present to the wind action is not rectangular (for instance, round tie bars presenting a cylindrical shape) the effect of the wind pressure might be modified accordingly (see p. 236, Part I.), though as a rule this is not taken into account in practice.

Sir Benjamin Baker's Forth Bridge experiments have shown that the average pressure on a large surface is much less than the maximum pressure exerted on small portions of it. Thus the

average pressure on a surface $20' \times 15'$ was found to be not more than 66 per cent. of that on a small surface $1\frac{1}{2}$ square feet.

The Committee of the Board of Trade, which considered the subject after the destruction of the Tay Bridge, recommended :— Board of Trade rules.

1. That in exposed situations the maximum pressure to be provided shall be 56 lbs. per square foot of surface.

2. That for open lattice work the surface on which this pressure acts should be from once to twice the front area, according to the openings in the lattices.

3. That for iron or steel work a factor of safety of 4 should be provided, and considering the tendency of the bridge as a whole to be overturned, a factor of safety of 2 should be provided.

Arrangement of wind bracing. We may now consider the arrangement of the bracing. The main girders should be braced together in a horizontal direction in such a way as to transmit to the abutments the horizontal pressure. Where the roadway of the bridge is on the upper booms it is possible to construct the bridge with horizontal bracing both on the floor of the bridge and between the lower booms, as well as vertical sway bracing between the panels. But where the flooring of the bridge is on the lower flanges or booms, it is only possible to have this double system of bracing when the depth of the girder is sufficiently great to allow of headroom over the roadway and under the upper bracing—a state of affairs which in English practice is comparatively rare. If the floor of the bridge consists of wrought-iron or steel troughing, of plates resting on cross girders, or brick arches, or any similar construction, such flooring in itself fulfils all the requirements of wind bracing.

When the floor consists of timber it will be necessary to introduce diagonal bracing, and with deep girders where the floor is on the lower booms; the top booms should be connected if there is sufficient headroom.

The actual design of lateral bracing will be exactly on the principles of the Howe or Warren girders, to be described in a later chapter. There is, however, this difference, that whereas in a vertical girder the weight of the bridge, and any passing loads, producing stresses in the various members, all act vertically downwards, in a lateral or horizontal system of bracing, the wind pressure may act in either direction, and it will, therefore, be necessary to provide for the difference in stress that may thus be produced.

Thus in *Fig.* 64, which shows the plan of a bridge of six panels, when the wind is blowing in the direction of the arrows, the bars

drawn in single lines are in tension, and in double lines in compression, a condition of things which would be reversed if the wind blew from the opposite direction. To obviate the disadvantages which would be produced by designing the various members to take either class of stress, counter-bracing is resorted to, as shown in *Fig.* 65, where the short bars are always in compression, the inclined bars drawn in continuous lines are in tension, and act when the wind blows from the left, but are inoperative when the wind blows from the right, the tension being then taken by the dotted bars.

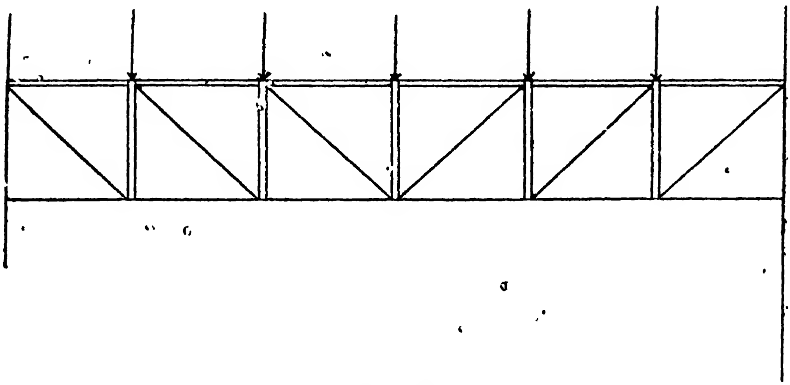
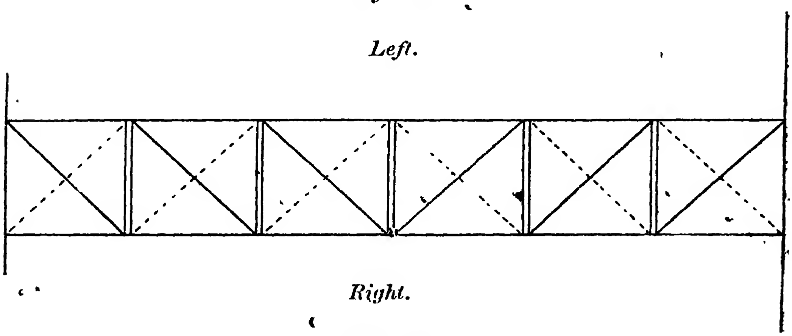


Fig. 64.

Left.



Right.

Fig. 65.

The method of calculating the stresses in each bar will be explained in Chapter V., where braced girders with parallel flanges are considered.

In some cases of timber bridges lateral stability is given by struts

outside the girders or railing, as shown on *Fig.* 66, or by a frame with sufficient headway to admit of traffic. See the drawings of the Hurroo Bridge and the Barra Bridge (*Plates* V. and XI., pp. 110 and 148).

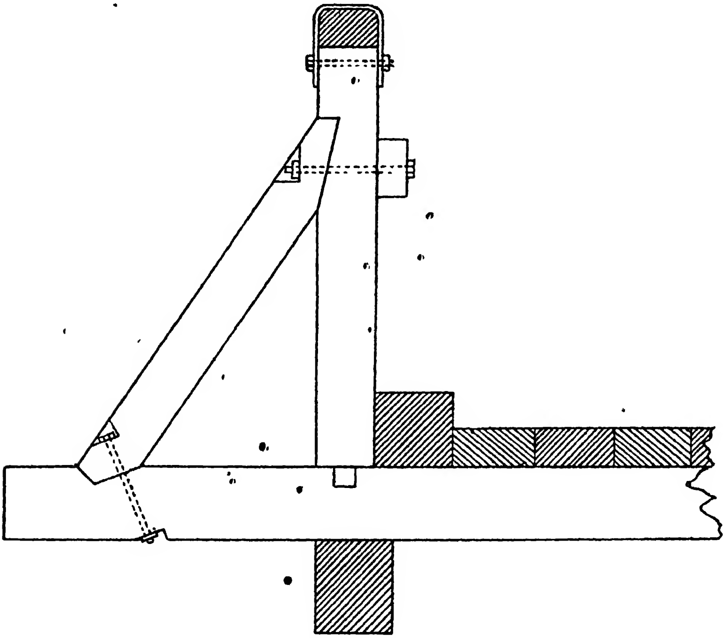


Fig. 66.

The subject of increase to the loads which should be made to compensate for the velocity of movement has been investigated by an Austrian Scientist, Professor Mélan (*Minutes of Proceedings, Institution of Civil Engineers*, Vol. CXVII., p. 411). His conclusions are that the actual rolling load should be increased by a percentage represented by the following formula :— Recent researches in Austria.

$$\text{Percentage increase} = 14 + \frac{800}{L+10}, \text{ where } L = \text{span in mètres.}$$

Thus the greater the span, the less relatively the effect of the rolling load.

This formula gives for the following spans—

Span in feet	6·5	13·12	16·4	32·8	49·2	65·6	98·4	131·2	262·4
Percentage increase	80	71·0	67·0	54·0	44·0	41·0	34·0	30·0	23·0

The state of our knowledge on this subject is far from perfect. There are many circumstances which modify the dynamic effect of rolling loads—(i.), the successive changes of form, which the load produces when it passes over a bridge, take place with a certain speed, and cause vibratory movements which in turn develop stresses; (ii.), the centre of gravity of the load in its passage describes a trajectory which is generally curved. This developes centrifugal forces, the action of which either adds or diminishes the load caused by gravity according to whether the trajectory is sagging or hogging; (iii.), the unevenness of the line, and shocks at repeated intervals (as previously pointed out) causing regular vibrations; (iv.), the differences in the vertical loads, produced by the balance weights on the wheels, or by alterations produced by other parts of the machinery in motion.

These can only be very approximately reduced to calculation. Practically, they indicate that, to ameliorate the results of the dynamic forces, the line should be as continuous as possible, the rolling stock in good order, interior cylinders should be used on locomotives, and the balance weights should be as small as possible.

APPENDIX.

Vibration of Beams from Impact.

The periodic time of vibration of beams from synchronous impacts been found to be

$$t = \cdot 0093 l^2 \sqrt{\frac{P}{EI}},$$

where t = time of vibration in seconds.

l = length of beam in feet.

P = total load in lbs. on the beam uniformly distributed.

I = Moment of Inertia of the section in *foot-units*.

E = Modulus of Elasticity in lbs. per square inch.

When the time of vibration coincides with any synchronous impact, the amplitude of the vibration rapidly increases until a comparatively small impact may by repetition produce serious deflections with their corresponding stress intensities. This is the reason why troops passing over a bridge should break step.

CHAPTER III.

PLATE GIRDERS OF IRON AND STEEL.

Method of Construction and of Calculation.—Market Sizes of Iron and Steel.—Practical Considerations of Depth, Width, etc.—Steps to be taken in Design.—Graphic Methods.—Theoretical Objections Discussed.—Example.—Various Forms of Bridge Floors.—Appendix.

PLATE GIRDERS. Principle already considered in rolled beams.

IN Part I., Chapters V., VI. and VIII., the principle of the *girder* where the flanges are designed to resist the transverse stresses, and the web to resist the shearing stress, has been fully detailed. The application of this principle, however, has been there confined to the narrow limits of rolled beams of iron or steel, and to rails. Rolled beams of the ordinary I-section are evidently not theoretically economical for *any* case, because the cross section, being constant, will give a Moment of Resistance which, if sufficiently great to resist the greatest Moment of Flexure, will be more than sufficient for the other parts of the beam where the M_f is less than the maximum. Hence at all sections other than that of greatest stress there is an excessive amount of metal in the beam. This is especially the case in supported beams, which, as compared with fixed or with continuous beams (under similar conditions of load, span, etc.), have greater differences in the value of M_f from the maximum to the minimum.

Restricted utility of rolled beams.

Thus we see that, although very convenient, *practically* economical, and for all ordinary house construction quite sufficient, rolled iron or steel beams of I-section are of very restricted utility when spans of any considerable length have to be bridged, and carry heavy loads. To resist the bending moments now brought into play, built-up girders must be employed, and the metal in these must be arranged so as to meet in the most economical manner the varying stresses brought to bear by the action of the external forces. Building up by means of plates, angle irons, etc., is expensive work, but this is compensated for by facilities which are afforded for the

economical arrangement of metal and the possibility of attaching secondary members.

As rivetting the parts is both a source of weakness and expense, it is desirable to use such forms of plates, angles, etc., as will suffice for the required work with a minimum of rivetting.

Plate girders, description. The plate girder is the first of these built-up beams that calls for examination. It consists of horizontal flange plates united to a thin vertical web plate by means of angle irons at top and bottom, rivetted to both web and flanges.

Functions of various parts. The M_r produced by the web plate is obviously small compared with that of the flanges, and is ordinarily neglected, although there is no reason why it should not be assumed to resist its due proportion of the M_f, since the web and flanges distort together as one solid beam. The web is considered as resisting the whole shearing stress. The function of the web is to transfer to the flanges through the rivets the stresses developed in the beam. The accurate calculation of the Moment of Resistance of the whole beam would be to include $\frac{1}{6}$ net area,* or $\frac{1}{8}$ gross area, of web, in addition to the area of the flange, as under direct stress.

Formula for Moment of Resistance. The flanges are generally made of the same size. The resistance of each is, therefore, considered the same, for wrought iron† or steel have practically equal resistances to tension and compression (*vide* Table XVI., p. 60, Part I.). Hence the Moment of Resistance of the cross section at any part is obtained by considering that the stresses in the two flanges constitute a couple in which the force is equal to the resistance afforded by one flange, and the arm is the distance between the centres of gravity of the two flanges. Thus if

r = the intensity of resistance in tons per square inch to tension,
a = *net* area of the flange (tension flange), having deducted the area of the rivet holes,
d = effective depth of girder at the section in question,

then $$M_r = rad.$$

In this equation d is usually taken as the total depth of the girder, though, properly speaking, it should be the distance between the centres of gravity of the flanges.

* See page 99, Part I.

† In wrought iron the intensity of resistance to tension is greater than to compression. As the rivet holes have to be deducted in tension flanges, the area of the two is made the same.

Another and still more accurate method of ascertaining the Moment of Resistance of the section is to calculate its Moment of Inertia, and apply the general formula $\frac{rI}{y}$. This method is generally followed by Continental engineers, but the approximate method is usually considered sufficient in England.

As regards the value of r, we must take into consideration the nature of the load, the span, and the ratio of live to dead load. On long span bridges the stresses due to the dead load may be very great compared with those due to any live load likely to be borne. On small span railway bridges subject to the rapid movements of heavy trains, as pointed out in the preceding chapter, the live load stresses may be far in excess of those due to the dead load. Hence in bridges of this class the range of stress coming on the metal would be very much greater than on those of the former class. As *fatigue* of the metal is produced by differences in the stress, or by repeated applications of load, a small unit stress must be allowed where the range of stress is large, and *vice versâ*. Value of r depends upon span and load as well as material.

For steel structures it is customary in England to allow a working value of 4½ tons per square inch for railway bridges of 20′ span and under, 5 tons for 30′ spans, 5½ tons for 80′ spans. No tensile stress on cross girders or rail bearers should exceed 4½ tons. For wrought iron the working stresses should be about 20 per cent. less than those for steel. Working values adopted in England.

With road bridges over railways, rivers, etc., these stresses might safely be increased to 7 tons for steel and 5 tons for wrought iron.

It is perhaps necessary to bear in mind that the steel used for construction of bridges should be that having a tensile strength of about 30 tons per square inch, and a 25 per cent. ductility in a length of 8 inches. Such a steel is about 30 per cent. stronger than wrought iron, and four times as ductile. The relation between the chemical and physical characteristics of various classes of steel is a most important and interesting matter, but is beyond the scope of this work.

Steel has also another advantage over wrought iron, in that there is no appreciable difference in the strength, in the direction of rolling, or transversely to that direction. Practically this is a matter of much importance, for in building up steel work the mechanic has not to consider the direction in which a bar or plate should be arranged.

Market Sizes of Iron.

Market sizes of iron and steel.

The next point of practical importance is to consider within what limits iron or steel plates and angles are obtainable in the market. Obviously the design should be based upon material ordinarily to be purchased, because special sizes entail a relatively higher price. Of course, sometimes there may be reasons for paying an extra price for a special size. Such instances would be exceptional, and need not be considered.

Improvements in manufacture have of recent years enabled much larger sizes of plates to be rolled than formerly were obtainable. In fact, the limit nowadays is determined not so much by the difficulty of obtaining large plates, but from the practical inconvenience of handling very thin plates of large area.

The following are limits of wrought iron published by one* firm of manufacturers :—

		W. Iron.
Area, maximum square feet	... ...	80
Length, maximum feet	.. ..	30
Width ,, ,,	... ...	6
Thickness between	... ...	$\frac{1}{16}''$ and $\frac{3}{4}''$

These may be compared with the following limits of steel plates published by the Steel Company of Scotland, and by the Dalziel Steel Company :—

		Steel.
Area, maximum square feet	... ...	250
Length ,, ,, ,,	... ...	50
Width ,, ,, ,,	... ...	10
Thickness between	... ...	$\frac{1}{16}''$ and $\frac{7}{8}''$

The maximum dimensions of area, length and width cannot all be obtained at once, because the plate has to be rolled from an ingot of a certain maximum weight. This limit of weight is ordinarily 10 cwt. in wrought iron, and from 40 to 60 cwt. in steel.

Steel plates, etc., rolled of larger size than wrought iron.

The fact that steel plates can be obtained of dimensions so much larger than wrought-iron plates is a strong argument in favour of its use, for not only is the strength greater, but the greater size of the plates enables fewer joints and rivets to be used than would be necessary under the same conditions with wrought iron.

* The Lilleshall Company.

Another firm gives the following :—

TABLE III.

Market Sizes of Steel Plates.

					$\frac{1}{2}''$	$\frac{9}{16}''$	$\frac{5}{8}''$	$\frac{3}{4}''$	$\frac{7}{8}''$	1″
Length	20′	23′	26′	29′	30′	32′	35′	35′	35′	35
Width	4′ 6″	5′	5′ 6″	5 10″	6′ 2″	6′ 6″	7′	7′	7′	7′
Weight, cwt.	7	10	13		21	26	32	40	45	50

"From this table for plates of a certain thickness, if the width of the plate is fixed, we can immediately find out from the maximum weight the maximum length, and if the length is fixed we can find the maximum width which can be obtained without extras. By payment of extra charges these lengths and widths can be increased. . . . . There is generally an extra charge for plates under 12″ wide, and some makers make an extra charge for plates $\frac{3}{8}''$ thick. By special arrangements plates of almost any required size can be obtained."*

Angle and T-irons.

Angle bars of steel under 11 united inches and over 6 united inches, and in lengths up to 50′ or 60′, and T-irons between 6 and 10 united inches, can be obtained without extra charge. "United inches" means the sum of extreme breadth and depth of the angle or T-iron.

The ends and edges of all plates should be planed, and must abut perfectly true, and the angles and T-irons should be neatly finished off at the ends. The planing of plates may be dispensed with if arrangement is made with the manufacturers to "roll" the edges. An extra price has to be paid for this, and the width of the plates so rolled is generally limited.

Ordinary market sizes should always be used in designs.

The necessity of using ordinary market sizes in design is a very important detail, and should always be borne in mind. There are a few other points of a practical nature which may also here be alluded to. The plates used should not be less than $\frac{1}{4}''$ thick, as any oxidation may seriously affect their strength. To lessen the evils

* Fitzmaurice, *Plate Girder Railway Bridges*.

of corrosion, it is advisable to add $\frac{1}{16}''$ to thin plates, or $\frac{1}{8}''$ when much exposed. In the webs of girders, a thickness less than $\frac{3}{8}''$ is seldom used.

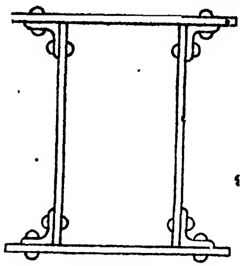


Fig. 67.

All parts of girder should be accessible.

As it is most necessary to paint ironwork periodically, the design should be arranged so that all parts are accessible. This puts box girders (*Fig.* 67) out of the category of economical designs, except those which are large enough for a man to get inside.

Practical Considerations of Depth, Width, etc., of Built-up Beams.

Depth of girders.

If we increase the depth, we see from the equation $M_r = rad$ that the flange stress ra is diminished. It will, no doubt, be advantageous, if heavy loads are expected on the bridge, to have deep girders. But in so doing the web will probably require stiffening. It must also be remembered that, where the load is carried on the upper flanges, head room below the proposed road level is frequently a matter of importance, while, on the other hand, if we carry the roadway on the lower flanges of the girder, the upper flanges of a deep girder may interfere with traffic, or else a wider bridge is necessary. Thus, in the case of a railway bridge of three girders, if the flanges of the central girder project more than 2′ 6″ above rail level. the width of the bridge will need to be greater than it would be if the depth is kept below that level. The depth is, therefore, often limited by local considerations.

Usual depth $\frac{1}{10}$ to $\frac{1}{12}$ span.

Other things being equal, it is always better to carry the roadway on the upper flanges. This construction brings the weight more directly on the girders, and saves trouble in fastening the decking or cross girders. Assuming that it is possible to adopt this form of construction, the usual depths adopted vary from $\frac{1}{10}$ to $\frac{1}{12}$ of the span for small spans (up to, say, 40′), but greater depths may be used for longer spans. Professor Johnson gives the following formula for the economic depth, where the M_r of the web is neglected :—

$$d = 1{\cdot}27\sqrt{\frac{m}{rt}},$$

where d = depth in inches.

m = central bending moment in inch-lbs. or inch-tons from dead and live loads.

t = thickness of web in inches.

r = allowable fibre stress in lbs. or tons, in conformity with m.

This gives a greater depth than is usual in English practice.

The *breadth* of the girder is governed by the considerations of fair bearing on the piers, and lateral buckling of the compression flange. It may be either taken as a proportion of the span, $\frac{1}{30}$ to $\frac{1}{40}$, or a convenient width between 12″ and 21″ may be assumed, as experience has shown that these limits are suitable for spans from 20′ to 60′. The effective length should be considered from centre to centre of bearings. **Breadth of girders.**

There should be as *few joints* as possible. These are always a source of expense, have the effect of reducing the Modulus of Elasticity, and introduce uncertainties of strength from variations in workmanship. The work generally should be as uniform as possible. **Joints should be few.**

One end of the girder should be fixed; the other should be free to expand and contract. A variation of temperature will produce in a beam which is immovable at the ends a definite stress per square inch of section without any increase to the load. For spans below 80′, planed cast-iron plates, lead, felt, or timber baulks are placed under the free end, so as to allow of easy longitudinal movement. To allow both ends to be free is dangerous, as, especially on an incline, the "creep" of the rails may work the girder off the abutments. This is particularly the case in hot climates, and instances have occurred of bridges having nearly worked off their bearings. For spans over 80′, plate girders are seldom used, but if used they should have rollers under the free end. **Expansion one end.**

Steps to be Taken in Design.

If the depth d of the girder is given (either by taking a ratio of the span or otherwise), the following steps should be taken in designing a web girder for a given span and loading (dead and live):— **Steps in design.**

(1). From the given span, weight, and depth, find the *flange stress ra* (where r = stress intensity of the metal and a = *area of the flange + angle irons*) at any point by equating the M_f at that point with *rad*. **Flanges.**

(2). Select suitable market sections of angle irons, and find their net area. This area will constitute a part of a; the remainder will be made up of plates.

(3). From the area of the plates thus ascertained, find the total *thickness of the flange* by assuming a width either $\frac{1}{40}$ span or = about 12″ to 21″.

(4). Having found the flange thickness, take market sections of

plates and settle the *number of plates* to be used. This will fix the thickness of each plate.

(5). The thickness of the plates will govern the *diameter of the rivets* (see p. 217, Chapter X., Part I.).

Joints (6). The considerations of limiting dimensions of plates will determine the *position of the joints*. Find the M_f at those joints.

(7). Find the flange stress ra at those joints by equating the M found in (5) with rad, where d is the known uniform depth, and ra is the flange stress at the joints.

(8). Find what *proportion* of that flange stress at the joints is borne by the plates and by the angle irons, respectively, by comparing their respective areas.

(9). From this flange stress find the *number of rivets* for the cover to the joint, both in shearing and bearing. If the angles require covers or "wrappers," the same procedure can be followed.

This completes the design of the flanges. Now we have to consider the web.

Web. (10). We must ascertain the *thickness of web* and the *number and size of rivets* required to fasten it to the angle irons. As the web is designed to resist the shearing stress, and as that stress is both vertical and horizontal, the number of rivets required in a given length to fasten the web to the angle irons, and the angle irons to the flanges, will be the number required to resist the horizontal shearing stress at the length in question.

Rivets. We have already ascertained the size of the rivets. The web must be thick enough to give a sufficient bearing area on the rivets. To do this we may proceed by trial and error. Assume that the web is, to begin with, the least practical dimension, say $\frac{3}{8}''$. Find the shearing stress at the supports, where it is a maximum, and divide the total shear by the depth, so as to find the intensity of shear per unit of length (generally 1 foot). In so doing we consider the shearing stress to be uniformly distributed over the whole of the cross section at the end, which, as we know, is not absolutely true; but the error is not very great, and for the purpose in view is on the safe side. That purpose is to find how many rivets per foot run are required to fasten the angle irons to the web. In practice, the usual pitch for the rivets is 4″. We may, therefore, instead of taking the web at the minimum practicable thickness, take this pitch, giving us three rivets per foot run, and find the thickness of the web required.

As the horizontal shearing stress at any point = the vertical

shearing stress, we can easily find either the number of rivets per foot run for a given thickness of web or the thickness of web for a given number of rivets per foot run.

Suppose the shearing force per foot run to be 4 tons, the web $\frac{3}{8}''$ thick, and that the steel plate can stand an effective bearing stress of 7 tons per square inch. The rivets will then have a diameter of $2 \times \frac{3}{8}'' = \frac{3}{4}''$, and if n be the number of rivets per foot run—

$$n \times \tfrac{3}{4}'' \times \tfrac{3}{8}'' \times 7 = 4 \text{ tons.}$$

$$n = 2{\cdot}03, \text{ say } 2 \text{ rivets per foot run.}$$

This would amply fulfil requirements of pitch (*vide* p. 217, Chapter X., Part I.), which are that the pitch should be at least twice the diameter of the rivet.

Suppose that we have the shearing force = 4 tons per foot per square inch, the rivets 4″ apart, and an effective bearing stress of 5 tons. To find the thickness of the web.

Assume rivets $\frac{3}{4}''$. Then there will be 3 rivets per foot.

$$3 \times \tfrac{3}{4} \times t \times 5 = 4\,;$$

$$\therefore \quad t = \frac{4}{11{\cdot}25} = \tfrac{3}{8}'' \text{ nearly.}$$

As the shearing stress is greatest at the ends of the girder, any conditions of rivetting which are sufficient there will more than suffice at other parts. It is not economical to introduce much alteration in rivets and pitch, so work is usually made uniform throughout.

(11). *Covers to the angle irons* or "wrappers," if required, may be similarly worked out from considering the direct stress in the angle irons at the place where the covers are required. This direct stress is obtained by equating the M_F at the place in question with rad, where d has constant values, and ra = stress on flanges + stress on angle irons. We thus find the value of proportion of stress on the angle iron. Covers to angle irons.

(12). To find whether *the web requires stiffeners*, we have to consider the thrust caused by the combined horizontal and vertical components of the shearing stress. The usual method of calculation is to consider these components replaced by their resultants acting at an angle of 45° with the horizontal. Let* ABCD (*Fig.* 68) be any Stiffeners to web.

* From General Wray's *Instruction in Construction*.

square particle in the web; let the arrows V, V′ represent the vertical shearing stress on opposite sides of the particle, HH′ the equal horizontal shearing forces. Replace these forces by double the number each of half the value, acting at the angles, as shown by the small arrows, and take the resultants of these forces along the diagonals of the square. The components are now P, P′ in compression, and T, T′ in tension, and the intensity of these tensions and compressions is equal to the intensity of the vertical and horizontal shearing stresses by which they are generated, for if BD = total thrust due to V and H = P, and BC = total shearing stress along the side BC = V, then $\frac{BD}{BC} = \frac{P}{V}$, whence $\frac{P}{BD} = \frac{V}{BC}$. But $\frac{P}{BD}$ = intensity of thrust along BC and $\frac{V}{BC}$ = intensity of shearing stress along BC.

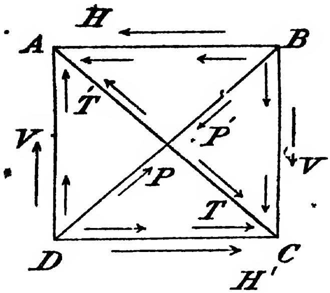


Fig. 68.

If, therefore, the shearing stress were uniformly distributed over any cross section (which may for all practical purposes be assumed), the whole of the shearing forces producing distortion might be replaced by a network of forces at right angles to one another inclined at 45° to the axes of the beam, these forces being tensions and compressions, under the latter of which the web tends to buckle in lines at right angles to the thrust.

The web must, therefore, either have sufficient strength in itself to resist this buckling, or it must be stiffened at such intervals as may be found necessary. For purposes of calculation, we consider the web made up of a series of strips one unit wide, of a length = the distance between the edges of the angle irons, $d \times \operatorname{cosec} 45°$ (see AC, *Fig.* 69), and loaded with a weight = intensity of the shearing stress. Such a strip may then be treated as a column fixed at the ends (see p. 168, Chapter X., Part I.), although it must be admitted that this is only an approximate method of treating the subject, for reasons which will be explained presently. When the distance S between the stiffeners is less than the net distance d between the angle irons, the length of the column = $S \times \operatorname{cosec} 45°$. Hence stiffeners, to be of any theoretical use for the prevention of buckling, must be closer together than the distance between the angle irons.

They would be more efficient if placed in an inclined position in the direction of the thrust, downwards towards the abutments.

As a matter of fact, the stiffeners are generally put in without calculation. They are generally placed, according to the experience of the designer, not more than 6′ or 7′ apart, and much closer to each other near the abutments than at the centre.* Vertical stiffeners are of great use in distributing external loads through the web. Hence they are often put in where cross girders join the main girders. Stiffeners usually put in by experience.

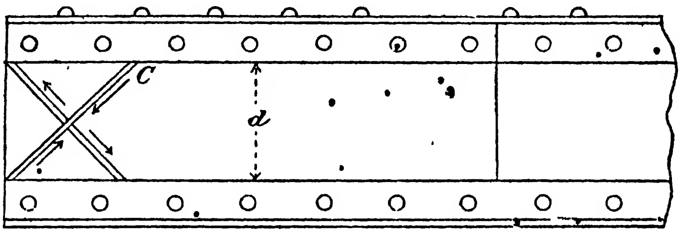


Fig. 69.

Graphic Methods.

In the above investigation we have hitherto not availed ourselves of the aid derived from any graphic methods, and as considerable practical assistance may be obtained therefrom, we must now devote attention to the subject. Graphic diagrams of moments and resistance.

In the calculation of the Moment of Resistance we have seen that $M_r = rad =$ a certain number of inch-tons. If a and d are constant for the whole span, as in the case of a rolled beam, this could be represented graphically by a rectangle of a length, on any scale, equal to the span, and of a height, on another scale (of inch-tons), representing M_r. In a built-up beam there will be at least one plate and the angle irons continuous throughout, so that, multiplying the flange stress (ra) on each by the effective depth d, we may get two rectangles, one representing the Moment of Resistance of the angle irons, and the other that of the plate. Thus if the net area (a) of the angle irons is 2 square inches, and the safe stress intensity (r) of the metal 5 tons, with a depth (d) of beam of 16″, we have the M_r of the angle irons 160 inch-tons. If the area (a_1) of the plate be 3″, we have in like manner, with the same depth, M_r for one

* Fitzmaurice, p. 77.

plate = 240 inch-tons. Lay off AB (*Fig.* 70) on any scale of inch-tons = 160, BC to same scale 240, and AD = span on any linear scale. If we add another plate of the same size at any part, its M_r will be represented by another rectangle added at the part corresponding to its position on the span AD.

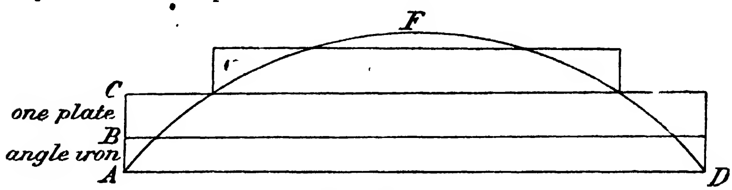


Fig. 70.

M_f diagra

Thus we see that by arranging the rectangles which represent the Moments of Resistance of given plates we may meet any varying conditions of the Moments of Flexure. If we draw to the same scale of inch-tons as AB, BC, the M_f diagram AFD for the actual loads which may be expected on the bridge, including allowance for rolling loads, dynamic effect, etc., we can arrange the plates and angle irons so as to cover the whole M_f diagram, and we can settle exactly the lengths of extra plates required. The larger the scale of moments the greater the accuracy.

Shearing stress diagram.

So also with regard to the web. We can draw the shearing stress diagram ACDB for the loads as in *Fig.* 71, and superimpose upon it

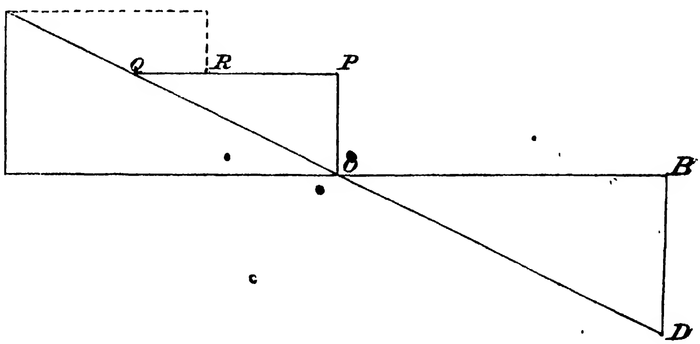


Fig. 71.

the safe shearing stress resistance OP represented at the centre (where the stress will be a minimum, probably) by the web plate of

least practicable thickness. The resistance afforded by such a plate will be the ordinates in a rectangle of which the length is measured from the centre of the span. The point Q where this rectangle cuts the shearing stress diagram for the loads will indicate the position where the thickness of the web must be increased. If practical considerations of limiting length of plates necessitate a joint as at R the thickness is sometimes increased there rather than at P. In these joints two covers, with combined thickness = that of the web, or one cover with thickness = web, must be used. The former arrangement is preferable to the latter.

When the load on the girder is concentrated at one or two points, it is usual to add stiffeners to the web at such points, for reasons stated above.

THEORETICAL OBJECTIONS DISCUSSED.

It may be objected that the method described above for finding the number of rivets required to fasten the angle irons to the web, and for finding whether stiffeners are required, is erroneous, in that it neglects— Objections to the theoretical treatment.

(1). The distribution of shearing stress over the cross section of the web; and

(2). The restraining influence of the tensile components of the vertical and horizontal shearing stresses.

As regards (1), the average intensity of shearing stress, which is here taken as the basis of calculation, is not very much greater than the intensity at the inner edges of the flanges. The case of a girder with wide flanges and a thin web is different from that of a rectangle. In the latter case the shearing stress intensity near the extreme fibre would be considerably less than the average intensity, as the whole of the shearing stress intensities are in that case represented by the ordinates to a parabola passing through the extreme edges and having its axis on the neutral axis and with a value = $\frac{3}{2}$ the average (*Fig.* 118, Part I.). In a girder the intensity of the shearing stress is found from the general expression given in the Appendix to Chapter VI., Part I., p. 123. The distribution curve would be somewhat as shown on *Fig.* 72,

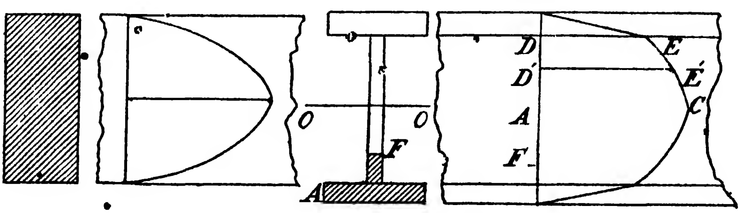


Fig. 72.

where the ordinate AC represents the maximum stress intensity, DE the stress at the flange, D'E' the average. The stress intensity at any section such as F can be easily calculated. For if b = breadth at that section, A = area of metal

between F and the outer edge, M = moment of that area about the neutral axis, I = Moment of Inertia of the whole section, then the stress intensity at $F = \text{whole shearing stress} \times \frac{M}{bI}$.*

This varies but little all the way down the web, since A increases but slightly compared with the constant bI; hence we are justified in taking the stress intensity at D as being uniformly distributed over the *web*. It will differ considerably at the inner and outer edges of the flanges, but that does not affect the question of finding the number of rivets required to fasten the web to the flanges.

(2). The restraining influence of the tensile components of the vertical and horizontal shearing stresses has never been determined (as far as the writer is aware), either theoretically or empirically. No doubt it does exercise some resistance to the compressive action just as the tension on a transversely loaded tie bar diminishes its deflection. No doubt, also, in ordinary cases there is a greater amount of metal in the web than is theoretically necessary. The exact amount is, however, not at present determinable. All that we can say is that the error which arises from calculating the stiffness of the web from the consideration of a long column points to the desirability in large spans, say over 80′, of replacing this form of girder by other forms where open bars are substituted for the continuous web. It is also true that no column formula is properly applicable to the web of a plate girder.

As an application of the principles which have just been considered, we may take the following :—

Example of plate girder for bridge over road.

EXAMPLE 8.—*Design a pair of girders to take a W.D. road across a double line of existing railway on a skew of* 60° (Plates III. and IV.). *The roadway to be wide enough to allow two carts abreast, and to bear a mass of people on the bridge, or the heaviest road engine, having* 9 *tons on driving wheels,* 5 *tons on leading wheels, wheel base* 10′ 8″, *and distance of centres of tyres* 6′ 2″. *The roadway to be borne by rolled steel cross girders* 12″ × 6″ *with jack arches built between, and to be carried on the top flange of the main girders. The jack arches to be* 9″ *thick, of Staffordshire blue bricks, with* 3″ *of Portland cement concrete above and* 12″ *of road metal.*

The main girders to be considered as supported only; the cross girders, however, may fairly be considered as fixed, or at least half fixed, which gives the same M_{ff}.

Width.

As sufficient width must be given to allow two carts to pass, the interior width of road between parapets will be a minimum of 16′, or we may take 17′ 8″ as the distance between centres of the main girders. This is made up of—roadway 16′, projection of parapet 6″, allowing 3″ on either side, and 14″ for the parapet walls (see *Fig.* 74, p. 80).

* See Appendix to this chapter.

(i.). *Distance apart of Cross Girders.*

We can approximately find the distance apart of the cross girders by assuming the whole area of the bridge to be densely crowded with human beings, bringing a live load of 80 lbs. (= 160 lbs. dead)* on each square foot. Cross girders.

With the flanges of the main girder 14″ wide, so as to support the 14″ parapet wall, the clear span of the cross girders will be $17' \, 8'' - 14'' = 16' \, 6'' = 16{\cdot}5'$.

The Moment of Inertia (I) for a $12'' \times 6''$ rolled joist has been shown to be 360 (see p. 97, Chapter V., Part I.). If we take a low value for r in the equation $M_r = \frac{rI}{y}$ we will probably get a resulting distance apart of the girders which will satisfy any requirements of road engines crossing them. This we can test subsequently. Take $r = 5$ tons (which gives a factor of safety of 6). Then y = half depth of beam = 6″, and $M_r = \frac{rI}{y} = \frac{5 \times 360}{6} = 300$ inch-tons.

Let x = distance apart of the joists. Then the road metalling and arches (altogether 2′ deep) may be taken as weighing on the average 1 cwt. per foot cube.† Therefore dead load on one joist

$$= 112 \times 1 \times 2 \times 16{\cdot}5' \times x = 3714x \text{ lbs.}$$

Live load at 160 lbs.

$$= 160 \times 16 \times x = 2560x \text{ lbs.} = 2{\cdot}8x \text{ tons.}$$

Total $$W = 6274x \text{ lbs.}$$

$$M_{ff} = \frac{Wl}{12} = \frac{2{\cdot}88x \times 16{\cdot}5' \times 12''}{12} = 46x \text{ inch-tons,}$$

16·5′ being the clear span, and the girder reckoned as being fixed, or half fixed.

$$M_{ff} = M_r, \text{ i.e., } 46x = 300, \text{ or } x = 6{\cdot}2 \text{ feet.}$$

If the joists, therefore, are 6′ apart, this distance will satisfy the requirements of the uniform load.

This may not, however, be sufficient for the rolling load. This

* Such a weight as this is in excess of that usually calculated for. A crowd such as this might possibly occur in the neighbourhood of a large town.

† The void below the soffit of the arch, which is not taken into account, will be compensated for by the extra weight of the joists themselves approximately.

will be in its most disadvantageous position when the traction engine is close to one parapet, and when the heaviest pair of wheels are directly over a cross girder.

It is true that the effect of the roadway will be to distribute the weight to an uncertain extent, through at least at an angle of 45°* on each side (*Fig.* 73), and the result of such distribution will be to reduce slightly the values of the M_{ff} under the wheels. If, however, we consider the weights as concentrated, we shall be on the safe side, and free from the uncertainty of the exact amount of distribution. It is also true that the increased deflection on one girder will tend to reduce the M_{ff} at that side (see Case 6, Chapter I., p. 14). It will be on the safe side, however, if we calculate the beam for the eccentric position of the load.

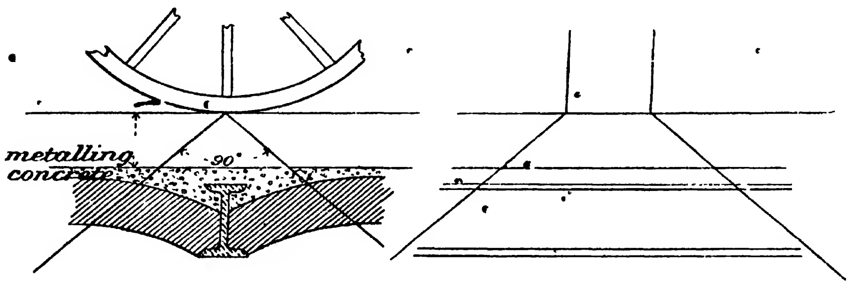


Fig. 73.

It may be considered that the engine would not usually be driven closer to the wall than the position shown in *Fig.* 74, where the centre of the left wheel is 3′ from the edge of the road. This would bring the axle close to the coping of the parapet.

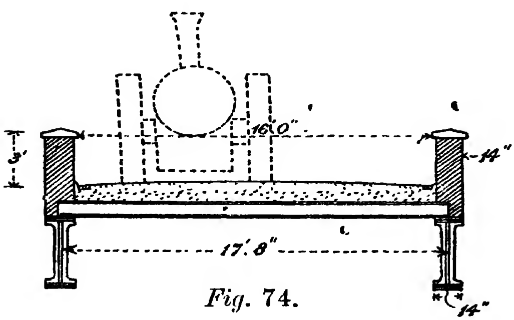


Fig. 74.

Draw the stress diagram for the left wheel (*Fig.* 75), then for the right wheel (*Fig.* 76), and combining them (*Fig.* 77), we see that

* Pascal, *Traité des Ponts Metalliques*.

the M_R for the combined load at the left point of fixing will be 210 inch-tons. (The effect of partial distribution may be to reduce this).

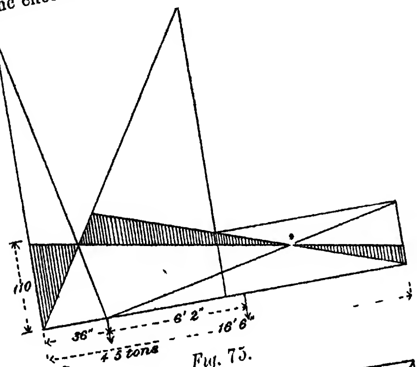


Fig. 75.

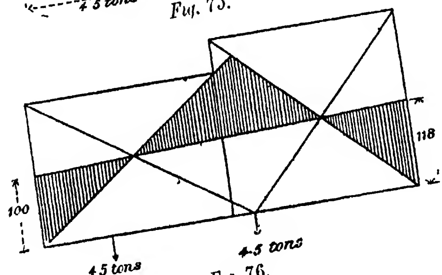


Fig. 76.

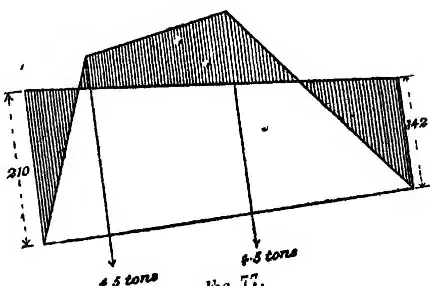


Fig. 77.

In addition to this, there is the M_{ff} for the dead load (which, as we have seen above, is $3714x$ lbs.). Hence the total M_{ff}

$$=210+\frac{3714x\times 16\cdot 5\times 12}{2240\times 12}=210+27\cdot 5x \text{ inch-tons.}$$

This must not be greater than 300, which, as noted above, is the value of M_1 with a factor of safety of 6.

$$\therefore \qquad 27\cdot 5x=300-210,$$

whence $\qquad x=3\cdot 3$ nearly.

With a factor of safety of 5, the cross girders might be upwards of 5 feet apart. If we make the distance 4′ we shall have a factor of safety of 5·3, which is good enough in practice. Make, therefore, 4′ apart.

Main Girders.

Main girders. The effect of the skew is to reduce the weights coming on the ends of the main girders. Were it not for the skew, the whole weight transferred from the cross girders to the main ones might have been regarded as uniformly distributed, and one might do so, even in the case of the skew, without appreciable error. For the sake of practice, however, we may adhere to the more accurate method. If w be the weight transferred from each of the cross girders to the main girders, the distribution will be somewhat as shown on *Fig.* 78. This represents the distribution in the upper main girder, as shown on the plan of the bridge (*Plate* IV.).

The diagram of moments for the *dead* load

$$(\text{where } w=(\tfrac{1}{2}3714\times 4)\div 2240=3\cdot 3 \text{ tons})$$

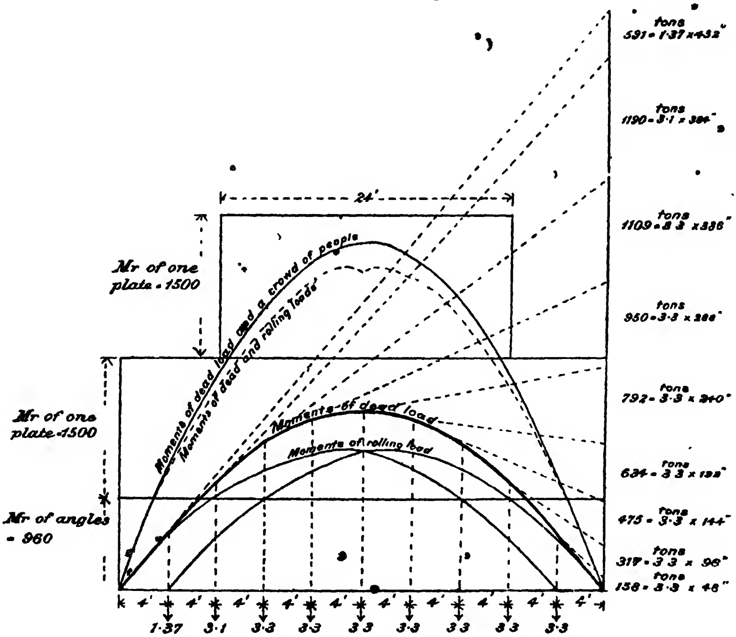
is as shown in *Fig.* 78.

The method of obtaining this by taking moments round the right abutment is shown on the right side of the figure. (Compare *Fig.* 52, p. 72, Part I.).

As regards the weight of the main girders themselves, Professor Unwin's formula is Weight of girders approximately

$$W_1 = \frac{Wl \times r}{cs - lr}$$

where W_1 = required weight.
W = total uniform load.
l = span.
r = ratio of span to depth, *i.e.*, 12.
c = a constant (in this case 1,400).
s = working stress, say 6 tons per square inch.



Scale of moments 2000 inch tons to an inch
Linear scale 16 feet to an inch

Fig. 78.

W, the total uniform load, when the bridge is crowded with a mass of people, and including the weight of parapets is

$$18 \times 40 \times 80 \times 2 \div 2240 = 52 \text{ tons} = 26 \text{ tons}$$

on each, and the dead weight of metalling, arches, etc., on each $= 3\cdot3^* \times 10 = 33$ tons. Hence $W = 26 + 33 = 58$ tons on each girder.

$$W_1 = \frac{58 \times 40 \times 12}{(1400 \times 6) - (40 \times 12)} = 3\cdot52 \text{ tons.}$$

The weight of this may be taken as distributed.

W at each point of junction of main and cross girders may be taken at $3\cdot3$ (as already found) $+ \frac{26}{10} + \frac{3\cdot52}{10} = 6\cdot25$ tons.

The ordinates of the curve of the combined load will be directly proportional to those of the curve of moments of the dead load in the ratio of 6·25 to 3·3, and can easily be read off a slide rule. They have been drawn on the figure so as to represent a curve with a comparatively great vertical scale. These are drawn on *Fig.* 78. The $M_{ff} = 3700$ inch-tons.

M_{ff} from rolling load.

This may, however, be less than the M_{ff} produced by the rolling load. From the previous chapter, Case 3, we see that if W_1 and W_2 be the weights on each pair of wheels, *i.e.*, 5 tons and 9 tons respectively, and a = distance apart of axles = 10′ 8″, the position of M_{ff} from the centre of the span will be

$$\frac{W_1 a}{2(W_1 + W_2)} = \frac{5 \times 10\cdot66}{2 \times 14} = 1\cdot9 \text{ feet,}$$

and

$$\frac{W_2 a}{2(W_1 + W_2)} = \frac{9 \times 10\cdot66}{2 \times 14} = 3\cdot42 \text{ feet.}$$

Hence, measured from one support—

$$x_1 = 20 + 1\cdot9 = 21\cdot9,$$
$$x = 20 - 3\cdot42 = 16\cdot58 \text{ feet.}$$

* Dead load on one joist $= \frac{3714 \times 4}{2240} = 6\cdot6$ tons. Hence 3·3 tons is brought on each side.

For one-half of the beam

$$M_{ft} = \frac{x}{l}\{(W + W_2)(l - x) - W_2 a\}$$

becomes

$$= \frac{16{\cdot}58}{40}\{(14 \times 23{\cdot}42) - (9 \times 10{\cdot}66)\} = 96{\cdot}3 \text{ foot-tons},$$

and for the other half of the beam

$$M_{ft} = \frac{x_1}{l}\{(W + W_2)(l - x_1)\} - \frac{W_1 a}{l}(l - x_1)$$

$$= \frac{21{\cdot}9}{40}\{(14 \times 18{\cdot}1)\} - \frac{5 \times 10{\cdot}66}{40}18{\cdot}1$$

$$= 115 \text{ foot-tons} = 1{,}380 \text{ inch-tons}.$$

This acts at a distance $= 21{\cdot}9$ feet from one support. As the load may pass either way, this will be the M_{ft} to be provided for on the main girders. Half of this bending moment would be brought on each girder if the load were gradually applied, and if the engine were in the centre of the road. But as the load will not necessarily be in the centre, and as the passage of the engine will be accompanied by impact and vibration, the total M_{ft}, 1,380 inch-tons, may be taken for each girder. The moments produced must be added to those of the dead load to find the moments on the bridge when the engine is crossing.

The diagram of combined moments is shown in dotted lines on *Fig.* 78. From it we see that the moments are just a little less than those calculated for a mass of people. Hence the latter must be taken. Yet if the traction engine had been a little heavier, its moments might have governed the actual design.

We have now to consider the Moments of Resistance.

It is proposed to build up the girder flanges of $\frac{1}{2}''$ steel plates, 14″ wide, the width 14″ being taken as the width of the brick wall above, connected to the web by $3'' \times 3'' \times \frac{1}{2}''$ angle irons. **Plates required in flanges.**

The rivets will be of mild steel $\frac{3}{4}''$ diameter.

The depth of the girder will be $\frac{1}{12}$ span $= 3'\ 4''$.

To find the M_r of the angle irons, the net area of one section is

$$(3'' + 2\tfrac{1}{2}'' - 2 \times \tfrac{3}{4}'') \times \tfrac{1}{2}'' = 2 \text{ square inches}.$$

Hence for two angle irons, with a depth of girder of 40″, and a safe tensile resistance of 6 tons per square inch, *rad* becomes

$$2 \times 6 \times 2 \times 40 = 960 \text{ inch-tons.}$$

The M_r of one steel plate $14'' \times \frac{1}{2}''$, with two rivet holes (any other rivets being arranged to break joint)

$$= \tfrac{1}{2} (14 - 2 \times \tfrac{3}{4}) \times 6 \times 40 = 1{,}500 \text{ inch-tons.}$$

Laying off rectangles with ordinates = 960 and 1,500 respectively on the same scale of moments as before (*Fig.* 78), we see that two plates will be required for a distance of 12′ on either side of the centre.

Joints in Plates.

Joints in plates.

Plates of 40′ long and 14″ wide are within market limits. For, from Table III., the maximum weight allowed for a $\frac{1}{2}''$ plate is 26 cwt. The weight per square foot of a $\frac{1}{2}''$ plate is 20·4 lbs. Hence the maximum length admissible is $\dfrac{112 \times 26}{20{\cdot}4 \times 1{\cdot}2} = 118$ feet.

For the sake of illustration, however, we may assume that there is a joint in the longer plate at the centre (see *Fig.* 3, *Plate* IV.).

The M_{ff} there is 3700 inch-tons (*Fig.* 78). The areas of the angle irons and 2 plates are 4, 12·5, 12·5. Hence the total flange stresses will be $\left(\dfrac{M_r}{d}\right)$, *i.e.*, $\frac{3700}{40} = 92{\cdot}5$ tons, and the proportions on angle irons and plates will be $\dfrac{4}{29}$ $\dfrac{12{\cdot}5}{29}$ and $\dfrac{12{\cdot}5}{29}$ of that amount. Hence the flange stress on one plate will be $\dfrac{12{\cdot}5}{29} \times 92{\cdot}5 = 40$ tons.

This stress must be transmitted from one side of the plate to the next by cover plates with $\frac{3}{4}''$ rivets. Let x = number of rivets. Then, for resistance to shearing (see Table XXVIII., Chapter X., Part I.), one rivet will stand 2·208 tons. Hence the number of rivets required will be $\dfrac{40}{2{\cdot}208} = 18{\cdot}12$.

For bearing, using a $\frac{1}{2}''$ cover plate, and the safe intensity of resistance being 6 tons—

$$x \times {\cdot}5 \times {\cdot}75 \times 6 = 40. \quad x = 17{\cdot}8.$$

Hence 19 rivets will be needed on each side of the joint if single covers are used.

Make 4 rows on each side of 5 rivets each (*Fig.* 4, *Plate* IV.).

The angle irons are sufficiently short to be independent of covers. If, however, they are required, they can be calculated on the same lines as the above.

Shearing Stresses.

For the rolling load the greatest shearing stress will be near an abutment when the engine has come on to the bridge, the heavier wheels being just clear of the abutment, and = 12·6 tons, or 6·3 on each girder, if the engine is fairly in the middle of the road. The stress diagram is as *Fig.* 79. Shearing stress.

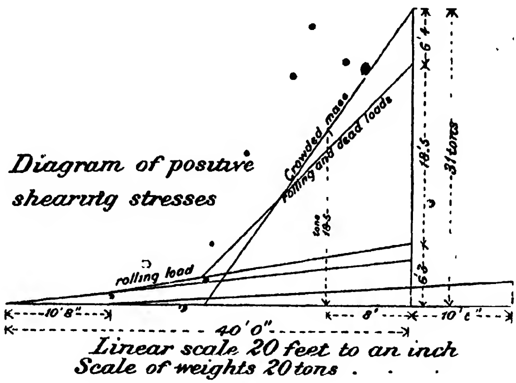


Fig. 79.

The shearing stresses produced by the *dead load* on each girder $=\frac{1}{2}$ total weight on it $=\frac{33+3\cdot52}{2}=18\cdot5$ tons. Therefore the maximum shearing stress due to rolling and dead load

$$=18\cdot5+6\cdot3=24\cdot8 \text{ tons.}$$

The shearing stress due to the uniform live load + dead load

$$=\tfrac{1}{2}\,(58+3\cdot52)=31 \text{ tons,}$$

which, being the greater, must be taken.

Let the thickness of the web at the abutments be $\frac{1}{2}''$, and the pitch of the rivets be (as is usual in practice) 4″. Then there will be 3 rivets per foot run, and as the depth of the girder is 3′ 4″, there will be 10 rivets to stand a shear of 31 tons in double shear, equivalent to 15·5 tons in single shear, or 1·55 per rivet. From

Table XXVIII., Chapter X., p. 216, Part I., this would necessitate $\frac{3}{4}''$ rivets.

Bearing area of rivets.

The bearing strength of the plate on the rivets must also be equal to the shearing force coming on them. That shearing force being 31 tons, the number of $\frac{3}{4}''$ rivets being 10, and thickness of plate $\frac{1}{2}''$, we have $r \times 10 \times \frac{3}{4}'' \times \frac{1}{2}'' = 31$. Whence $r = 8{\cdot}2$ tons. This is just within safe limits for steel plates, giving a factor of safety of nearly 4. In comparatively shallow girders, as in this case, there is always a difficulty about the bearing area of the rivets near the abutments, a difficulty which increases with the load. To obviate the difficulty we must either (1) increase the number, and, therefore, diminish pitch of the rivets, or (2) increase their diameter, or (3) increase the thickness of the plate. Of these alternatives (3) is the one adopted in this case, as we might otherwise have used a $\frac{3}{8}''$ plate throughout.

The thickness of the web at the ends is $\frac{1}{2}''$. Its net sectional area is $40'' \times \frac{1}{2}'' - 10 \times \frac{3}{4}'' = 12{\cdot}5$ square inches. It has to stand a shearing stress of 31 tons, or $\frac{31}{12{\cdot}5} = 2{\cdot}48$ tons per square inch, which is well within safe limits.

This thickness might be reduced to $\frac{3}{8}''$ at 8 feet from one abutment. Theoretically, it might still further be reduced, but practically it is not advisable to have a less thickness than $\frac{3}{8}''$. At 8′ from the abutment the shearing stress is (from *Fig.* 79) 18·5 tons. Hence the bearing stress r_b is found from the following —

$$10 \times \tfrac{3}{4}'' \times \tfrac{3}{8}'' \times r_b = 18{\cdot}5.$$

Whence $r_b = 6{\cdot}4$ tons, the safe limit being about 8·25.

Whether stiffeners are required.

To ascertain, finally, whether the web needs stiffeners, we take the shearing resistance per foot run at the end, $\frac{31}{3{\cdot}3}$ tons $= 9{\cdot}4$ tons, as the force coming on a column of length = the distance between angle irons $(40 - 6 = 34) \times \operatorname{cosec} 45^\circ\ (= 1{\cdot}414) = 48{\cdot}2''$, of breadth $12''$, and thickness $\frac{1}{2}''$, and with ends fixed.

Thus $$\frac{l}{h} = \frac{48{\cdot}2}{{\cdot}5} = 96{\cdot}4.$$

Hence, in Gordon's formula—

$$P = \frac{r_c A}{1 + a\left(\frac{l}{d}\right)^2}$$

(see Chapter IX., Part I., p. 168). $P = 9{\cdot}4$ tons, $A =$ sectional area

on column $= 12'' \times \frac{1}{2}'' = 6$, $a = \frac{1}{2480}$ (Table XXV., p. 169, Part I.), and $\frac{l}{d} = 96 \cdot 4$, the value of $r_c = 6 \cdot 6$ tons per square inch. This is within safe limits, and therefore it appears that stiffeners are not necessary to provent buckling of the web, though practically it would be desirable to introduce T-irons at intervals below the cross girders, say every cross girder for the first 4 cross girders at each side and at every alternate one for the rest.

Fig. 80 represents the main girder in elevation.

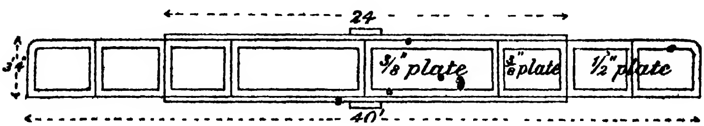


Fig. 80.

If the limiting value of r be taken at 6 tons per square inch, and the calculated value 6·6 be considered inadmissible, as being beyond the limit, the distance apart of the stiffeners would thus be ascertained.

The shearing stress being 9·4 tons per foot run, and the intensity being $\frac{9 \cdot 4}{12 \cdot \times \frac{1}{2}} = 1 \cdot 56$ tons per square inch, we see that this corresponds, in Table XXVI., p. 176, Part I., to a value of $\frac{l}{k}$ of 219. As the assumed strip is rectangular in cross section, we see from p. 177, Part I., that $\frac{l}{k} = 3 \cdot 5 \frac{l}{h}$, h being the least dimension, and l the unknown length. Therefore $l = \frac{219}{2 \times 3 \cdot 5} = 31 \cdot 3$ inches, which is the least length measured at an angle of 45° between the edges of the angle irons which will stand without buckling. Hence the distance measured horizontally will be $\frac{31}{1 \cdot 414} = 22$ inches.

Practically, the stiffeners would be put when the cross girders join the main girders.

Deflection.

The deflection of the cross beams, which are (1) fixed at both ends; (2), of uniform cross section; and (3) uniformly loaded, will be Deflection. Cross girders

found from the general formula $v = \frac{nWl^3}{EI}$, the symbols in which will have the following values:—

$$n = \cdot 0026,$$
$$W = 3714 \times 4 = 14{,}856 \text{ lbs.} = 6\cdot 64 \text{ tons},$$
$$l = 16\cdot 5 \times 12 = 198 \text{ inches},$$
$$E = 13400 \text{ tons and } I = 360.$$

Hence $$V = \cdot 02716 \text{ inches}.$$

Main girders.

In the main girders we may either use the same formula, in which case the values of I would need to be calculated, or we may use the formula $V = \frac{1\,rl^2}{Ed}$, where the value of I is not required, but from which we can ascertain the deflection, which would be produced when the stress intensity is at the maximum for which the girder is designed, *i.e.*, when the value of r in the expression rad is $= 6$ tons per square inch. The girder has been designed so that this value shall nowhere be exceeded by any conditions of loading which are likely to come upon it. The formula is expressed in a simple form on p. 131, Part I., viz.:—

$$V = \frac{3rl^2}{dE},$$

where d and l represent the depth and length in feet, for a girder (such as this is) of uniform strength and depth. Taking the value of E at 13000 (slightly less than that above on account of the defects of workmanship)—

$$V = \frac{3 \times 6 \times 1600}{3\cdot 3 \times 13000} = 0\cdot 67 \text{ inches},$$

or about $\frac{1}{710}$ of the span.

This is rather more deflection than is generally allowed for girders, though less than the proof load given by Professor Rankine (see p. 134, Part I.). The deflection in this case, however, has been calculated on the basis of a *supported* beam, while in reality the girder is partially fixed, and the deflection would be considerably less than 0·67 inches.

Cost of bridge.

The design would then be as shown on *Plates* III. and IV. The cost of such a bridge in the south of England would be about £1,100, including all masonry and earth filling, etc. The head room to rail level at the bottom of the girder would need to be at least

14′ 6″. This consideration in many cases has led to the cross beams being placed on the lower flange, thus saving a height in wing walls, abutments and bank equal to the depth of the girder. The cross girders are then rivetted to the web at the ends by angle irons.

Wind or other lateral bracing is, in English practice, not considered necessary, with such short spans, although in France it is given in even shorter spans.

The wing walls are designed by methods usual in English railways. No description of this need be given, as the drawing indicates the procedure.

Various Forms of Bridge Floors.

In the foregoing example the roadway is supported on rolled joists and brick arches. This method of construction, though not uncommon, is perhaps less usual nowadays than some form of steel decking. In railway bridges for double lines the usual type of bridge, up to 40′ span, is 3 main girders, the central one of which is proportionately stronger than the other two, with trough floors, as shown in *Fig.* 81. A saving is effected by using 3 main girders for double lines, as the width of the roadway necessitates an intermediate point of support for the floor, of whatever nature it may be. *Fig.* 82 shows the method of fastening the trough girders used for the flooring to the longitudinals. Bridge floors.

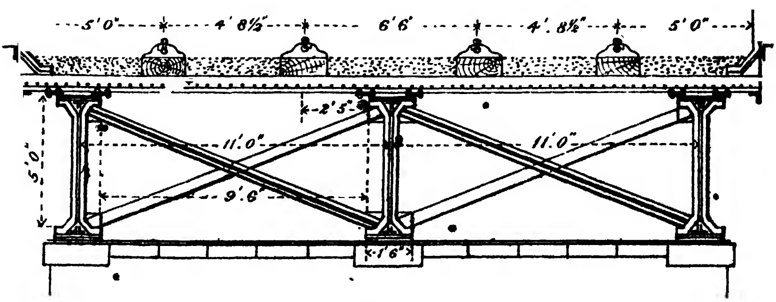


Fig. 81.

The advantages of trough floors are, that they do away with all cross girders, rail bearers, etc., are easily erected, occupy little room, Trough girders.

are easily drained, and are of moderate cost. They may be even used for public roads without any main girders at all, as in *Fig.* 83. This figure is taken from Messrs. Dorman, Long & Co.'s catalogue, which gives much information regarding this particular kind of trough, which is known as Lindsay's Patent.

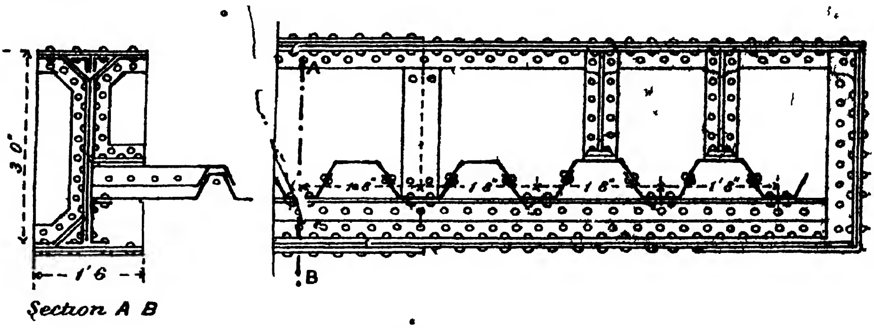


Fig 82

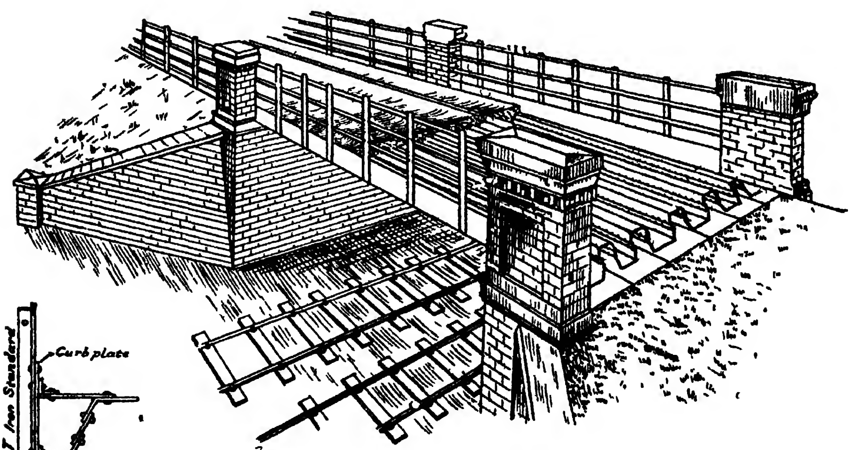


Fig 83.

To ascertain the Moment of Resistance of any trough girder section, the graphic method of areas of equal resistance as shown in Chapter V., Part I., may be followed. As the values of the M_r for each form in inch-tons are given in manufacturers' catalogues, such calculation is usually unnecessary.

In some English railways buckle plates of iron or steel are riveted to cross or longitudinal girders, forming a continuous metal floor on which the ballast, sleepers, etc., are laid. Other forms of flooring are Hobson's Patent, shown in *Fig.* 84, and corrugated plates, as used in the Indian State Railways; the latter, however, are not intended to take much weight.

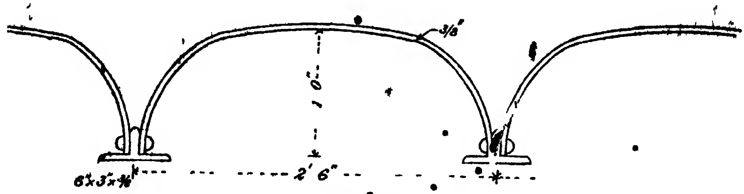


Fig. 84.

Fig. 85 shows another form of trough something like Lindsay's Patent, which is often used

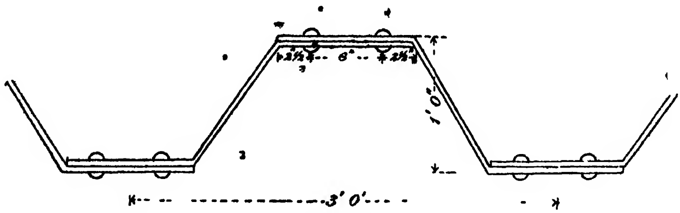


Fig. 85.

Stamped plates of steel may also be used with economy between rolled beams or troughs. As regards the weights they can carry, the following are the results of some experiments :—In France a stamped plate 5·8′ long, 4·9′ broad, 0·31′ thick, camber 2½″, was fixed at the ends, covered with 5·8″ of sand, and then loaded. The deflection under 6,000 kilogrammes (5·9 tons) was 0·15″, and when the load was removed the plate resumed its original form (Pasçal, *Ponts Metalliques*). Stamped plates

[Note.—*Figs.* 81, 82, 83 are copied, by the kind permission of Messrs. Dorman, Long & Co., from their catalogue.]

APPENDIX.

THE general expression for the value of the intensity at any point in the cross section of a beam of any form is

$$q' = \frac{S}{Ib'} \int_{y'}^{y_1} ybdy,$$

where q' = intensity of shearing stress on any plane.
S = total shearing stress on the section.
I = Moment of Inertia of the cross section about its neutral axis.
b' = breadth of the section where the shearing stress $= q'$.
y' = distance of the plane where the shearing stress is q' from the neutral axis.
y_1 = distance of extreme fibre on that side of the neutral axis from that axis.
b = breadth of section at distance y from the neutral axis.

Now $\int_{y'}^{y_1} ybdy$ is the statical moment of the area outside the plane when the shearing stress intensity is q_1 about the neutral axis. Call this moment M.

Then
$$q' = \frac{SM}{Ib'}.$$

But in the web b' is constant, and I and S for the whole cross section being constant, we see that q' varies as M. The increments of area of the cross section are so slight for planes taken in succession from the neutral axis to the ends of the web that we may consider q' to be practically constant.

CHAPTER IV.

TRUSSED AND STRUTTED BEAMS.

The Design of Simple Braced Structures.—Deformation and Redundant Bars.—Triangular Trussed Beam.—Trussed Beam with two Struts.—Effect of Bracing and Counter-Bracing.—Strutted Beams.—Deflection in Braced Structures.—General Remarks on Trussed Beams.—Appendix.

TRUSSED BEAMS.

FROM the consideration of the built-up girders under review in the preceding chapter we might go on at once to the investigation of stresses in those built-up girders where the web, instead of being a solid plate, is a series of bars meeting the flanges at some angle. Before doing so, it will be desirable to consider some principles of the design of braced or framed structures, and the application of those principles to the simplest cases of trussed beams.

Simplest cases of braced structures.

If we take a simple triangle, such as ABC (*Fig.* 86), formed of 3 bars fastened to one another at their ends, we see that, however it is loaded, it cannot undergo any change of form, without altering the length of the sides, or by giving way at the joints. With a quadrilateral figure, however, this is not the case, for we see in *Fig.* 87 that the sides AB, BC, CD can move into a position such as AB′C′D shown in dotted lines, without altering the lengths of the

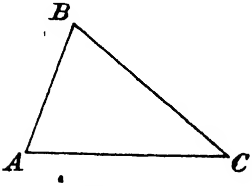


Fig. 86.

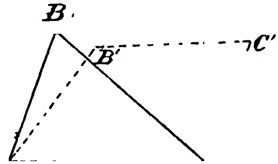


Fig. 87.

bars. Such deformation would be impossible if the quadrilateral is divided into two triangles by the introduction of a diagonal such as

BD. The other diagonal might be added, but it would be what is called a "redundant member," and would be unnecessary under ordinary circumstances, though in certain cases in bridge construction these redundant members are designed to perform the useful function of counter bracing, a subject which will be hereafter investigated. If, however, counter bracing is not required, or is already sufficiently applied, the redundant members are a waste of material, add to the weight to be carried, and are in some cases a hindrance to the duty of the necessary members. In the case of a quadrilateral we can see, if we draw a stress diagram, that one diagonal only is necessary. Let the frame (*Fig.* 88) be loaded at the left hand top corner with a weight W. Then the stress diagram is as shown on *Fig.* 89. In this case the bars BD, DF are redundant. If, however, CD had been omitted, the stress polygon would not close. If the

Redundant bars.

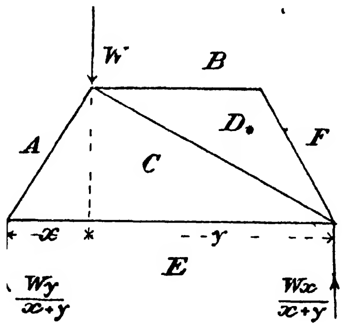


Fig. 88.

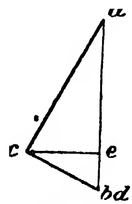


Fig. 89.

frame had been as shown in *Fig.* 90, the stress would be as shown below, every member would be doing a certain part of the work, and there would be no need for another diagonal.

The broad principle in the case of any truss is to divide it into a series of triangles which should be as few as possible, consistent with practical requirements of tension and compression members, and certain other considerations.

Formula for ascertaining whether a beam is properly trussed.

If the truss is properly braced the following formula will hold good. Let S = number of bars of the truss, A = number of apices (where two or more bars meet), then $S = 2A - 3$.

If this does not hold good the truss is either insufficiently or else

redundantly braced. Take the beam shown in *Fig.* 91. The sides are 8, the apices are 6. S = 8, but 2A − 3 = 9, hence there is one bar wanting. If such is applied as shown by the dotted line, the frame will be properly trussed. If two diagonals are given, S will = 10, and there will be a redundant bar.

In this truss, however, the diagonals are sometimes wholly omitted, where the load is uniform. The tendency to change the form is prevented by extra breadth in the short struts *a*, *a*. Sometimes also both diagonals are given in cases where the load may be moving across the truss, and counter-bracing is desirable.

The equation S = 2A − 3 does not indicate the best system of trussing, nor does it show the smallest number of triangles that may be used. All that it does is to show in any given case whether the requirements of trussing are fulfilled.

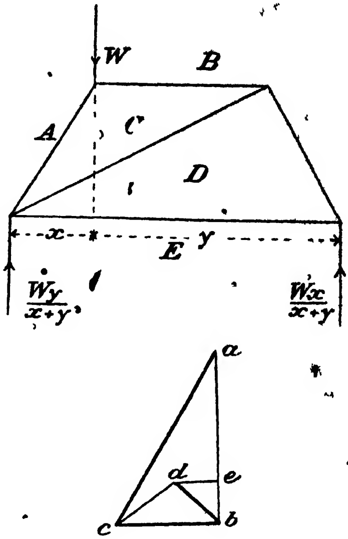


Fig. 90.

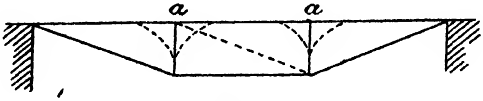


Fig. 91.

Trussed Beams in Detail.

The simplest form of trussed beam is that shown in *Fig.* 92. **Triangular trussed beam.**

The compression members, *i.e.*, the horizontal beam and vertical post, are usually of wood (sometimes the latter is of cast iron), and the inclined bars are of wrought iron or steel. It will be evident that this beam, when loaded either uniformly or with a single weight at any part other than the central joint, is under *combined* stress, both transverse and compressive.

As the beam is continuous over two spans, we see, from the

principles laid down in Chapter I., that the central load, at the point $C = \frac{5}{8}W$, where $W =$ the total distributed load. If w be the load per unit of length, then the bending moment at $C = \frac{wl^2}{32}$*, which for a rectangular section $= \frac{rbd^2}{6}$. Hence $r = \frac{3}{16}\frac{wl^2}{bd^2}$, r being the stress intensity at the extreme fibre produced by the uniformly distributed transverse load.

Transverse stress.

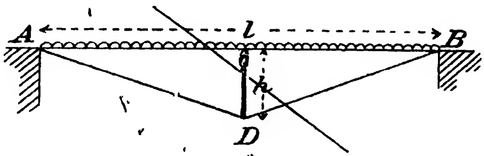


Fig. 92.

Direct stress.

Now the direct compression in the beam may readily be ascertained by the method of sections described in Chapter XII., Part I. Drawing a section as in *Fig.* 92, through AC, CD, BD, and taking moments round D, we have stress in $AC \times CD =$ net reaction at $B \times BC$, or calling stress in $AC = P$ and $CD = h$.

$$Ph = \frac{5}{16}W \times \frac{l}{2} = \frac{5wl^2}{32}, \text{ since } W = wl,$$

hence r', the stress intensity due to compression, only—

$$\frac{P}{bd} = \frac{5}{32}\frac{wl^2}{bdh}.$$

Stress intensity in main beam.

Hence

$$r + r'\dagger = \frac{3}{16}\frac{wl^2}{bd^2} + \frac{5}{32}\frac{wl^2}{bdh} = \frac{1}{32}\frac{wl^2}{bd^2h}(6h + 5d) \ldots \ldots\ldots(\text{i.}).$$

This must not be greater than the material can stand.

* $M_{ff} = \frac{W'l'}{8}$ where $W' = \frac{wl}{2}$ and $l' = \frac{l}{2}$.

Hence $M_{ff} = \frac{wl^2}{32}$.

† It is true that r and r' are different stress intensities. The former is the Modulus of Rupture, which is not the same as the unit resistance per square inch to direct crushing r'. In Part I., Chapter IX., p. 178, the combined stress is considered with reference to the direct crushing only.

The tension in the tie bars will be = net reaction × cosec θ, where θ = angle of inclination. But the net reaction is $\frac{5}{16}\,wl$, being $\frac{1}{2}$ the total weight $\frac{wl}{2}$ minus the $\frac{3}{16}\,wl$ borne at the end of the beam, and Tension in tie bars.

cosec $\theta = \frac{\sqrt{l^2+4h^2}}{2h}$. Hence tension

$$= \frac{5wl}{32h}\sqrt{l^2+4h^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii.)}.$$

Example of trussed beam.

EXAMPLE 9.—*Design two trussed beams to carry a road bridge across a span of 16 feet, the roadway to be of 3″ planks laid close together, and the width to be 10′. The timber to be spruce (A. excelsa).*

Main beams.

The weight on an ordinary road bridge being a maximum of 80 lbs.* per square foot, and the width of the roadway being 10′, the load coming on each beam will be $5 \times 16 \times 80 = 6{,}400$ lbs. The weight of the roadway on each beam will be $16 \times 5 \times 3'' \times 30$ lbs. (weight of spruce per foot cube) = 600 lbs. Total weight 7,000 lbs. on 16 feet or 36·5 lbs. per lineal inch = w. Then $l = 192''$, and the compressive strength of spruce being on the average 6,400 lbs., with a factor of safety of 5 we get $r_c = 800$.

Assuming a length of strut = 4′ 2″ or 50 inches, we have from (i.), substituting values—

$$800 = \tfrac{1}{32}\,\frac{36{\cdot}5 \times 192 \times 192}{bd^2 \times 50}\,(6 \times 50 + 5d).$$

If $d = 8$ this gives us a value of $b = 5{\cdot}58$. Make $8'' \times 5\frac{3}{4}''$.

Struts.

Struts.

The value of $P = \frac{5}{8}W = \frac{5}{8} \times 6{,}400 = 4{,}000$. Ratio of $l:d$ is $50:4$ or $12{\cdot}5:1$. $\therefore P = \frac{1}{2}r_c A$,† or $4{,}000 = 400 \times A$. Hence $A = 10$. Make $4'' \times 4''$.

Tension Rods.

Tension rods.

Substituting values in (ii.) we have tension

$$= \tfrac{5}{32} \times \frac{36{\cdot}5 \times 192}{50}\sqrt{192^2 + 4 \times 50^2} = 4{,}720 \text{ lbs.}$$

Wrought iron will easily stand 5 tons = 11,200 lbs. per square inch.

* In ordinary highway bridges a total load of 80 lbs. or live load of 40 lbs. per square foot is considered sufficient. In France 41 lbs. is the government rule.

† See Formula (4), p. 160, Part I.

Hence a $\frac{3}{4}''$ bar will stand $\cdot75 \times \cdot7854 \times 11{,}200 = 6{,}600$ lbs., and will be sufficient.

The bridge would then be as in *Fig.* 93. A thin iron plate, say $\frac{1}{4}''$ thick, will be required at the end of the strut, so as to distribute the pressure. The tie bars may be secured to the beams by links and $1\frac{1}{2}''$ bolts. The shearing stress at the ends AA will be $\frac{6400}{2} = 3{,}200$ lbs., and the net area to resist shearing will be $(8 - 1\frac{1}{2}) \times 5\frac{3}{4} = 37\cdot25$ inches. Taking the safe resistance to shearing across the grain at 500 lbs, this gives 18,625 lbs., which is ample.

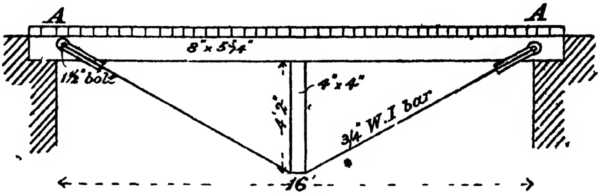


Fig. 93.

Weight the bridge could carry.

This bridge would be strong enough to stand a rolling load approximately of one half the distributed load. The latter being altogether 6,400 lbs., the rolling load would be 3,200 lbs. on each beam. This is not equal to a traction engine, but is more than field artillery or ordinary carts To calculate such a bridge for a traction engine, one would need to compare the M_f and shearing stress diagrams produced by such an engine in its passage with the similar diagrams for the distributed load. If the former gave large results the unit for the uniform load could then be increased, and the truss re-calculated on the basis of the increased load. It would also be necessary to consider the effect produced by the engine in certain positions on the span, and to calculate by stress diagrams, or by the method of sections, the stress on each member of the truss.

Trussed Beam with Two Struts.

Trussed beam of trapezoidal form.

Although a trussed beam with two struts, and no diagonals, is not a perfectly braced structure, as we have seen above, it is frequently so constructed, and its stresses therefore demand investigation. The beam of the truss serves (1) to resist the direct compression produced by the pull of the tie rods ; (2). it resists transverse bending ; and (3) it serves to distribute the load over the whole truss.

Let it be assumed that the load is supported by struts $\frac{1}{3}l$ from each end. A load W rolling across the beam produces a distortion as shown in *Fig.* 94. Single load.

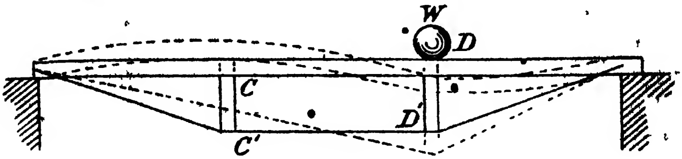


Fig. 94.

Let h = height of truss, b and d, as usual, the breadth and depth of the beam.

If then W be over any one support the reaction will be found in exactly the same way as for any supported beam, whether trussed or not. Hence Reaction.

$$R_A = \frac{W}{3},\ R_B = \frac{2W}{3}.$$

A part of W is carried by the strut directly below it, the remainder supported by AB, and is brought to bear at D.

Since the distortion is small, and the tie bars AC' and BD' are practically of the same inclination, their horizontal components are equal, being equal to the stress in C'D'. Hence their vertical components must also be equal.

But the vertical components in these tie rods are equal to the vertical compressive stresses in the struts CC' and DD'. Hence the compressions in these struts are equal.

From the symmetry of the trussing, if the point D drops a certain amount the point C will rise the same amount. But the force which causes C to rise is the thrust in CC', and the force which causes D to drop is the difference between the load W acting at D and the compression in the bar DD', the latter tending to prevent deflection. Hence the remaining portion of the load W, after the upward thrust of DD' is deducted, is equal to the thrust on CC', or the thrust on DD'. That is, W is divided into two (approximately) equal parts, one half carried by the strut, the other resisted by the transverse stresses in the beam.*

* This proof, as well as some others in this chapter, is taken from Professor Johnson's *Modern Framed Structures*. Another proof, by Major H. D. Love, R. E., is given in the Appendix to this chapter.

Compression in main beam.

Hence compression in AB = tension in C′D′

$$=\frac{W}{2}\cot\theta=\frac{W}{2}\times\frac{l}{3h}=\frac{Wl}{6h}. \qquad \text{(iii.)}.$$

Tension in tie bars.

Tension in inclined bars = vertical thrust in CC′ or DD′ × cosec θ

$$=\frac{W}{2}\times\frac{1}{h}\sqrt{\frac{l^2}{9}+h^2}=\frac{W}{6h}\sqrt{l^2+9h^2}\ldots\ldots\ldots\ldots\ldots\text{(iv.)}$$

Transverse stress in main beam.

Since the central point of the beam F is the point of contra-flexure, there is no bending moment there, and it may be taken as a free supported end. The shear at that point, taken with reference to the right support, is $\frac{2}{3}W - W = \frac{1}{3}W$, which is the same as the shear referred to in the left support. Hence M_f at C or D

$$=\frac{1}{3}W\times\frac{1}{6}l=\frac{Wl}{18}\ \ldots\ldots\ \ldots\ldots\ \ldots \qquad \text{(v.)}.$$

As this, in a rectangular beam, must be

$$=\frac{rbd^2}{6},$$

we get

$$r=\frac{Wl}{3bd^2}. \qquad \text{(vi.)}.$$

And as the intensity of the direct stress in AB from (iii.) is

$$r_1=\frac{Wl}{6hbd},$$

$$r+r_1=\frac{Wl}{6hbd^2}(2h+d)\ \ \ldots\ldots\ \ldots\ldots\ldots\text{(vii.)},$$

which gives the maximum compressive stress per square inch in the beam, under a load W.

If either of the beams shown in *Figs.* 92 and 94 are inverted, as in *Figs.* 95 or 96, of course the same formulæ hold good, only the tensions become compressions, and *vice versâ*.

Case of uniformly distributed weight.

If the *weight is uniformly distributed*, over a beam with two struts, $\frac{11}{30}wl$ comes over each intermediate support, and the bending moment at that support

$$=\frac{wl^2}{90},^{*}=\frac{rbd^2}{6},$$

Transverse stress on main beam.

hence

$$r=\frac{1}{15}\frac{wl^2}{bd^2}.$$

* See Table II., p. 33.

The direct stress = net reaction × cot θ Direct compression on main beam.

$$= \frac{11wl}{30} \times \frac{l}{3h} = \frac{wl^2}{90h}$$

Hence $$r' = \frac{11wl^2}{90n \times bd},$$

and $$r + r' = \frac{wl^2}{90hbd^2}(11d + 6h) \text{.......... (viii.).}$$

This gives the maximum compressive stress per square inch in the beam under a uniformly distributed load *wl*.

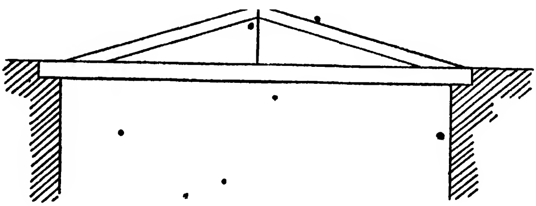


Fig 95.

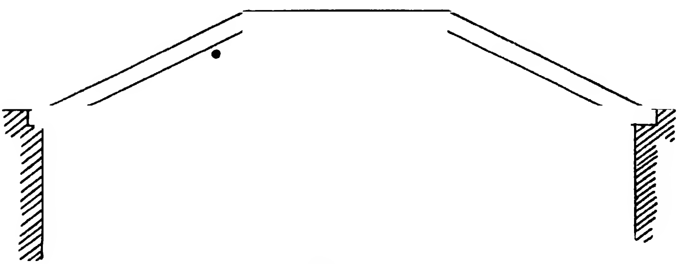


Fig. 96.

The tension on the tie bars = net reaction × cosec $\theta = \frac{11}{30} wl$ cosec θ Tension in tie bars

$$\frac{11wl}{90h}\sqrt{l^2 + 9h^2} \text{.. ....(ix.).}$$

EXAMPLE 10.—*Design a pair of trussed beams, with two struts, to take a cart road* 10′ *wide over a span of* 30′. *Timber, red fir.* Example of trussed beam with two struts.

The weight of the roadway being taken, as in Example 9, at

36·5 lbs. per lineal inch, we have for the main beam, if we take a length to each strut of 4′ 2″ from (viii.)

$$r = \frac{wl^2}{90hbd^2}(11d + 6h) = \frac{36{\cdot}5 \times 360^2}{90 \times 50 \times bd^2}(11d + 300).$$

Main beam.

Take $r = \frac{6000}{5} = 1{,}200$, and take $d = 8''$. This gives a value to b of 4·22. Make $8 \times 4\frac{1}{2}''$. If 2″ bolts are used, the safe resistance to shearing of the timber at the ends will be $6'' \times 4{\cdot}5 \times 500 = 13{,}500$ lbs. The stress will be $\frac{36{\cdot}5 \times 360}{2} = 6{,}570$, so that the beam is quite safe against failure by shearing.

Tie bars.

Tension on the bars from (ix.)

$$= \frac{11wl}{90h}\sqrt{l^2 + 9h^2} = \frac{11 \times 36{\cdot}5 \times 360}{90 \times 50} \times \sqrt{360^2 + 9 \times 2{,}500}$$
$$= 12{,}500\text{lbs.} = 5{\cdot}58 \text{ tons}.$$

As wrought iron will stand a safe pull of 5 tons per square inch, if d = diameter, $\frac{\pi d^2}{4} \times 5 = 5{\cdot}58$, whence $d = 1{\cdot}22$, say $1\frac{1}{4}''$.

Greatest single weight beam can bear.

To find the *greatest weight that the beam can bear* from (vii.)—

$$r + r_1 = \frac{Wl}{6hbd^2}(2h + d),$$

becomes $$1{,}200 = \frac{W \times 360}{6 \times 50 \times 272}(100 + 8),$$

whence $$W = 2{,}518 \text{ lbs on each beam}.$$

This is more than the weight on one wheel of a loaded cart in ordinary traffic.

Struts.

The stress on the *struts* will in each case respectively be either $\frac{11}{30}wl = 4{,}830$ lbs. or $\frac{W}{2} = \frac{2518}{2} = 1{,}259$ lbs. Taking the former, we have 4,830 lbs. (= P) as the stress coming on $6'' \times x$, *i.e.*, assuming that the least dimension of the strut is 6″. As the net length of the strut is 50·5″, $\frac{l}{h} = 8{\cdot}33$. Hence from Formula (3), p. 160, Part I., $P = \frac{5}{6}f_cA$, or $4{,}830 = \frac{5}{6} \times 1{,}000 \times 6 \times x$, whence x might = 1 inch. If we make the strut $4'' \times 4''$ it will be strong enough. *Fig.* 97 shows the trussed beam.

Beams considered continuous.

It will be observed that in these examples the beams are reckoned as continuous, just as the rafters in roofs (Chapters XI. and XII., Part I.) are so reckoned. This is strictly accurate only when the supports are all on one level; the effect of changes of temperature would be to alter the stresses to a certain extent. The formulæ, however, would be as nearly accurate as is possible to obtain, and the latitude allowed by the factors of safety would cover minor variations.

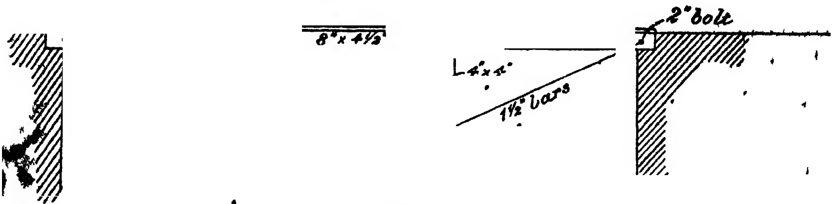


Fig. 97.

Economical to make struts as long as possible

It is also noteworthy that the greater the length given to the struts, the less will be the stress on the horizontals and tie bars. Hence it is decidedly economical, where circumstances admit, to give considerable length to the struts. This is practically the same as saying that the greater the depth given to a truss, the less will be the stress on the members.

Trapezoidal truss with diagonal bars.

When the beam is trussed by cross bars, as in *Fig.* 98, the investigation is simple. Each joint of the truss may be investigated (as far as direct stresses are concerned) by a stress diagram, or by the analytical value of the stresses, as shown in *Fig.* 98 for a particular case. *Fig.* 99 shows the stress diagram for such a case. This will give the direct stresses on each member of the truss, and the combined bending and direct stress on the upper horizontal beam can be easily worked out on principles previously investigated and exemplified. All the other stresses are direct only.

Alternate stress on diagonal bars.

It will be noticed that in *Fig.* 98, if there is one bar as EF, it is in tension when the load is above DE. If the load roll as far as FG, EF will now become a strut. In the design, therefore, we have two alternatives—(*a*), we may either make the bar EF of such a section as will resist either tension or compression, taking care to give the metal a sectional area based upon the consideration of the reversal of stress, and the consequent fatigue produced; or (*b*) we may give a second diagonal bar as shown in dotted lines, which,

although inoperative while the load is at DE, will come into tension when the load is at FG, and will relieve EF of all stress when the load is at FG.

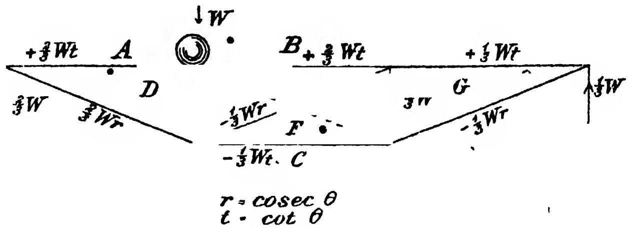


Fig. 98.

The second alternative is the preferable, for not only is it possible to use light tension rods for both diagonals, but the permissible unit stress on these diagonals will be at least twice as great as the permissible unit stress on a single bar, subject to reversal of stress. The principle of putting a second redundant bar will be, in a subsequent chapter, alluded to under the heading of "Counter-bracing." It is well, however, to understand the principle thoroughly here as applied to a simple case.

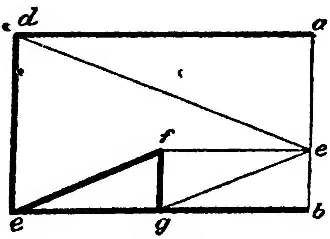


Fig. 99.

In the above investigation we have assumed that the whole structure is jointed. This is not strictly accurate, because the beam is continuous. It is the same general principle as adopted in the investigation of roofs (see Chapters XI. and XII., Part I.).

The M_{ff}, due to a load rolling across, will be $\frac{Wl}{24}$ (because the central third may be regarded as a fixed beam of length $= \frac{l}{3}$, and we know that the M_{ff} in a beam of length L and weight W, is $\frac{WL}{8}$), and will occur at the junction of the struts and beam, and in the centre of the spans.

Strutted Beams.

Strutted beams.

These are the converse of trussed beams, the stresses produced and the investigation of those stresses being analogous. The beam

may be either strutted, as in *Fig.* 100, corresponding with the triangular truss in *Fig.* 93, or may have a straining beam as in *Fig.* 101, corresponding to the truss in *Fig.* 94.

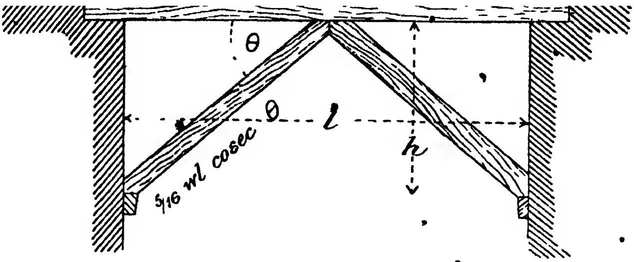


Fig. 100.

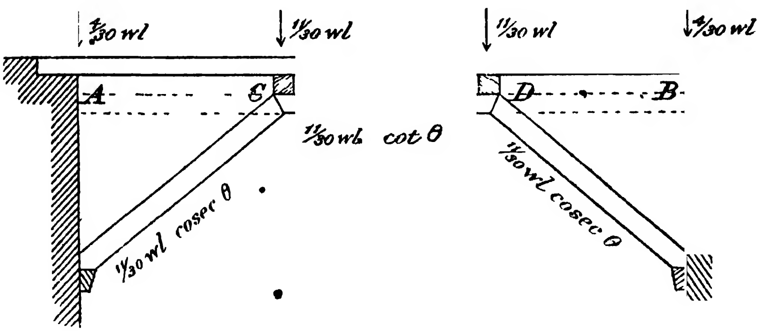


Fig. 101.

The triangular form in *Fig.* 95 or in *Fig.* 100 has a thrust down each inclined strut of $\frac{5}{16}$ W cosec θ, W $= wl$ being the total distributed weight on the span l. There is, therefore, at the foot of this strut a horizontal thrust of $\frac{5}{16}$ W cot θ, and an equal thrust along the horizontal beam itself. This thrust is combined with the transverse stress produced in the beam itself, which produces, as we have seen above, a maximum stress intensity of $\frac{1}{16}$ W$l \div bd^2$. Hence, as in Equation (i.), $r + r = \frac{1}{32}\frac{Wl}{bd^2h}(6h + 5d)$, and the thrust on the struts $= \frac{5W}{32h}\sqrt{l^2 + 4h^2}$. Triangular form.

When the beam is supported by inclined struts below, as in *Fig.* 100, the cleats or blocks at the foot of the struts must be designed to bear a vertical shearing stress of $\frac{5}{16}$ W.

Various forms in trapezoidal-strutted beam.

With the trapezoidal form of strutted beam we may either (*a*), have the form shown in *Fig.* 101 ; or (*b*), that in *Fig.* 102.

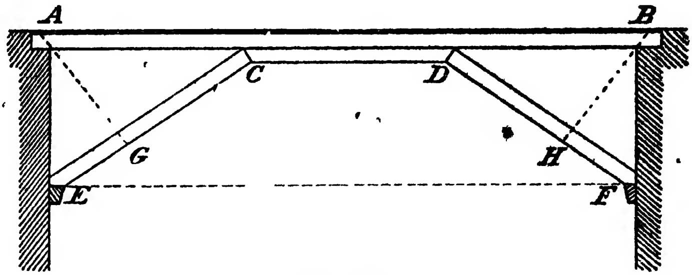


Fig. 102.

In the former case there are transverse joists c, c' transferring the weight to the frame below. AB is simply a beam continuous over three spans. As regards the struts and straining piece, they are all under direct stress only The stresses are as written on the members.

There is no object in having horizontal bars AC, DB.

The second case (*Fig.* 102) may be considered to be a beam continuous over two spans, where the central support (CD) is very wide, and the central pier consists of two inclined struts, EC and DF, and a straining beam CD The reaction at the central pier is approximately $\frac{2}{3}wl$, where w = weight per unit, and l = length of the beam. Hence CD is under combined stress, viz., the transverse stress of the weight $\frac{2}{3}wl$, and the direct stress $\frac{1}{3}wl \cot\theta$. As CD, regarded as a beam under transverse stress, may be reasonably considered as fixed at the ends, its M_{ft} will be $\frac{1}{12}W'l'$ (W' being the total distributed load

Stress on straining beam.

$=\frac{2}{3}wl$ and l' its total length, CD $=\frac{1}{3}l$) $=\frac{1}{54}wl^2$. As the $M_r = \frac{rbd^2}{6}$,

$$r = \frac{1}{9}\frac{wl^2}{bd^2}.$$

As r', or the intensity of direct compression

$$= \frac{wl}{3bd}\cot\theta = \frac{wl}{3bd} \times \frac{l}{3h} = \frac{wl^2}{9bdh},$$

we have

$$r + r' = \frac{wl^2}{9bd^2h}(h+d) \qquad \text{(x.)}.$$

Compression on struts $= \frac{wl}{3} \operatorname{cosec} \theta$ — Inclined struts.

$$= \frac{wl}{9h} \sqrt{l^2 + 9h^2} \quad \ldots \text{(xi.)}$$

Main horizontal beam.

The main horizontal beam AB is under transverse stress only, each of its outer segments AC, DB being in the condition of a beam supported at one end and fixed at the other, in which, when uniformly loaded, the M_B may be proved (Case 7, Chapter I.) to be $\frac{Wl'}{8} = \frac{wl^2}{72}$, and as $M_r = \frac{1}{6} r bd^2$,

$$r = \frac{wl^2}{12bd^2} \quad \ldots \text{(xii.)}$$

Diagonal pieces.

Sometimes diagonal pieces are given, as shown in dotted lines AG and BH, *Fig.* 102. The only object of these is to reduce the length of the struts EC, DF, and prevent buckling in the plane of the bridge.

If a tie rod be given as shown by the dotted line EF, its tension will be equal to the thrust on the abutments $= \frac{1}{3} wl \cot \theta$.

Example of strutted beam.

EXAMPLE 11.—*Design a pair of strutted beams, as in* Fig. 102, *to take a cart road* 10′ *wide, over a span of* 30′. *Timber, red fir. The struts to rise from a point* 8′ *below the horizontal beam.*

The value of w is taken as in former examples at 36.5 lbs. per lineal inch, $l = 360$ and $h = 96$ inches.

Main beam.

The maximum compression in the *main beam* AB is found from (xii.)—

$$r = \frac{wl^2}{12bd^2},$$

or

$$1200 = \frac{36.5 \times 360^2}{12 \times bd^2},$$

from which

$$bd^2 = 328.$$

Make

$$5 \times 8\tfrac{1}{2}.$$

Straining beam.

In the *straining beam* CD, from (x.)—

$$r + r' = \frac{wl^2}{9bd^2h}(h + d),$$

making

$$d = 8', \; b = 7\tfrac{1}{2}'.$$

Make

$$8'' \times 8''.$$

Struts.

In the *struts* CE and DF, the length is about 128″. The least dimension may be taken at 8″, so as to frame with CD.

The stress on the strut is, from (xi.)—

$$\frac{wl}{9h}\sqrt{l^2+9h^2},$$

whence, substituting values

$$\frac{36{\cdot}5\times 360}{9\times 96}\sqrt{360^2+9\times 96^2}=7012.$$

Since

$$128:8::16:1,$$

$$P=\tfrac{1}{2}\,r_c{\cdot}A \text{ or } 7012=600\times A,$$

∴ A = less than 12 square inches.

For practical reasons the struts may be made 8″ × 8″.

The Hurroo Bridge (*Plate* V.), built by General Sir A. Taylor, G.C.B., R.E., is an instance of this form of structure applied on a somewhat large scale. The spans are 40′ and the thrust on the piers is taken by wooden tie-beams. To prevent deflection in these, they are supported by iron queen-rods. The stress in the latter is very slight.

Deflection of Framed Structures.

Deflection of framed structures.

To ascertain the deflection of a trussed beam or any framed structure may appear almost impossible owing to the variations in the elasticity of the materials. The best method appears to be that proposed by Prof. Johnson, which is as follows:—

Proposition I.—The external work of distortion in a framed structure is equal to the internal work of resistance.

Professor Johnson's "pull over E" formula.

Proposition II.—The deflection D of any point in a framed structure subjected to a given load is given by the formula

$$D=\Sigma\frac{pul}{E}\quad\ldots\ldots\ldots\ldots\ldots(a),$$

where p = stress per square inch in any member for given load.
l = length of any member.
u = factor of reduction.

This is conveniently called the "pull over E" formula. The quantity $\frac{pul}{E}$ is computed for every member, and D = the algebraic sum.

Proposition III.—When there are two or more paths over which a load may travel to reach the support, the load divides itself among the several paths strictly in proportion to the rigidity of the parts.

"The relative rigidities of the paths are indicated by the relative loads required to produce a given deflection, or they are inversely as the deflection

produced by a given load which is required to pass wholly over each path in succession. Having established this proposition and computed the rigidities of the paths by Proposition II., we can write enough quotations of condition to enable us to solve for any number of redundant members. This proposition applies to all structures whether framed or composed of masonry, as in the case of a curved masonry dam."*

If we call W the external load, P the stress in any member, Z the distortion produced by P, and d the deflection in the truss produced by the distortion of that one member, then

$$W:P::Z:d;$$

for we may assume the structure to be perfectly rigid except one member, then all the deflection in the truss will be due to the distortion in that one member. Hence the total deflection D = the sum of all these deflections.

Now since E = ratio of stress to strain = the total stress on the member ÷ total increment produced by that stress—

$$E=\frac{pl}{Z} \quad \text{or} \quad Z=\frac{pl}{E}.$$

Also since W may be any load placed at the loaded point we may make it one pound, and then calling the resultant stress in the member due to this one pound u, we have

$$\frac{P}{W}=\frac{u}{l}.$$

Hen[c]e

$$\frac{P}{W}, \quad Z=\sum\frac{pul}{E}.$$

In applying this we know p, l and E, but not u. "Since this is a pure ratio equal to the stress in that member for one pound placed at the point, we simply put 1 lb. at the point whose deflection is desired and find the resulting stress in every member, either analytically or graphically." The formula of course takes no account of bending stresses.

To apply this to the simplest case, viz., the trussed beam in Example 9. Application to simplest form.

The problem is to find the deflection, when the stress intensity in the timber, $p_c = 800$ lbs., and in the iron rods 5 tons, say 10,000 lbs. p_t. Then the unit stresses in the various bars in the structure for a load of 1 lb. placed at the centre of the span are as shown on *Fig.* 103.

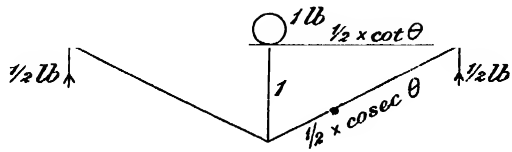


Fig. 103.

Then calling E and E′ the moduli of elasticity for timber and steel (say

* *Theory and Practice of Framed Structures*, p. 220.

1,400,000 and 30,000,000 lbs. respectively), l the length in inches and h the length of the strut or total depth of the truss, we have for the upper bar

$$\Sigma\frac{pnl}{E}=p_c\times\frac{l}{2h}\times\frac{l}{2}\times\frac{1}{E},$$

for the strut $\qquad\qquad$,, $=p_c\times 1\times h\times\frac{1}{E},$

for the tie rod $\qquad\qquad$,, $=p_t\times\frac{l2\times 4h^2}{8h}\times\frac{1}{E'}.$

Hence total

$$=\frac{p_c}{E}\left(\frac{l^2}{4h}+h\right)+\frac{p_t}{8hE'}(l^2+4h^2)=(l^2+4h^2)\left(\frac{p_c}{4hE}+\frac{p_t}{8hE'}\right).$$

Substituting values

$$=(192^2+4\times 50^2)\left(\frac{800}{4\times 50\times 1400000}+\frac{10000}{400\times 30000000}\right)=\cdot 518 \text{ inches.}$$

The objection to the above treatment is that it involves taking average values for the tensile and compressive stresses (p_t and p_c). These can only be approximately computed, but as the assumed working loads are generally greater than the actual loads, it is a justifiable method of working.

General Remarks on Trussed Beams.

Struts generally large for practical reasons.

The struts in trussed beams are generally made much larger for practical reasons than is necessary to satisfy the requirements of theory. From the formulæ in this chapter it will be readily seen that the greater the depth given to the truss as a whole, and consequently the greater the angle between the horizontal beam and the tension bars the less will be the stresses on all the members.

Travellers.

Figs. 104 and 105 shows forms of travellers, which are sometimes made with three struts. The principles of construction would be the same as those considered above. Travellers which are built up of continuous plate webs, and flanges, would probably be more frequently used where heavy weights had to be handled, rather than the braced form here shown.

Braced purlins.

Where purlins are braced, a form of construction which can at times be used in roofs with considerable economy, there is no necessity for counter-bracing, as the load is uniformly distributed. In warehouses for carrying heavy weights the floor girders may sometimes be advantageously strengthened by bracing.

Camber.

Camber is given to trusses chiefly for the sake of appearance. The amount given is generally a proportion of the span, but should theoretically = the deflection when fully loaded, so that when the truss is loaded it may not have an appearance of weakness. Giving a camber not only adds no strength to the truss, but if the truss is

tightened up in order to produce the camber there are initial stresses produced in all the members which must be deducted from their available capacity to resist the stresses produced by loads.

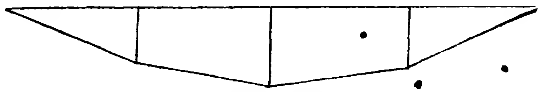


Fig. 104.

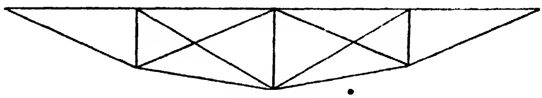


Fig. 105.

The various forms of beams considered in this chapter present, in the crudest and most elementary forms the principal methods of bridge design. (1). The trussed beam with load above and trussing below (*Figs.* 92 and 93), is the simplest type of what are known as *deck* bridges, where the weight is carried on the upper flanges. (2). The trussed beam with trussing above the line of the load (*Figs.* 95 and 96), is the simplest form of *through* bridges. And (3), although in the strutted beam we have only shown two forms (*Figs.* 100 and 102), one with the strutting in two, and the other in three pieces, yet it is evident that the number of strutting pieces might be indefinitely increased, forming ultimately the *arch*. There is still another method (4), the converse of the strutted beam where the load is suspended by tension bars or chains, as in *Fig.* 106, instead of being strutted from beneath. This gives the simplest form of *suspension bridge*.

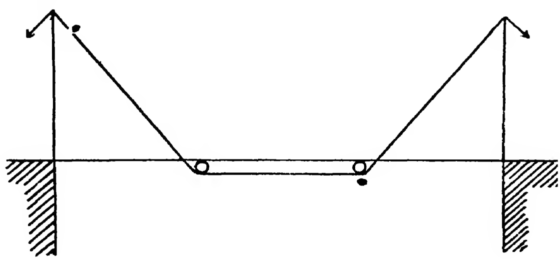


Fig. 106.

It will be our task to examine the detailed development of these various forms in following chapters.

APPENDIX I.

Major Love's proof of loaded trussed beam.

THE following investigation of the trussed beam is given by Major H. D. Love, R.E., in the *R.E. Journal* for August, 1897 :—

The loaded point B of the continuous beam ABCD (*Fig.* 108) is deflected and falls below the horizontal line AD, and from the symmetry of the truss the point C must rise an equal distance above this line. Hence there is a point of inflection or virtual joint at O. The M_f diagram in the beam is shown in *Fig.* 107. The outline of the truss AEFD forms a M_f diagram suited to a certain loading (Case 8, p. 72, Part I.). But the M_f diagram due to the load W on the truss is a triangle AGD, and the two diagrams must intersect in the points A, O′, O, and D, where the M_f in the beam = 0. Hence the ordinates of the shaded area give the bending moments which must be resisted by the beam itself.

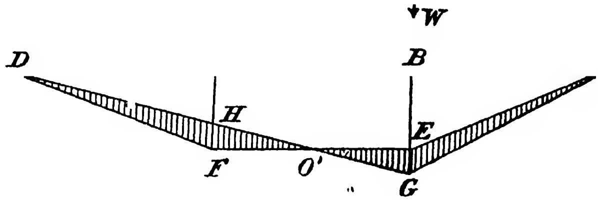


Fig. 107.

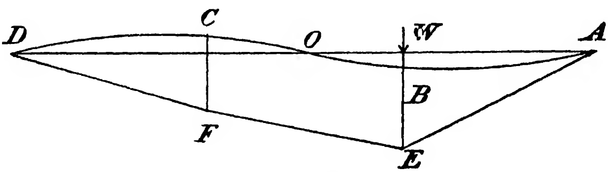


Fig. 108.

It is easily seen that EG or FH

$$= \tfrac{1}{4}\,BG = \tfrac{1}{4} \times \tfrac{2}{3}\,W \times \frac{l}{3} = \frac{Wl}{18}.$$

But this is the M_f which would be produced by a load $\frac{W}{2}$ at B in a supported beam of a length $\frac{l}{2}$. The stress in the strut BE is therefore $= \frac{W}{2}$. Also since the M_f at C = M_f at B and the former is due to the thrust of the strut CF, it follows that the stress in this strut must $= \frac{W}{2}$.

If θ be the inclination of the tie-rods to the horizontal and h the depth of the truss at the centre—

$$\text{Tension in sloping ties} = -\frac{W}{2}\operatorname{cosec}\theta.$$

$$\text{,, horizontal ties} = -\frac{W}{2}\cot\theta.$$

$$\text{Thrust in beam} = +\frac{W}{2}\cot\theta = \frac{W}{2}\,\frac{l}{3h}$$

The stress in the horizontal tie may be checked by taking moments about O. Call this stress P

$$\tfrac{2}{3}W \times \frac{l}{2} - W\frac{l}{6} = P \times h\,.$$

$$P = \frac{Wl}{6h}$$

The maximum compressive stress in a rectangular beam of section b, d, assuming that the points BC do not deflect sensibly, is

$$\frac{M_f y}{I} + \frac{Wl}{6hbd} - \frac{Wl}{18} \times \frac{6}{bd^2} = \frac{Wl}{6bd^2h}(2h + d),$$

which is the same as Equation (vii), p. 102.

CHAPTER V.

Braced Girders, with Parallel Flanges.

Alternative Methods of Enquiry.—Types usually found in Practice.—Methods of Investigation.—Simple Method for Symmetrical Loads.—Rules for ascertaining Stresses.—Application of Rules to Cantilevers.—Fixed and Continuous Braced Girders.—Unsymmetrical and Rolling Loads.—Calculation of Strength of a given Braced Girder.—Example of Simple Road Bridge.—Major Bate's Portable Bridges.

Braced Girders

Methods of investigation.

In the investigation of built-up girders with parallel flanges we may pursue one of two alternative methods of enquiry.

(A). We may regard the structure as one beam, and proceed by following the laws for transverse stress, regarding the booms or flanges as taking the whole of the resistance to the Moment of Flexure, and the members which form the bracing as resisting the shearing stress.

(B). We may view the whole as a braced structure pure and simple. We may dismiss all ideas of transverse stress on the structure as a whole, and proceed on exactly the same lines as we should adopt in the case of a braced roof.

Evolution of the type.

The former method of investigation is representative of English practice. The simple beam has, by a process of evolution, become refined, first into a rolled beam of I section, then into a plate girder, with the material arranged economically in the flanges, but with an excessive quantity still in the web, and, lastly, into the lattice web or braced girder form, the whole being fastened together as securely as possible by rivetting at the joints.

In America, the braced girder was evolved from wooden or composite trusses, and the term "truss" is still used in that country for what is in England called a girder. Each member in the truss is treated as having its own individual function to perform, and the joints are usually pin and link connections.

In Chapter III. we have investigated the girder in its penultimate stage of development, and in Chapter IV. we have examined the simplest forms of braced structures, as applied to bridges. We are therefore now in a position to use either mode of investigation in the examination of braced girders, and will find that there are advantages in thus having two avenues of approach to the same subject.

A few of the forms which are ordinarily used for this class of design are shown on *Plate* VI. The N, Whipple-Murphy, or Pratt truss, is shown on *Fig.* 1. All the verticals in this truss, as shown, are struts, the diagonals ties.* If the diagonals were sloping the other way, as in *Fig.* 2, they would be struts, and the verticals ties. The former method has the advantage that in it the struts are of minimum length. On the other hand, with timber and iron bars, the joints are more easily made with the form in *Fig.* 2. This form is one of the best types of braced girder for temporary work. The roadway may be either on the upper or lower boom, the former for preference, if there is sufficient headroom below. Ordinary forms

A combination of two or more systems of triangulation of this form is often used, as shown in *Fig.* 3. This is sometimes called the Linville truss. See also *Fig.* 8. Linville girder.

Where all the braces are inclined at the same angle, and there is one system of triangulation, the type shown in *Fig.* 4, called the Warren girder, is produced. This is much used in India, and is there the standard type for railway bridges from 100′ to 150′ span. Warren girder.

Two Warren girders, as in *Fig.* 5, form the simplest type of lattice or trellis girder. Here there are two web systems of triangulation, but there are sometimes more than two systems. The infinite multiplication of such systems would, of course, eventually become the continuous plate girder.

Sometimes the Warren girder is made with verticals at intervals, as in *Fig.* 6. The form shown in *Fig.* 7, and called the Howe truss, is also frequently found, but either the vertical bars, or else half of the diagonals, are redundant, except as regards rolling loads. The investigation of this presents peculiar difficulties, which will be noticed later. Howe truss.

These are the usual types found in practice. Other types would be investigated on the same lines as these.

* In the diagrams of this and other chapters struts are shown in thick lines, sometimes in red. Ties are shown in thin lines, or else in blue.

Methods of Investigation.

When we come to consider the method of ascertaining the stresses in the various members, we find that there are several ways of doing so. These may be briefly stated as follows:—

1. Lathom's method. 1. *Lathom's Method.*—The reaction produced at each support by each portion of the load acting at each panel* point is first ascertained by the ordinary rules for reaction in supported beams, and the effect of that reaction on every bar of the truss considered. These effects are tabulated, and the algebraic sum taken as the resultant stress on the bar in question. This method is very laborious, and though it was some 30 years ago universally used, is now as universally discarded.

2. Clerk Maxwell's method. 2. *Clerk Maxwell's Method.* — This consists in drawing stress diagrams for the truss in the same way as described in Chapters XI. and XII., Part I., for roof trusses. This, although convenient for dead loads, would involve, in the case of rolling loads, a fresh diagram for every position of the load, and would therefore involve much labour.

3. Claxton Fidler's method. 3. *Claxton Fidler's Method.*—This consists in regarding the girder as an extension of the trussed beams described in Chapter IV., and applying certain simple rules for the stresses.

4. Diagrams for M.; 4. By regarding the whole as a beam, and using the diagrams for the bending moments and shearing stresses, as in the case of a plate girder. This is the method used by French engineers.

5. Method of sections. 5. By the method of sections, as described in Chapter XII., Part I. This is in all cases useful as a check, and is largely used by American engineers.

We may, of course, use any combination of the above that may help us to obtain the required information as quickly and as exactly as possible.

The first and second methods may be put on one side as being tedious. The fourth and fifth methods are very useful, but need not be specially described, as the subject has already been discussed for other forms of structures. The third method is very simple and easy for all uniform loads, and for loads symmetrical with reference to the centre, and may now be described in general terms.†

* Panels are the divisions into which the girder as a whole is divided by the intersection of the longitudinals with the braces.

† The process here described is not exactly that given by Mr. Claxton Fidler, but is based on his methods. It is partly derived from French writers.

In *Fig.* 109, if we conceive the weight W to be suspended from the apex of the triangle *abc*, which in its turn is suspended from the points *d* and *e* of the trapezium *hdef*, we see that the weight W is transmitted in equal parts to the abutments at *h* and *f* by the bars *ba*, *ad*, *dh*, and *bc*, *ce*, and *ef*. We see also that the bars *ba*, *bc*, *dh* and *ef*, which slope upwards towards the centre, are struts, and that the vertical bars are in tension. **Single weight on a system of bars.**

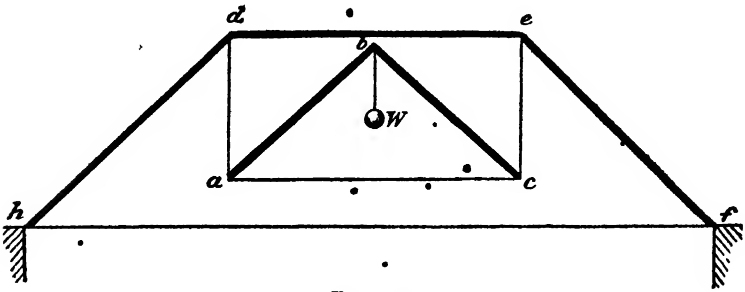


Fig. 109.

If, however, the framework be inverted, as in *Fig.* 110, the *amount* of the stresses remains the same, but the *nature* of those stresses is altered. The bars which slope downwards towards the centre are ties, and the verticals are now in compression. The path along which the weight W is transmitted to the abutments is the same as before, and we still see that the upper horizontal bar of the framework is in compression, the lower in tension.

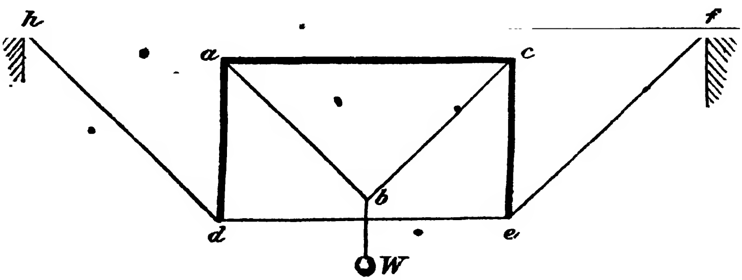


Fig. 110.

If we take the arrangement of bars represented in *Fig.* 111, which is a further extension of the same principle, we see that the triangular

beam *abc* is supported by vertical bars *ad*, *ce* on the trapezium *hdef*, and that this, in its turn, is supported by vertical bars at the points *i* and *g* on the trapezium *jigk*.

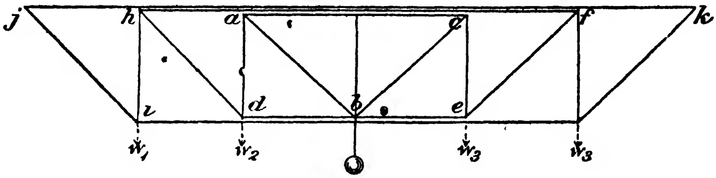


Fig. 111.

Further, we see that the stresses on all the *inclined* and *vertical* bars are simply those due to the portions of the weight W transferred to the abutments, but that if we consider the *horizontal* members as bound together, or otherwise united into one, the stress between *a* and *c* will be the greatest in the whole structure, and those on *jh*, *id*, *cg* and *fk* will be the least.

It will be noticed that this is in exact accordance with the rules for Moments of Flexure and shearing stress in a supported beam.

Several equal loads equidistant from centre.

If we were to put equal loads w_1, w_2 .....at the points *i*, *d*, *e* and *g*, it is evident that the stresses produced by such loads must be additional to those already existing in *ih*, *hd*, *da*, *ab*, *bc*, etc. Further, any part of the weight w_2 conveyed from *d* towards the right abutment will, in effect, be neutralized by an equal portion of the weight w_3 at *e* conveyed in the opposite direction towards the left abutment, or (to express the same fact more scientifically), $\frac{2}{6}$ of w_2 will cause compression in *ab*, while $\frac{2}{6}$ of w_3 will cause tension in *ab*. These will neutralize each other. Again, $\frac{4}{6}$ of w_2 will cause tension in *hd*, while $\frac{2}{6}$ of w_3 will also cause tension in *hd*, hence we may consider *all* the weight w_2 to travel along *hd*, *hi* to the left abutment, and *all* the weight w_3 at *e* may be considered to pass along *ef*, *fg* to the right.

But although on this reasoning the increments of weight w_1..... w_4, will have no effect on the central web bars, they have an appreciable effect on the central horizontal bars. For any increment in the stresses in *ef*, *gk* will produce corresponding increments in *hf* and *jk*, and consequently in *ac*, if we consider the horizontal bars all bound together in one.

Stresses may be written on any number of symmetrically loaded girders.

From these premises we deduce the important practical fact that *in any braced girder with symmetrical loads we can by simple addition write down the amount of the stresses on any member of the girder, and by inspection can ascertain the nature of that stress.*

In the case which we have been considering, let us suppose the girder loaded with weights w, w......at h, a, o, c and f, and with $\frac{1}{2}w$ at j and k. Then, starting with the web bars from the centre, we see that the stress on ob (*Fig.* 112) will be w, and that half of this will be transferred to each abutment by the bars ba, bc. The stress on each of these will consequently be $\frac{1}{2}w \times$ the cosecant of the angle which the bar makes with the horizontal. At ca fresh increment of weight comes in, hence the stress in ce is $1\frac{1}{2}w$, and in like manner the stress in ef is $1\frac{1}{2}w$ cosec θ. Similarly, the stresses in fg and gk are $2\frac{1}{2}w$ and $2\frac{1}{2}w$ cosec θ respectively.

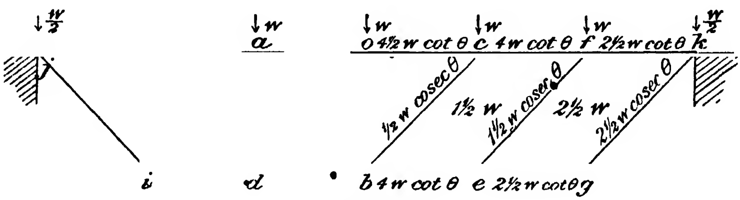


Fig. 112.

As the stress in gk is $2\frac{1}{2}w$, the stress in kj, and also in ge, must, for equilibrium, be equal to $2\frac{1}{2}$ cot θ, from the principle of the triangle of forces. The stress on cf must be the sum of the stress in fk and the horizontal component of the stress in cf - $2\frac{1}{2}w$ cot $\theta + 1\frac{1}{2}w$ cosec $\theta \times \cos\theta = 2\frac{1}{2}w \cot\theta + 1\frac{1}{2}w \cot\theta = 4w \cot\theta$.

For the same reason this will also be the stress in b . The stress in oc will, on the same reasoning, be $4\frac{1}{2}w \cot\theta$.

To ascertain the *nature* of the stresses, we know that all bars sloping downwards to the centre must be in tension, that therefore the verticals will be in compression, that the upper boom will be in compression, and the lower will be in tension.

Rules for Ascertaining Stresses in Symmetrically Loaded Girders.

General rules for stresses in various members.

The general rules for stresses in symmetrically loaded braced girders with parallel flanges may be, therefore, as follows :—

(1). Write on each inclined bar cosec θ (or any symbol such as r), and on such horizontal bar cot θ (or any symbol

such as t), θ being the angle of inclination of the bar to the horizontal.

(2). Working along each series of triangles, for each inclined or vertical bar write the vertical weight between it, on its own series, and the centre of the span. In the case of inclined bars this will be multiplied by cosec θ, already written.

(3). In any horizontal bar write the sum of the horizontal components of all the bars meeting the bar in question at the end nearest the abutment.

(4). All bars sloping upwards to the centre are struts, all sloping downwards to the centre are ties. Vertical bars are struts when the sloping bars are ties and "vice versa." They change sign, however, in the centre (i.) with distributed loads; and (ii.) at the weight, in the case of a concentrated load. The upper horizontal bars are struts, and the lower horizontal are ties.

It must be remembered that these rules *do not apply to cantilevers*, nor to beams continuous over several spans. These would be investigated by similar methods, and will be explained presently.

The girders shown in *Figs.* 2, 4 and 6, *Plate* VI., show the stresses written against the members according to these rules.

Rules correspond to those for M_f and shearing.

These rules correspond, it will be observed, with those for shearing stresses and Moments of Flexure. The inclined bars, taking the shearing stresses, transfer the horizontal portions of those stresses to the booms, and the vertical portions to the abutments. They are a minimum in the centre, a maximum at the abutments, corresponding with the ordinates in the diagram of shearing stress in a symmetrically loaded beam (*Fig.* 113), whereas the stresses in the booms are a minimum at the abutments, and a maximum at the centre corresponding with the ordinates to the M_f curve, as in *Fig.* 114. This curve will, indeed, give us the stress at any point of the booms, for we know that the ordinates show the M_f which is = the Moment of Resistance = ad, and as d, the depth of the girder, is constant, the ordinates give us graphically the values of the varying stresses, provided we make on any convenient scale the central ordinate $= Wl \div 8d$, where W is the total weight on the beam, l and d the length and depth.

The shearing stress diagram (*Fig.* 113) shows, by its ordinates, the value of the vertical or horizontal shearing stress at any point. In order, however, to find the actual stress coming on any inclined bar

such as *bc*, *ef*, *gk* (*Fig.* 112), we draw from *b'*, *e'*, *g'* (points in the shearing stress diagram corresponding to *b*, *e*, *g*) lines *b'c'*, *e'f'*, and *g'k'* parallel to the bars *bc*, *ef* and *gk*. The lengths of these lines represent the stresses on *bc*, *ef* and *gk*.

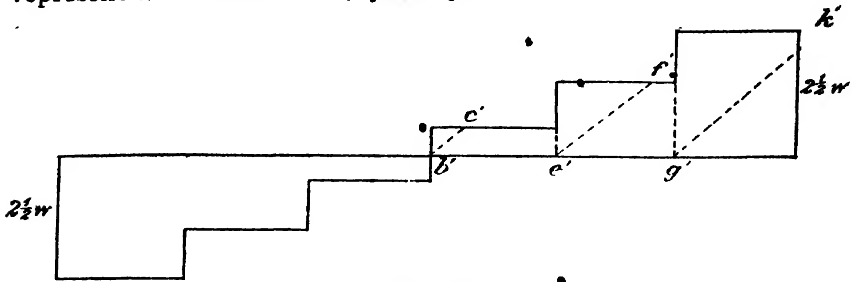


Fig. 113.

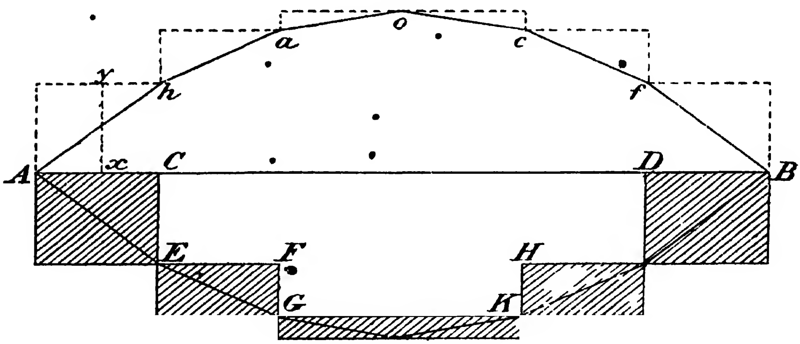


Fig. 114.

Similarly, *Fig.* 114 represents the M_f diagram. Let this be drawn both above and below the beam line AB. Through the panel points on the M_f diagram *above* AB (viz., *h*, *a*, *o*, *c*, *f*) draw dotted lines parallel to the beam line and meeting verticals through those panel points. Then the ordinates to the dotted lines represent the stresses in the various members of the girders, *e.g.*, any line *xy* between A and C represents the compression in the upper boom of the girder in the end panel ($2\frac{1}{2}w \cot \theta$). For the lower part of the diagram the stresses in the members of the lower boom are represented by the ordinates to the diagram CEFG......D, where the shaded rectangles represent by their ordinates the horizontal components of the stress in the inclined bars *gk*, *ef*, etc. (*Fig.* 112).

It may be objected that the ordinates of a M_f diagram represent *moments*, not forces or stresses. But in the case of a girder with parallel flanges, the M_f diagram is also a flange stress diagram, because $M_f = M_r = md$, and as d is constant $\frac{M_f}{d} = m$. Hence any ordinate, such as CE, divided by the constant depth d, represents the flange stress.

Explanation of some points in rules.

There are one or two matters in connection with the rules given above which may require explanation. Rule 2 regards each series of triangles as a path along which the vertical loads are transmitted to the abutments. Thus in *Fig.* 115, which represents a half elevation of a Warren girder with two series of triangulations (one drawn dotted to distinguish it from the other), the stress on AB is $2w$ cosec θ, because $2w$ is the weight between AB and the centre, on the series of triangles (dotted), of which AB is a part. If the girder had been as in *Fig.* 116, with exactly the same loading, etc., the stress on AB would now be $4w$ cosec θ, because there is now one series of triangles, and the weight between AB and the centre on that series of triangles is $4w$. In *Fig.* 116 the stress on AB is, therefore, twice that in *Fig.* 115.

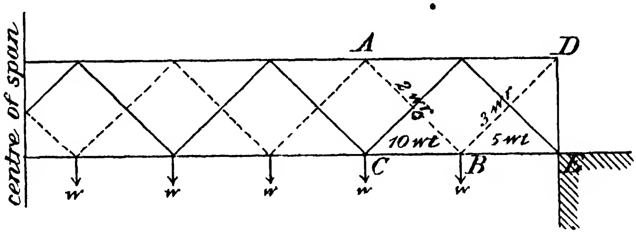


Fig. 115.

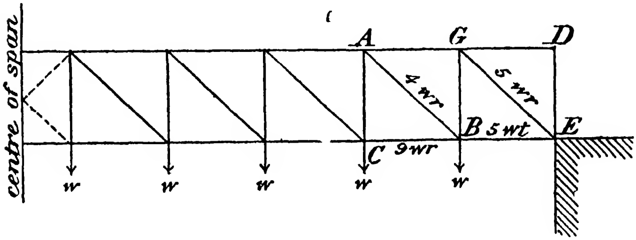


Fig. 116.

But, comparing the stress on CB in both cases, we see that it is

much more nearly equal, *i.e.*, in the first case it is $10w \cot \theta$, and in the second case it is $9w \cot \theta$. In *Fig.* 116 the bars GD, DE are superfluous.

If the same girder were designed as shown in *Fig.* 117 (which is not a good form owing to the increased length of the compression bars FA, AD, etc.), the verticals AC, FG simply transmit to the points A and F the weights suspended from them, and do not form a part of the path by which the weights are transmitted to the abutments. Here the stress on AB would be again $4w \operatorname{cosec} \theta$, and the tension on CB will now be $14w \cot \theta$. (See also *Fig.* 6, *Plate* V.).

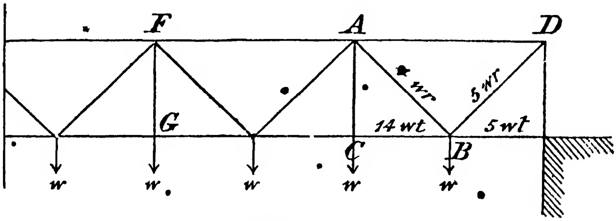


Fig. 117.

When a double series of triangles are used in the N girder, as in *Fig.* 118 (a common design in large bridges, *e.g.*, the Attock Bridge over the Indus, and the Dufferin Bridge over the Ganges), there is generally one bar at each end, as DB, which is at a different inclination from the others. The tension in this bar is $5\frac{1}{2}w \operatorname{cosec} \theta$, since there are $5\frac{1}{2}$ times w between it and the centre of the bridge on its N girder with double series of triangles.

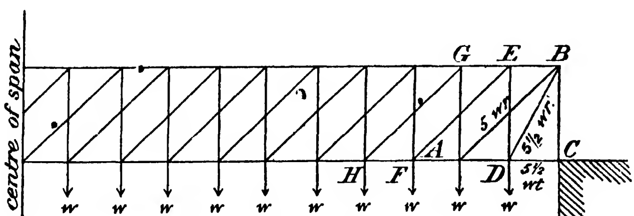


Fig. 118.

own series of triangles. Similarly, the tension on AB is $5w \operatorname{cosec} \theta$. The compression on the end pillar CB is $10\frac{1}{2}w$, and the tension on the lower boom is $5\frac{1}{2}w \cot \theta'$, while the compression on EB is $5\frac{1}{2}w \cot \theta' + 5w \cot \theta$.

The tension on AD is equal to the compression on EB, while the compression on GE = tension on HF $= 5\frac{1}{2}w \cot \theta' + 9\frac{1}{2}w \cot \theta$. The summation goes on on both booms in the same way, $5\frac{1}{2}w \cot \theta'$ being a constant increment in each case.

Check by stress diagram

In case of doubt, one can easily verify one's result by making a stress diagram of the forces acting at the point in question. For instance, taking the forces acting at B (*Fig.* 118), we know that $\theta = 45°$ and $\theta' = 64°$ (nearly), and that BC is $= 10\frac{1}{2}w$. Drawing the forces as in *Fig.* 119, with the usual notation, EB is the only unknown. It is found from the stress diagram (*Fig.* 120), where

$$\begin{aligned} eb &= ef + fb = ed \cos \theta + cd \cos \theta' \\ &= 5w \operatorname{cosec} \theta (\cos \theta) + 5\tfrac{1}{2}w \operatorname{cosec} \theta' (\cos \theta') \\ &= 5w \cot \theta + 5\tfrac{1}{2}w \cot \theta'. \end{aligned}$$

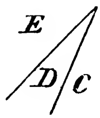


Fig. 119.

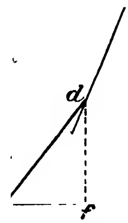


Fig. 120.

Application of Rules to Cantilevers.

Rules for cantilevers.

In cantilevers the rules given above require to be modified as follows:—

1. Same as given before.

2. Working along each series of triangles, for each vertical or inclined bar write the vertical weight between it, on its own series, and the free end of the cantilever.

3. In horizontal bars work from the free end towards the abutment, and write on each the sum of all the horizontal components of all the bars, meeting the bar in question at the end furthest from the abutment.

4. All bars sloping upwards to the abutments are ties, and downwards to the abutment are struts. Vertical bars are struts when the adjacent sloping bars are ties, and *vice versâ*. The upper horizontal bars are ties, and the lower horizontals are struts.

Fig. 121 shows a cantilever with a load $= w$ on each panel point, and stresses written on each bar.

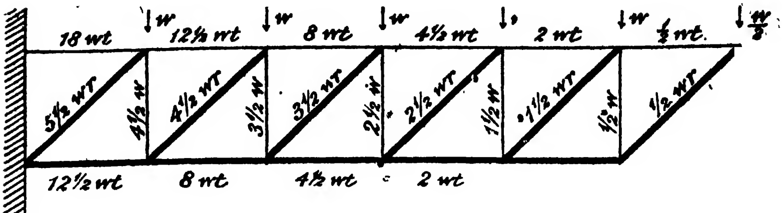


Fig. 121.

Parallel flanges not economical in cantilevers.

Cantilevers are rarely built with horizontal flanges because of the maximum stress in the booms at the abutments necessitating extra plates. This will be apparent if we compare *Figs.* 123 and 125. *Fig.* 122 represents a cantilever with parallel flanges of 7 panels, each panel load 2 tons and a weight of 4 tons at the end. *Fig.* 124 represents a cantilever with the *same* number of panels, the *same total length* of arm, and the *same loads*, but with the depth increasing from the extremity to the abutment. To a casual observer it might appear as though *Fig.* 122 were a more economical construction than *Fig.* 124. But when we examine *Figs.* 123 and 125, which

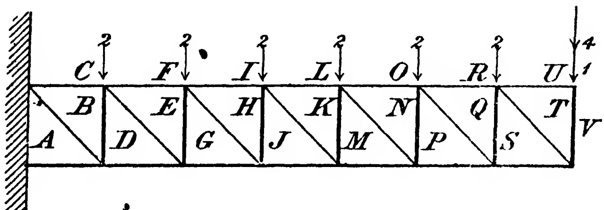


Fig. 122.

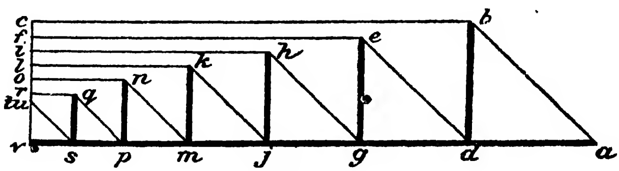


Fig. 123.

represent the respective stress diagrams drawn to the same scale, and with the letters corresponding in each, we see that, whereas the

maximum stress on any member such as AV is, in *Fig.* 122, 77 tons, in *Fig.* 124 it is only 19 tons, while the stresses on all the other members are correspondingly less.

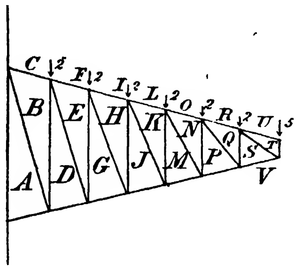


Fig. 124.

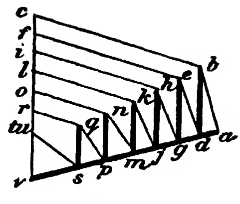


Fig. 125.

This is the reason why, in bridges built on the cantilever principle, such as the Forth Bridge, the Lansdowne Bridge over the Indus at Sukkur, etc., the cantilevers are not built with parallel flanges.

The principle is, in fact, based on the same grounds as the modifications for roof trusses, mentioned in Part I., p. 283.

Fixed and Continuous Braced Girders.

Fixed and continuous girders.

The investigation of fixed and continuous beams, built as braced girders with parallel flanges, follow from the consideration of the Moments of Flexure and shearing stresses in these beams.

The shearing stresses are the same in fixed beams as in beams under similar conditions of load, but supported only. Hence the stresses on the web bars, whether inclined or vertical, of a braced girder with parallel flanges are the same under similar conditions of load and span, whether the beam be fixed or supported. The stresses in the booms will be a minimum at the points of contra-flexure, and, working from those points to the centre and to the abutments respectively, they may be written down as in the case of supported beams and cantilevers.

Take for instance the fixed girder AB with 10 panels, shown in *Fig.* 126, with a load of w at each panel point. We first find the points of contra-flexure C and D. These we know to be distant from A and B nearly* $\frac{1}{5}$ of the span (see Table I., p. 4). We there-

* Between 0·211 and 0·25.

fore write down the stresses on the girder between C and D as if supported at those points, and on the cantilevers AC, BD as being loaded with half the weight on CD at their extremities, as well as the load w at each panel point.

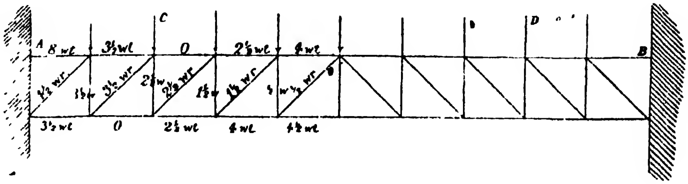


Fig. 126.

The loads are written for each member, for half the girder, on *Fig.* 126. *Fig.* 127 shows two cantilevers AC and BD with a central girder CD. The stresses in the members are the same in the one case as in the other, the two structures being designed on similar principles.

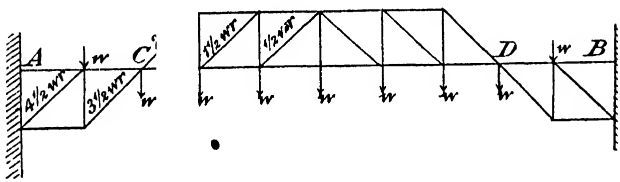


Fig. 127.

A similar procedure would be carried out in the case of continuous beams. Or the M_f diagrams and shearing stress diagrams might be drawn for the whole continuous girder (as in *Plate* II), and on those loads might be super-posed the diagrams for the Moment of Resistance of the plates, which it is proposed to use, in exactly the same way as pointed out in the previous chapter for plate girders.

With beams subject to moving loads, the shifting of the points of contra-flexure is such a serious practical drawback that it is comparatively rare that braced girders with parallel flanges are designed on the continuous principle. This disadvantage is not quite so great practically as theory might lead one to suppose, because the practical limits of thickness of plates, etc., demand some minimum sizes of boom at the points of contra-flexure, and these

Shifting of points of contra-flexure.

minimum sizes may be sufficient to cover the variations in the position of the points of contra-flexure. M. Pascal, in his very useful *Traité Pratique des Ponts Metalliques*, gives various examples of both road and railway bridges, built on the lattice girder principle continuous over several spans, where the metal is arranged so as to include by its Moment of Resistance the varying Moments of Flexure produced by loads at various positions. But it may be doubted whether much real economy is effected by the use of continuous girders in bridges of spans below 150′.

Case where continuous beams may be used.

Sometimes, where the span is necessarily great and not subject to rolling loads, *e.g.*, the roof purlins of a railway station, braced girders on the continuous principle, built up with parallel flanges, can be most economically employed.

Unsymmetrical Loads and Rolling Loads.

Rolling loads.

If we take the Warren girder shown on *Fig.* 1, *Plate* VII., loaded with a uniform load *w* at each lower apex, we have the stress diagram shown on *Fig.* 2. If we consider the effect of a single load W at any one of these apices (*Fig.* 3 and *Fig.* 5), we see that the magnitude of the stresses in all the members will vary with the position of the load, and also the *nature* of the stresses in some of the members will vary. If the load be in the centre, the stress diagram will be as shown in *Fig.* 4, all the bars sloping upwards towards the centre being in compression, and those sloping downwards towards the centre being in tension, as in the case of a uniform load. But when the load has moved to the apex nearest to the abutment on the right (*Fig.* 3), the stress diagram is as shown in *Fig.* 5. In this case we must first find the reactions at each support according to the ordinary rules on the subject, and having found those reactions, the drawing of the rest of the stress diagram (*Fig.* 5), is easy. We see that *if a vertical line be drawn through the weight, all bars sloping upwards towards this vertical line are in compression, and "vice versâ."* The bars *km*, *no*, which were formerly in tension, are now in compression, while the bars *mn*, *oq*, formerly in compression, are now in tension. If, now, *Fig.* 2 represents to the same scale the esses for the dead load on the girder, it appears that there will be some bars towards the centre of the span which may be in tension or compression, according to the position and magnitude of the rolling load. For example, the bar *mk* has a constant amount of tension due to the dead load (*Fig.* 2), and it has, under certain circumstances

of the rolling load, a definite amount of compression. If, then, the latter is greater than the former, it is clear that the bar must be designed to bear either class of stress.

This is the simplest case of what is called counter-stress. Where the dead load on a structure is great in comparison with the moving load, the numbers of bars which may be called upon to bear either class of stress will be small, and *vice versâ*. It will be clear that where rolling loads are anticipated on a structure there will be in the central bars a reversal of stress, which will necessitate a high factor of safety, on account of the fatigue of the metal produced. **Counter-stress**

To find the limits of the counter-bracing required in any given case, perhaps the simplest plan is to draw a shearing stress diagram, since the shearing stresses are borne by the web bars. In the case considered above, where there is a single rolling load W and a uniform load wl, due to the weight of the girder and superstructure, all that we have to do is to draw AB to any scale representing the length of the girder, AC on any scale of weight $= \frac{1}{2} wl =$ BD, AE = BF = W on the same scale. Join CD (*Fig.* 6), EB, AF. Draw CC′ and DD′ = AE and BF, and join C′D and CD′ cutting AB at G and H. The points G and H will indicate the limits within which counter-bracing is required, because within those limits there may be either positive or negative shearing stress. This reasoning applies to all classes of rolling loads. **Limits of counter-bracing. Single**

Let us examine this matter a little further. The loads which we usually have to consider in a bridge are uniformly distributed, coming from one side or the other until the whole bridge is covered. From the principles explained in Chapter II. we can draw the shearing stress diagram for such a load, and this diagram must be added to the shearing stress diagram for the uniform load produced by the actual weight of the bridge.

For instance, *Fig.* 128 represents the shearing stress diagram for a beam AB of weight $W = wl$, producing a shearing stress diagram ACODB. If this beam be subject to a passing load also of weight w per unit of length, the positive shearing stress will be represented by AEH, and the negative shearing stress by GBF. Between G and H there may be either class of stress. If the load advances from left to right the shear at the head will be negative, if from right to left the shear will be positive, hence between the limits, where either class of stress may occur, there should be counter-bracing. To show how the positive and negative shearing stresses affect the inclined bars, let ABCD represent (*Fig.* 130) a panel in a girder, and let the shearing **Distributed rolling load.**

stress on the girder at the panel be positive, as shown by the arrows. The bar AC is evidently in compression. If the shearing stress were negative, the bar AC would either have to be capable of bearing tension, or another bar BD, shown dotted, would have to be introduced as counter-bracing.

If the rolling load weigh $2w$ per unit, the stress diagram will be as on *Fig.* 129, and the points G and H are further apart than in *Fig.* 128.

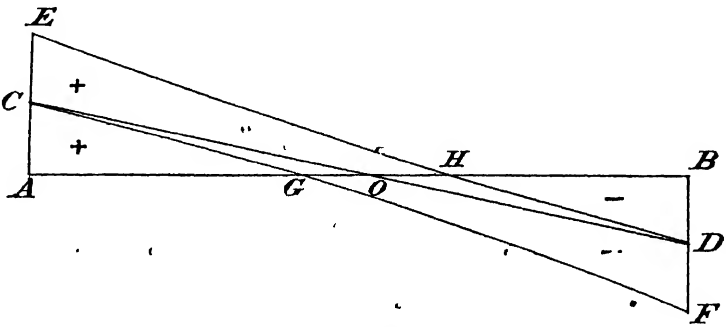


Fig. 128.

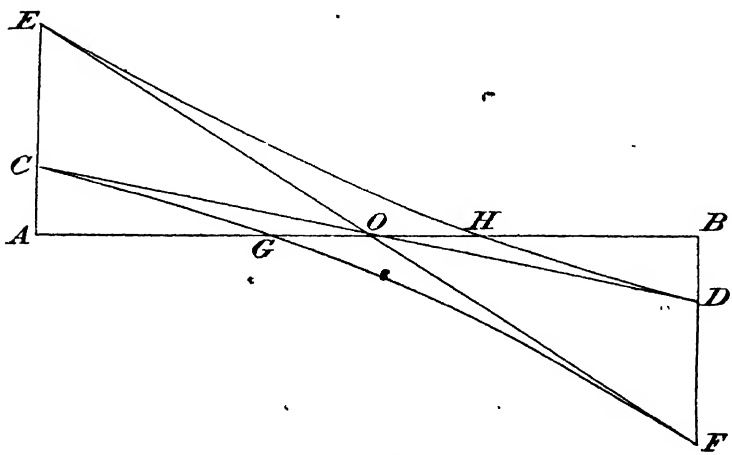


Fig. 129.

If we join E and F (*Fig.* 129), we get EAOBF, the shearing stress diagram for the beam covered all over with the dead load and the rolling load. This does not differ much from the diagram of

stresses produced by the rolling load in its passage, and the difference, such as it is, is chiefly in the centre, where, from the practical limits imposed by market sections of metal, a certain size of member must in any case be used. The shearing stress diagram representing the resistance of the section of minimum size will be in nearly all cases approximate to the theoretical requirements of the maxima stresses produced by the rolling load in its passage. Hence, in almost all cases, we may ascertain the stresses in each member, from the consideration of the whole span being entirely covered. This reduces the calculation to the simple rules given in the former part of this chapter.

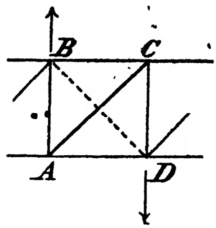


Fig. 130.

Calculation of the Strength of a Given Braced Girder with Parallel Flanges.

Strength of existing girder.

It is frequently necessary to ascertain the safe strength of an existing girder. There is no difficulty about this, if the main principles are borne in mind, viz.:—that the booms furnish the Moment of Resistance (usually expressed by the formula *rad*, where r = intensity of resistance per square unit, a and d representing the area of the flange and depth of the girder, generally expressed in inches), and that the braces resist the shearing stresses. If these braces are at an angle to the horizontal, the stress they may be called upon to resist will be the shearing stress at the position they occupy in the beam × cosec θ, where θ is the angle of inclination.

For instance take the girder shown in *Fig.* 131 (from *Instruction in Military Engineering*, Vol. III., *Plate* 19, *Figs.* 4 and 5), where the

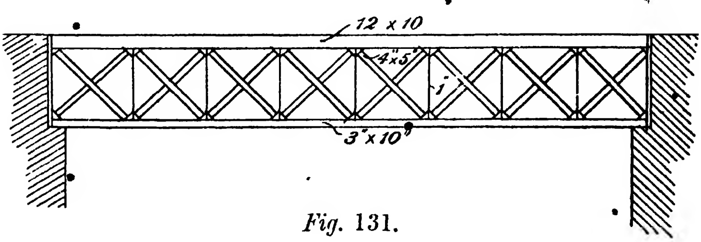


Fig. 131.

span is 40′, depth from centre to centre of booms 5′ 6″ = 66″, braces 4″ × 5″ at an angle of 45°, verticals say 1″ diameter (the thickness

is not specified). The timber is red fir, the ultimate resistance of which is—for compression 5,375, and for tension, 12,000 lbs. per square inch.

Then the expression rad (allowing a factor of safety of 5), becomes :—(1). With the tension, or lower boom—

$$\frac{12000}{5} \times 30 \times 66 = 4{,}752{,}000 \text{ inch-lbs.}$$

(2). With the upper boom—

$$\frac{5375}{5} \times 120 \times 66 = 8{,}514{,}000 \text{ inch-lbs,}$$

which is far greater than (1). The minimum safe available Moment of Resistance is therefore 4,752,000 inch-lbs.

The maximum shearing stress will be at the supports, and will there be resisted by a brace or strut, about 6′ 8″ long and 5″ × 4″ in section. The ratio of length to least dimension will be 80″ to 4″ or 20 to 1″. Hence by Equation (4), p. 160, Part I., $P = \frac{1}{2} r_c A$, where P is the greatest load the strut can bear, r_c is the safe resistance to crushing $= \frac{5375}{5} = 1{,}075$ lbs. per square inch, and $A = \text{area} = 20$ inches.

Hence $P = 10{,}750$ lbs, and the vertical or horizontal component of this will be $\frac{10750}{\operatorname{cosec} 45^\circ} = 7{,}620$ lbs.

The iron bars, which are capable of bearing a safe tensile stress of 5 tons per square inch, and are 1″ diameter, can stand

$$1'' \times \frac{\pi}{4} \times 2{,}240 \times 5 = 8{,}820 \text{ lbs.}$$

This is rather more than 7,620, hence the latter must be taken as the available resistance to shearing.

Thus we gather that the girder can resist a weight producing a M_r of 4,752,000 inch-lbs., and can resist a shearing stress of 7,620 lbs.

To find the unit weight per foot run, we can write

$$4{,}752{,}000 \text{ inch-lbs.} = 396\,000 \text{ foot-lbs.} = \frac{wl^2}{8},$$

where w = required unit weight per foot run and $l = 40$.

$$\therefore \quad w = \frac{8 \times 396000}{40 \times 40} = 1{,}980 \text{ lbs. per foot run.}$$

The total load would be $1{,}980 \times 40 = 79{,}200$ lbs., and the shearing stress produced by such a load uniformly distributed would be

39,600 lbs., which is far more than the 7,620 lbs. that the braces are designed for. The strength of the latter, however, would be largely increased by fastening them together where they cross. This would double the effective strength, as the ratio of l to h would now be reduced.

The counter-bracing of the Howe truss type of girder here shown throughout presents difficulties. Where there may be either class of shearing stress, counter-bracing is necessary, as we have seen. But where there is no chance of both kinds of shearing stress, the problem of how much of the direct stress is borne by the two sets of braces is a very indeterminate one. It will depend upon the deflection of the structure as a whole, and the relative rigidities of the different sets of triangulations. This may be worked by the $\frac{pul}{E}$ formula, explained in Chapter IV., on the assumption that the load divides itself equally between the systems of sets of triangulation. This assumption must only be true when the workmanship is good, and where each joint is arranged so that the various members are equally doing their proper share of the work, a condition of things which would be rarely attainable in practice. Counter-bracing of Howe truss.

It is true that in some forms of bridge there may be either positive or negative shearing stress at any point. An instance of this is the stiffening girder of a suspension bridge (see Chapter VII.). In such a case the Howe truss is the best form to adopt. In ordinary cases the Howe truss presents a certain number of redundant members.

*Application of Simple Braced Girders to a Road Bridge.**

As an example of the application of a simple braced girder to a road bridge, we may take the following:— Example of small braced girder.

Example 12.—*Design a road bridge for a span of 50 feet, of the N or Whipple Murphy truss form, the timber to be spruce and the tie bars of round iron.*

Width of roadway 10′; weight to be allowed for 80 lbs. per foot super, load all over, exclusive of the weight of the girders and superstructure.

* This is an example for a small road bridge such as might be required in the hills in India. For a large span wooden bridge on this principle, see *Minutes of Proceedings, Institution of Civil Engineers*, Vol. CXXVIII., where there is a description of a bridge of 3 spans each of 110′ across the Wurrumbiggee river at Wagga-Wagga, N.S.W. The timber used in this case was iron bark (see Part I., pp. 49, 52).

Take a depth of 5 feet, let the load be on the upper booms, and let the two girders be at 6 feet central intervals. The form of the bridge will be shown as in *Fig.* 2, *Plate* VIII., each panel being 5′ wide.

For spruce r_c = (average) 4,000, r_t = (average) 10,000. Modulus of Rupture (average) = 8,000 lbs. per square inch.

Planking.

The *road planking* rests on cross baulks 5′ apart. We may take each plank to be at least 15′ long and nailed at the ends, hence it may be reckoned as fixed. The weight uniformly distributed is 80 lbs. per square foot, the width of each plank may be taken at 12″, and clear span = 5′ = 60″, $r = \frac{8000}{5} = 1{,}600$.

Hence

$$\frac{Wl}{12} = \frac{rbd^2}{6} \text{ becomes } \frac{80 \times 5 \times 60}{12} = \frac{1600 \times 12 \times d^2}{6}\ ;\ d = 0{\cdot}8.$$

Make $1\frac{1}{4}$″, so as to give extra stiffness.*

Baulks.

Baulks.—The M_R will occur when the 6′ central portion is loaded and the outer parts unloaded. Here

$$W = 6 \times 5 \times 80 = 2{,}400 \text{ lbs}, \text{ and } l = 6 \text{ feet} = 72''.$$

Hence

$$\frac{Wl}{8} = \frac{rbd^2}{6} \text{ becomes } \frac{2400 \times 72}{8} = \frac{1600}{6} \times bd^2.$$

$$\therefore \qquad \times\, bd^2 = 81.$$

Make $3\frac{1}{4}'' \times 5''$.

The weight of the superstructure will then be

10 baulks $15' \times 3\frac{1}{4}'' \times 5'' \times 30$ lbs.	... ...	= 507
Planking $50' \times 10 \times 1\frac{1}{4}'' \times 30$	... ..	= 1,562
Railings $2 \times 2 \times 50 \times 4'' \times 3'' \times 30$	... ...	= 500
Struts $2 \times 10 \times 4' \times 3'' \times 3'' \times 30$	... ..	= 150
Posts $2 \times 10 \times 3' \times 3'' \times 4'' \times 30$	... ..	= 150
Add weight of girders, say	... ...	2,000
		4,869

* This planking would only stand a safe central load of 640 lbs., which would give a deflection of 0·25″. If the bridge be required for cart traffic, the planking should be thicker or baulks placed closer, as shown in dotted lines on *Fig.* 3, *Plate* VIII.

say 5,000,* or $\frac{500}{2} = 250$ lbs. per panel on each girder, which must be added to the $5 \times 5 \times 80 = 2,000$ lbs. live load on each panel point.

Booms.—The scantlings of the booms may be made uniform, hence we need only take the required scantling at the place where the stress is a maximum. This will be at the centre, and will be $= 12wt$ in the case of the upper boom, and $12\frac{1}{2}wt$ in the case of the lower boom (*Fig.* 2). As t or $\cot\theta = 1$, the stresses are $12w$ and $12\frac{1}{2}w$ respectively. Booms.

For the upper boom $12w = 12 \times 2,250 = 27,000$ lbs. The length of any one panel is 60″, and the safe unit stress r_c is $\frac{4000}{5} = 800$. Make 8″ × 8″. This will bring the least dimension less than $\frac{1}{8}$ the length (see Formula 2, p. 160, Part I.). These dimensions will be sufficient to admit of additional cross baulks, as shown in dottted lines, to strengthen the roadway.

For the lower boom the greatest stress is $12\frac{1}{2}$ cwt., where $w = 2,250$ and $t = 1$. Then stress $= 28,125$ lbs., and $r_t = \frac{10000}{5} = 2,000$. The section may be $\frac{28125}{2000} = 14$ square inches. Make the tie beam double of two planks each 8″ × 1″.

Tie Rods.—Tie bars (verticals) greatest stress $3\frac{1}{2}w$, *i.e.*, $3 \cdot 5 \times 2,250 = 7,875$. A 1″ tie rod, with a stress of 5 tons per square inch, will stand 8,800 lbs. Therefore the end rods may be 1″. Tie rods.

The next rod has to stand $2\frac{1}{2}w$, *i.e.*, $2 \cdot 5 \times 2,250 = 5,625$ lbs. This will require a $\frac{7}{8}$″ tie rod. The remaining bars may be $\frac{3}{4}$″, since that dimension is the least that is advisable.

Struts.—The inclined braces are all struts about 7′ long. The greatest stress on any is $4\frac{1}{2}w \operatorname{cosec}\theta = 4\frac{1}{2} \times 2,250 \times 1 \cdot 414 = 14,320$ lbs. Taking the least dimension as not less than 4″ $\frac{l}{h} = 21$, and $P = \frac{1}{2}r_c A$ (see p. 160, Part I.), *i.e.*, $14,320 = 400 \times A$. $A = 35 \cdot 8$. Make 6″ × 6″. Inclined struts.

The next struts have a stress $3\frac{1}{2} \times 2,250 \times 1 \cdot 414 = 11,180$. Make 5″ × 6″.

The next struts have $2\frac{1}{2} \times 2,250 \times 1 \cdot 414 = 8,000$ nearly. Make 5″ × 4″.

The two central panels may be made the same. The stress on the greater being $1\frac{1}{2} \times 2,250 \times 1 \cdot 414 = 4,772$. Make 4″ × 4″. This is more than is theoretically necessary, but it would be practically inadvisable to reduce them beyond this.

It only remains to determine how far the counterbracing should be added.

* A wheel guard shown in drawing might be added. The central rail is $1\frac{1}{2}$″ × 3″. The total weight of superstructure would be just about 5,000 lbs.

The dead load consists of the superstructure as found above (less approximate weight of girders there taken into calculation) + actual weight of girders The latter amounts to for each girder :—

Compression boom $40 \times 8'' \times 8'' \times 30$	... ...	$= 533$
Tension „ $40 \times 8'' \times 2'' \times 30$	... ...	$= 133$
2 braces at $7' \times 6'' \times 6'' \times 30$	... ...	$= 105$
2 „ „ $7' \times 6'' \times 5 \times 30$	.. .	$= 87$
2 „ „ $7' \times 5'' \times 4'' \times 30$	... ...	$= 58$
4 „ „ $7' \times 4' \times 4'' \times 30$	.. ..	$= 93$
1″ tie bars $2 \times 6' \times 2\ 62$ lbs	.. .. ..	$= 31\ 4$
$\frac{7}{8}''$ „ „ $2 \times 6' \times 2{\cdot}011$	... .	$= 25{\cdot}3$
$\frac{3}{4}''$ „ „ $7 \times 6' \times 1\ 47$	. ...	$= 61{\cdot}8$
For each girder	... ...	1,127·5
To which we may add for wind bracing and counter-bracing	.. ..	300
		1,427·5

The superstructure weighs 1,434 lbs. on each girder. Hence total dead load $= 1{,}427 + 1{,}434 - 2{,}861$, say 3,000 lbs.

The total live load at 80 lbs. per foot on each girder is

$$40 \times 5 \times 80 = 16{,}000 \text{ lbs.}$$

We may draw the shearing stress diagram for the dead load, and superpose on it that for the live load as in *Fig.* 1 ; we see that the four central panels require counter-bracing.

Wind pressure. For safety against wind, lateral bracing must be given and vertical bracing between uprights. Very light scantling would do for this, say $4'' \times 4''$.

The wind pressure for the girders alone, taken as horizontal, would amount to

Booms* $2 \times 2 \times 40 \times 8'' \times 30$ lbs	... ...	$= 3{,}200$ lbs.
Braces (taken at 6″ each) $2 \times 14 \times 7 \times 6'' \times 30$		$= 2{,}940$ „
		6,140 lbs.

acting with a leverage of $2\frac{1}{2}$ feet.

* One surface only of the lower boom is here taken, as the two halves are close together.

Hence the overturning moment for the girders alone would be $6,140 \times 2 \cdot 5 = 15,350$ foot-lbs.

The moment resisting overturning will be the weight of both girders $\times 3 \cdot 3''$, *i.e.*, $6,000 \times 3\frac{1}{4} = 19,500$ lbs., which is more than the overturning moment. The calculation, however, has not taken into consideration the wind pressure underneath the roadway, and on the lateral braces, hence to prevent the bridge being wrecked in a gale, it should be secured by iron cramps or bolts at the ends, built into the masonry.

The details are shown on *Plate* VIII.

Major Bate's Portable Bridges.

Major Bate's bridge. The principle of building up a composite girder with comparatively small component parts has been applied by Major C. McG. Bate, R.E., in the portable steel bridges which he has introduced for railway and other engineering work in the field.

Component parts. The component parts used are shown on *Plate* IX. They consist of 3 principal bars, viz.: —

A, a bulb angle 15′ 0″ long: $9'' \times 3\frac{1}{2}''$ in section.

B, a bulb angle of similar section but 10′ 0″ long.

C, a flat bar, 5′ 9″ total length, but 5′ 0″ between centres of pin holes. Section $6\frac{1}{2}'' \times \frac{5}{8}''$, but the net section $4\frac{1}{4}'' \times \frac{5}{8}''$. Gross area of section 4·06 square inches, but net area = 2·65 square inches.

Other parts are three sizes of bolts $2\frac{1}{4}'$, $1\frac{1}{4}'$, and $\frac{7}{8}''$, with corresponding washers and nuts.

Advantages, etc. of bulb angle. The bulb angle is unusual in structural design. In most permanent works a channel iron would be used instead, but the bulb angle has the advantage of occupying less room when packed in the hold of a vessel for transport.

It is in itself weaker than a channel iron as a strut, but when used in pairs with distance pieces this disadvantage disappears.

* The cross section of the $9' \times 3\frac{1}{2}''$ bulb angle has an area of 8·1 square inches, and vertical Moment of Inertia of 80. The transverse or horizontal Moment of Inertia is only 5·5, hence the least radius of gyration (or $\sqrt{\frac{I}{A}}$, see p. 163, Part I.), would have a value of only 0·825″ if the bulb angle were used alone. In pairs,

* These have been taken from Messrs. Dorman, Long & Co.'s book of market sections, and may possibly be a little different from those used by Major Bate in his calculations.

with a bolt or distance piece between, as shown on *Fig.* 132, the value of the radius of gyration will be $\sqrt{\frac{80}{8 \cdot 1}} = 3 \cdot 16''$.

Mr of pair of bulb angles.

The value of the Moment of Resistance of two bulb angles thus placed with distance pieces, is found from the expression $2 \times \frac{r\mathrm{I}}{y}$, where r = the safe intensity of stress per square inch for steel (say 6 tons for rapidly moving loads), I = 80, and $y = 4''$. Hence

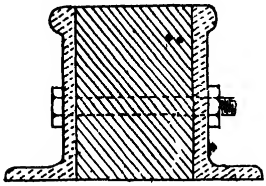


Fig. 132.

$$\mathrm{M}_r = \frac{2 \times 6 \times 80}{4} = 240 \text{ inch-tons.}$$

On a span of 14′ in the clear, or 15′ centre to centre between bearings, this would give a total distributed load W, found from equating M_f with M_r :—

$$\frac{\mathrm{W}l}{8} = 240, \text{ or } \frac{w \times 15 \times 12}{8} = 240,$$

whence W = 10·66 tons on each rail, or 21·3 tons on a pair of rails.

If the value of r be taken at $4\frac{1}{2}$ tons only, the maximum distributed load will be about 16 tons, which is Major Bate's estimate.

In building up these simple lengths into composite girders pin fastenings are used, arranged so as to come on the mean fibre of the material. The struts are formed of two of the C bars with distance pieces, a very good form of strut, theoretically.

The tension members are formed of one or more C bars placed vertically in pairs, with the braces brought between them and secured by the pin fastenings on the mean fibre.

The compression booms are formed of the bulb angles A or B, with C bars added at the sides if necessary.

Joints in the compression booms are covered by C bars, used as cover plates.

Application to span of 50′ feet.

EXAMPLE 13.—*To take as an example the 50′ span bridge shown on* Plate IX. *Find the safe load that such a girder can carry.*

Booms.

The maximum compression is on the upper boom and amounts to $24\frac{1}{2}\ w \times \cot\theta$, from *Fig.* 133, which shows a skeleton diagram of half the truss. As $\theta = 60°$, $\cot\theta = \cdot 57$.

The actual section consists of 2 A bulb angles with C cover plates.

The 2 A bulb angles have a total section of 16·2 square inches, united together at intervals, so that they may be reckoned as one

piece with a least radius of gyration in the direction of the depth, the value of that radius being

$$\sqrt{\frac{2I}{2A}} = \sqrt{\frac{2 \times 80}{2 \times 8 \cdot 1}} = 3 \cdot 16$$

The length of each panel is 5′ or 60″, hence $\frac{l}{k}$ or ratio of length to least radius of gyration $= \frac{60}{3 \cdot 16} = 19$. Hence, since the safe intensity of stress in a steel column with ends fixed, and $\frac{l}{k}$ of value 19 is 5·29 (see p. 175, Part I.), the greatest safe stress which the two bulb angles are capable of bearing is $5 \cdot 29 \times 16 \cdot 2 = 85 \cdot 6$ tons.

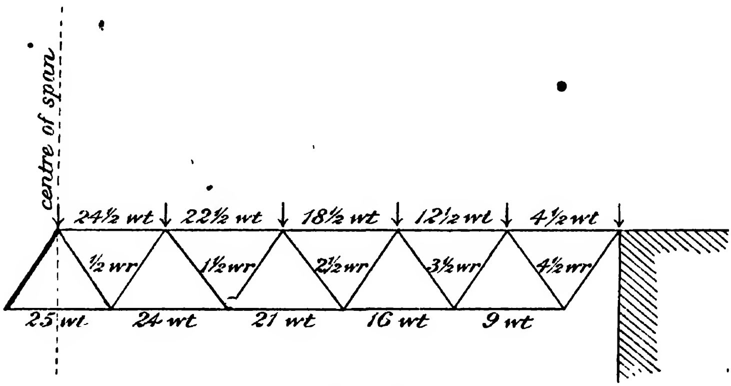


Fig. 133.

Hence $85 \cdot 6 = 24\frac{1}{2}\ w \cot \theta - 24 \cdot 5 \times \cdot 57 \times w$, whence $w = 6 \cdot 1$ tons. As w is the load on each 5′ panel, the weight per foot run is $\frac{6 \cdot 1}{5} = 1 \cdot 2$ tons, or 61 tons on the whole girder.

In this calculation the strength has been based on Table XXVI., Part I., giving results from Mr. Claxton Fidler's formula, which again, is based on the most unfavourable conditions of eccentricity of load, and inequality of material. The ratio of $l : k$ is so small that one might almost consider that the strength of such a column would be that of a small cube of the material, *i.e.*, 7 tons per square inch safe intensity. This would admit of $7 \times 16 \cdot 2 = 113 \cdot 4$ tons on the whole section of two bulb angles, and thus W, the weight on

each panel, would be $\frac{113 \cdot 4}{24 \cdot 5 \times \cdot 57} = 9 \cdot 56$ tons, or 95·6 tons would be the distributed load on the whole span

Diagonals

The greatest stress on any diagonal in tension or compression is $4\frac{1}{2}\, w \operatorname{cosec} \theta$. As $\theta = 60°$, $\operatorname{cosec} \theta = 1 \cdot 15$

Each strut or tie is formed of two C links. In tension each link can bear 2·65 (net area of cross section) × 6 (intensity of resistance per square inch of steel with a factor of safety of 5) = 15·90 tons. Hence two such bars can take 31·8 tons. Therefore $4\frac{1}{2}\, w \times 1 \cdot 15 = 31 \cdot 8$, $w = 6 \cdot 13$, which gives 61 tons on the whole girder as before.

Each strut has an area (gross area of the two links) of 8 square inches, and is arranged with $1\frac{1}{4}''$ distance pieces at 10′ intervals. We may place the links as far apart as the length of the bolt H ($4\frac{1}{4}''$) will allow, by using washers. By using $2\frac{5}{8}''$ washers between the links, we obtain a total thickness of $2\frac{1}{2}''$ of the strut. According to Table XXVI, pp 176—77, Part I, $\frac{l}{k} = 3 \cdot 0 \frac{l}{d}$, where d = the least dimension of the strut and l is the length. In this case $d = 2 \cdot 5$ and $l = 60$. Hence

$$\frac{l}{k} = \frac{3 \cdot 0 \times 60}{2 \cdot 5} = 72$$

For this value, a strut of mild steel, ends fixed, has a safe stress intensity of 4·63 tons per square inch. Hence the strut can bear 4·63 × 8 (gross area) = 37·04 tons which must be $= 4\frac{1}{2} w \operatorname{cosec} \theta$

Hence
$$w = \frac{37 \cdot 04}{4 \cdot 5 \times 1 \cdot 15} = 7 \cdot 15,$$

giving a slightly larger value than 6·13 above.

In the centre the links are somewhat larger than is theoretically necessary.

Weight of Girders

As regards the approximate weight of girders, Professor Unwin's rule is as follows :—

$$W_1 = \frac{Wrl}{Cs - rl},$$

where W_1 = required weight of main girder, W = total distributed load in tons (exclusive of W_1), s = stress per square inch on boom at centre, r = ratio of span to depth, l = length in feet, C = a coefficient

varying with the class of girder being 1,500—1,400 in plate girders, and 1,400—1,300 in braced girders.

For Mr Campin's rules, see Molesworth's Pocket Book, p. 199 (Ed 1893)

Professor J. B Johnson gives the following rules for single-track railway bridges

Plate girders,	$w = 9l + 120$,	where l = span in feet,	w = dead load per foot run, in lbs	
Lattice ,,	$w = 7l + 200$,	ditto	ditto	
Howe ,,	$w = 6 \cdot 5l + 275$,	ditto	ditto	

The above apply to steel railway bridges For ordinary road bridges they would give too high results

CHAPTER VI.

BOWSTRING GIRDERS AND ARCHED RIBS.

Economy of Bowstring Girders.—Uniform Loads.—Diagonal Bracing necessary with Rolling Loads.—Example of Bowstring Bridge Subject to Passing Loads.—Barra Bridge.—Hog-back Girders.—Arched Ribs.—Bending Moments.—Relation between Equilibrium Polygon Stresses at any Section.—Braced Iron Arches.—Appendix.

BOWSTRING GIRDERS.

THE parabolic bowstring girder is the most economical form in which a composite beam can be designed, because in this form, as compared with the parallel type, there is a minimum amount of stress on the web, and the material in the flanges is disposed to the best advantage.

Importance of web in parallel girders.

In a parallel girder, whether of the plate type or braced, we have seen that *the web* performs a most indispensable function. The flanges being horizontal, the vertical shearing forces can only be transmitted to the flanges by some inclined path, either, as in the case of the plate girder, through a continuous web, or, as in the case of a lattice girder, by inclined braces. If we were to omit the inclined bars in the latter case, not only would the structure be imperfectly braced, but its strength would be only that of a single flange considered as a beam under transverse stress. The flanges could offer no horizontal resistance to the vertical stresses unless these are transmitted through the web or inclined bars. Hence under all circumstances of load the web or inclined bracing is absolutely necessary in parallel girders.

Area of flange varies in parallel girders.

Again, in parallel girders, the depth d being constant, and the arrangement of flange plates being such that r, the stress intensity, is also practically constant, we see that the *area of the flange* will vary according to the ordinates of a parabolic curve, since $M_f = rad$, and M_f has its locus, for a uniform load, on a parabolic curve, the equation being $M_f = \frac{wx}{2}(l-x)$ (see Part I., p. 73).

Now if we design a bridge with an elevation on a parabolic curve, we see that the horizontal components of the stress on the curved member, and the stress on the horizontal member, will be a constant, being $= ra$, since the variations in M_f are now met by variations in d. Further, we see that, as the shearing force is measured by the inclination of the M_f curve (see p. 112, Part I.) at each point, the vertical component of the stress on the curved member will meet the vertical shearing stress, leaving no work to be done by the inclined braces, so long as there is no passing load. Area of flange almost constant in bow-string girders.

Hence in such a bridge *subject to a uniform load* the diagonal bracing may be entirely omitted, as in *Fig.* 134 (1). Web unnecessary with uniform loads.

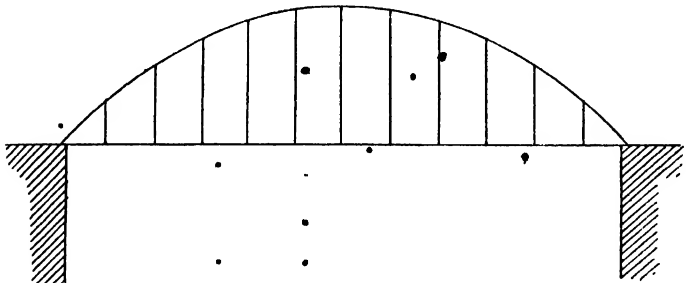


Fig. 134.

In the horizontal member of such a bridge the stress is entirely due to the thrust of the ends of the curved member, and (2), might be replaced by the thrust of abutments as in *Fig.* 135. This is the principle of *the arch*. Various fo[rms] of curved girders.

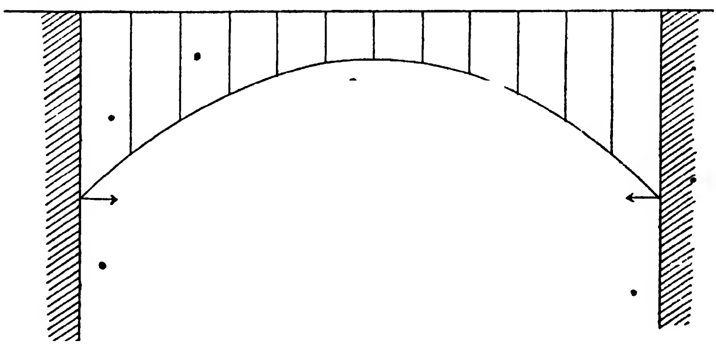


Fig. 135.

If the bridge have the curved member below the horizontal (3), the same reasoning applies, except that the stresses are reversed, the horizontal member being now in compression, as in *Fig.* 136. Or (4), the horizontal member may be replaced by anchorages at either abutment, as in *Fig.* 137. This is the principle of the *suspension bridge*.

Fig. 136.

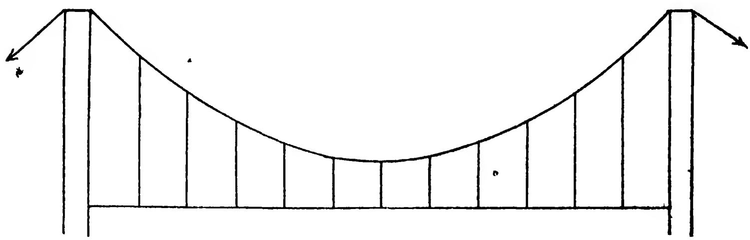


Fig. 137.

In all these cases the diagonal bracing has no function to perform, and may be omitted

Bracing necessary with variation in load.

But when variations in the load occur, deformation in the curved beam will tend to take place, and this must be met. In the bowstring girder it is met by diagonal bracing, in the arch and in the suspension bridge by other devices, which will be investigated in due course.

Curve will always be a parabola, with uniform load.

Meantime it is well to note that with a uniform load, and uniform horizontal stress, the curve of a bowstring girder, or an arch, or a suspension bridge, must be a parabola In the case of a suspension bridge the cables, if flexible, will adjust themselves to that form, but the fundamental principles differ in no way in this case from the others, and if the construction in the case of arches, etc., uniformly loaded, differs from the parabolic elevation, equilibrium and safety can only be obtained at the expense of engineering economy.

Where the loads, instead of being distributed throughout the

entire length, are carried (as is frequently the case) on panel points, the same principles hold good, the M_f diagram here being a polygon inscribed in a parabola (see p. 74, Part I.).

Bowstring Truss Subject to a Moving Load.

Example 14.

To trace the effect of a uniform moving load on a parabolic truss we may take that shown on *Plate* X., where the span is 50′, and divided into 5 panels. The rolling weight per foot run on each girder may be taken at 4 cwt., *i.e.*, with a 10′ roadway this would be nearly 90 lbs. per square foot. This gives a panel load of 10′ × 4 cwts. = 2 tons, and the total weight on each girder when the bridge is covered would be 4 × 2 = 8 tons (not 5 × 2, since half a panel load will be borne directly at each abutment). **EXAMPLE 14.** Bowstring girder subject to moving load.

Drawing the diagonals as shown (DE, FG, etc., *Fig.* 1), we may proceed to the investigation of the load travelling from left to right.

Fig. 2 gives the diagram when the first panel point is loaded, *Fig.* 3 when the second is loaded, and so on till we get *Fig.* 5, when the load covers the bridge.

Examining these we see that all the inclined bars are in tension, the verticals are sometimes in tension, sometimes in compression, and that each inclined bar has its maximum value when the bridge to the left of the panel point where the bar joins the horizontal is fully loaded, and when the part to the right is unloaded. For instance, DE is a maximum in *Fig.* 2. FG is a maximum in *Fig.* 3. HI is a maximum in *Fig.* 4. Further, if we compare the lengths of the bars, and their maximum values, the latter are directly proportional to the former.

If the load were coming the other way from right to left, we can see that the bars DE, FG, HI would be in compression. Hence, as it is desirable to have all the bars tension members (so as to avoid the use of long compression bars), we insert the dotted bars as counterbraces. The whole of the members would now be tension, and certain of them would be redundant, according to the position of the load.

If the load were a single one rolling across, and the structure had not been designed with dotted diagonals, then with the load at the panel point 2, the bar DE would be in compression.

The following very expeditious graphic method is given by Mr. Claxton Fidler:— Mr. Claxton Fidler's graphic method.

"At each end of the span AC (as in *Fig.* 6, *Plate* X.), set up

the vertical lines AG and CK each = 4 times the depth of the parabola, so that the lines GC and KA would form tangents to the parabolic curve at A and C. Now let the heights AG and CK represent on a suitable scale of tons the upward reactions of the abutments when supporting the entire live load wL, *i.e.*, make $AG = \frac{wL}{2}$ (in this case 5 tons). Then using the same scale, the length of the chord or flange AC will represent the maximum stress in that member. The length of any diagonal will represent the greatest direct stress in that diagonal, and the length of any vertical post will represent its greatest compressive stress. To find the direct stress in any member of the parabolic or polygonal flanges, such as ED, it is only necessary to continue the line ED until it intersects the verticals AG and CK in the points R and S, when the length RS will at once measure the direct stress in the member. This very expeditious method will be equally applicable to the upright or the inverted bowstring, or to a girder of the Saltash type,* and will be equally correct whether the girder is divided into an odd or an even number of panels, and whether the panels be wide or narrow, or of regular or irregular widths." For proof see Appendix to this chapter.

In the above it will be noticed that this graphic method gives the maximum *compressive* stress on the verticals. In most cases, however, these are in tension, the amount of which is not given by the graphic method, but it can be easily estimated, as it is usually the weight of $\frac{1}{2}$ one panel of the floor + $\frac{1}{2}$ live load + $\frac{1}{2}$ the weight of one panel of the girder.

In the example only the stresses due to the moving load have been estimated. Those due to the dead load of the bridge and roadway would have to be considered in addition. This will vary with the span, and can, as a rule, be approximately estimated.

This will give us all the stresses in the bridge. These are written against each member in *Fig.* 6.

Barra Bridge

Plate XI. shows the Barra Bridge on the Lahore and Peshawar Road (the late Major-General Sir James Browne, R.E., Executive Engineer); a good example of the bowstring principle adapted to timber and iron. A full description of this is given in Vol. II. of

* Where there are two parabolic ribs, one arched and the other inverted, as in the Saltash Bridge at Plymouth, the Mainz Bridge over the Rhine, etc.

the *Roorkee Treatise on Civil Engineering*,* but the following points may be noted here. The tie beam (10″ × 16″) is under combined tensile and transverse stress. Lateral bracing is given at intervals by rectangular frames. The curvature was given to the planks of the arched ribs by first immersing them for 8 to 10 days in water, then they were taken out, placed on edge before a bright chip fire, and oiled on both sides until well heated throughout. They were then bent to the required curve by means of a rack-stick and rope looped over their ends.

The scarf joints in the tie beam showed a tendency to split at the angles. A fish joint with straps and bolts might have been better.

The bridge was tested by a dead load of 200 lbs. per square foot of roadway.

Before leaving the subject of bowstring bridges it may be well to point out that the reason why it is not possible to apply the M_f and shearing stress diagrams, and arrange the material accordingly, as in other cases, is because the shearing stresses with moving loads are partly borne by the curved bow and partly by the diagonals. There is no difficulty whatever in ascertaining the stresses in the bow and the string, the only difficulty lies in the diagonals, and in their case the exact determination of the actual stress in any given case would largely depend on the initial tension of the bars meeting at any one panel point.

In the Barra Bridge the diagonals are designed for compression, and the verticals for tension only. The calculation would be similar to that which we have previously considered, except that for any given rolling load, instead of considering only all bars sloping *downwards* towards the load, as in *Figs.* 1 to 5, *Plate* X., regarding tl others as redundant, we would consider those as sloping *upwards* to a vertical line through the weight. In *Fig.* 1, *Plate* X., for instance, the bars shown in dotted lines would be calculated for a weight rolling from left to right, until such weight arrived at the panel point 3.

In practice, it would be necessary to tighten up the verticals, to be careful about the exact fitting and bearing of the struts, and provision should be made for taking up the shrinkage of the timber by such means as wedges at the joints.

* From which *Plate* XI. has been copied, by permission of the Secretary of State for India.

Advantages and disadvantages of bowstring bridges. To sum up, it may be said that while the theoretical advantages of bowstring bridges are very great, owing to the very slight variation in the stresses, there are certain practical disadvantages arising from the necessity of more careful workmanship and fitting at the joints, and from the difficulty of lateral bracing, which have made the adoption of this form of bridge less frequent than it might otherwise have been in railway construction. For road bridges of timber the above disadvantages do not, relatively, produce so much effect as in iron or steel, and the type is, therefore, one which can be economically applied to timber work. The depth is usually in railway bridges about $\frac{1}{8}$ of the span, but in very wide spans greater depth would be economical.

Hog-back girders. Intermediate between the parabolic and the parallel girders there are various designs in which one flange is more or less curved (*Fig.* 138). These are common in railway construction, and the investigation of the stresses is carried out by methods already

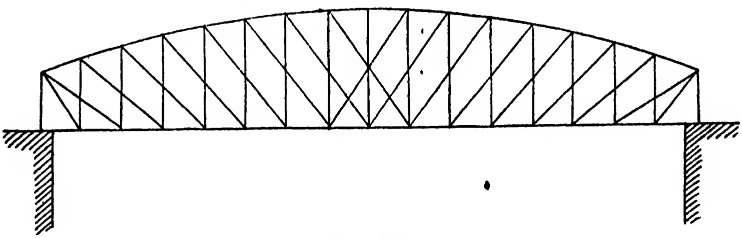


Fig. 138.

described. These girders, usually called "hog-backed," have a greater flange stress at the ends, but less stress in the bracing, than parallel girders of the same depth. When compared with parabolic girders of the same depth in the centre, the flange stress is less at the ends, but the stress in the bracing greater.

Arched Beams.

Arched beams. The thrust of the bow in the foregoing investigation might be met by the resistance of abutments, as has already been pointed out. This principle results in the construction of the *arch*.

Value of thrust in arch. As long as the load is uniformly applied along a horizontal line, whether above or below the parabolic arch, the horizontal component

H of the thrust at all parts is uniform, and is equal to $\frac{wl^2}{8d}$, where w is the weight per unit of length, l is the span, and d the rise. The actual thrust at any point will be H sec θ, where θ is the angle of inclination to the horizontal of the tangent to the curve at the point in question.

The actual design of the curved metal rib would then be according to the principles for struts. Usually the material is arranged in flanges, as in the case of a beam under transverse stress, not only because this is a good form of cross section for a pillar, but because there are bending moments, as we shall see, to be resisted.

Where the uniform load is very great in comparison with any passing loads, *e.g.*, in the case of a highway bridge such as that over the Medway at Rochester, it is unnecessary to consider the effect of transverse stress, but where the passing load is considerable, special arrangement must be made to prevent deformation, and to meet the unsymmetrical stresses brought into action.

When the weight over the arch is uniform we have seen that H, the horizontal component of stress in the rib $\frac{wl^2}{8d} = \frac{Wl}{8d}$. By a similar process of reasoning, if there be a weight W in the centre producing a M_{ff} of $\frac{Wl}{4}$, the value of H due to that weight will be $\frac{Wl}{4d}$. Similarly, no matter how the arch is loaded, we can find from the value of the M_{ff} the value of the horizontal reaction at the abutments. **Horizontal reaction at abutments.**

There is, however, this assumption in the above, that the arch at the abutments is not fixed, but free to turn so that the resulting thrusts will pass through the axis of the arch. This is not always done in practice, though if the arch is not constructed with hinged or rounded bearings the question becomes extremely complicated. Still further complications will be produced if, as is almost invariably the case, the elasticity of the material varies throughout the rib. **Assumption that ends are free.**

Hence elaborate mathematical investigation applied to a fixed arch, hinged neither at the springing nor the crown, can only be approximately true. The full investigation of this subject involves the consideration of the deflection of curved beams, which also depends on uniformity of elasticity. For present purposes we may consider the elasticity uniform.

Relation between the Funicular Polygon and the Stresses at any Section of an Arch.

Funicular polygon, Culmann's principle.

If the M_f curve produced by *any* loading on the span of the given arch corresponds with the curve of the arch, there will be no stress in the arch other than direct compression, just as in a flexible cord, capable of adjusting itself to various loads, there is no stress other than direct tension. If the arched rib were, like a cord, capable of adjusting itself to various loads, there would be no cross bending produced, but if, as is usually the case, it is incapable of doing so (within certain limits), there will be bending induced in the rib, as well as direct compression, *in every case where the M_f curve does not coincide with the curve of the given arch.* To find the amount of this bending stress we may proceed as follows:—

On pp. 70, 71, Part I., is given Culmann's method of finding the M_f at any point of a loaded beam in terms of the vertical intercepted at the position of the point in question in the funicular polygon, and the polar distance.

Let w_1, w_2... ..w_{11} (*Fig.* 139), be a series of loads acting on a straight line supported by a parabolic arched rib. Let *Fig.* 140 represent the polar diagram where the polar distance Ox is taken,

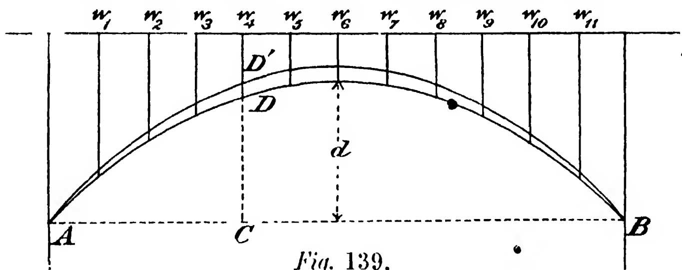


Fig. 139.

on the scale of loads, representing the horizontal component of stress $= \frac{M_{fl}}{d} = H$. (As we can take the polar point anywhere, we may give the polar distance any definite value). Draw the funicular polygon starting from the point A (similarly to that described on pp. 70, 71, Part I.). If the polygon corresponds with the form of the rib there is no bending stress anywhere, for, from Culmann's principle, at any point C, $M_f = CD \times H$.

But if the polygon does not correspond with the rib, but if it is CD′, then the moment excited in the arched rib will manifestly be the difference between the Moment of Flexure produced by the

external force and the Moment of Resistance supplied by the form of the structure, *i.e.*—

$$= CD' \times H - CD \times H = H\,(CD' - CD).$$

In other words, the bending moment at any point of an arched rib is proportional to the difference between the ordinates of the rib and of the funicular polygon for the loads on the arch.

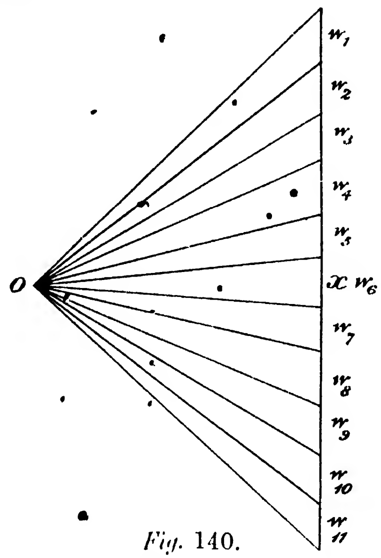


Fig. 140.

Fig. 141 represents an enlarged portion of any part of the rib in *Fig.* 139. T represents the direct compressive stress in the rib, R the stress in the funicular polygon, *bc* the direction of the side of the polygon.

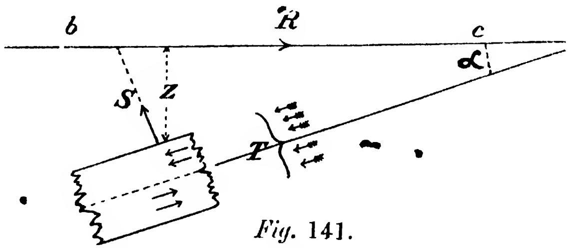


Fig. 141.

In addition* to the direct thrust T (*Fig.* 141), and bending moment M_f, there is a shearing stress S on the arch. If α is the Stresses on arched rib.

* From Prof. Johnson's *Modern Framed Structures*.

angle between the side of the funicular polygon and the tangent to the neutral layer (*Fig.* 141) and R the stress in the funicular polygon at that point—

$$R \cos \alpha = T \ldots\ldots\ldots\ldots (a),$$

$$R \sin \alpha = S \ldots\ldots\ldots\ldots (b),$$

and we have seen that

$$H \times z = M_f \ldots\ldots\ldots\ldots (c).$$

Thus the thrust, shear and moment are readily found if the funicular polygon is drawn, and the design of the section can be made accordingly, the rib being either a braced beam, or built up with flanges.

Arch hinged at springings and at crown.

To find the funicular polygon, however, we must know the magnitude of H and the direction of the abutment reactions. Hence it is desirable to have the arch hinged both at the springing and at the crown, both because we ensure by this method that the directions of the reactions shall be ascertainable, as they will pass through the hinged crown, the abutment hinges, and some point on the line of the load at any given point, and also because by this means we provide for expansion and contraction under changes of temperature.

In such a case the drawing of the equilibrium polygon for any load such as P (*Fig.* 142) is simple, for the arch being free to turn

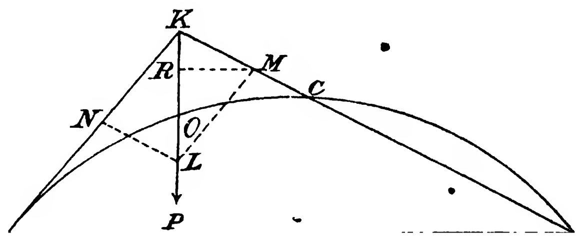


Fig. 142.

at A, B and C, the moments there must be equal to 0, and the thrust produced by P at D must be transmitted through C. The right half of the arch BC is acted on by two forces only (neglecting the weight of the half-arch itself), viz., the pressure of the left half at C, and the abutment reaction at B. As these hold the half arch in equilibrium they must be equal and opposite to each other. Hence if we join BC and produce it to meet the vertical through P at K and join AK, we have the polygon.

The stresses for any part of the rib can then be found by the principles pointed out above. Analytically the stresses may be found by the ordinary methods of moments as follows :—In *Fig.* 143 let ACB be the rib, then if V_1 and V_2 be the vertical components of the reactions, c = half span, d = rise, and b = distance of load P from centre C measured positively towards the right –

$$V_1 = P\frac{(c-b)}{2c} \dots\dots\dots\dots\dots\dots (\alpha),$$

and

$$V_2 = P\frac{(c+b)}{2c} \dots\dots\dots\dots\dots\dots (\beta).$$

Taking moments about C we have for loads on CB, $H \times d = V_1 \times c$,

$$\therefore \qquad H = \frac{V_1 \times c}{d} = P \times \frac{(c-b)}{2d} \dots\dots\dots\dots (\gamma).$$

Similarly for loads on AC

$$H = P\frac{(c+b)}{2d} \dots\dots\dots\dots\dots\dots (\delta).$$

The components of the reactions can thus be computed for each load and the results added. Graphically the components may be found by making (*Fig.* 142) KL = P on any scale, and drawing LM parallel to KN, and NL parallel to KM. Draw MR at right angles to KL, then MR = H and KR, RL = the vertical components at B and A. The reactions being known, the stresses can be found by the ordinary method of moments.

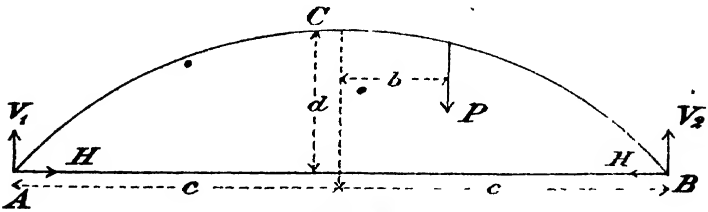


Fig. 143.

The rib will therefore be under combined stress except at those places where the $M_f = 0$, *i.e.*, where the funicular polygon cuts the neutral axis of the parabolic rib. As these points where the $M_f = 0$ will vary for various positions of the load, it will be necessary to consider the whole rib as, more or less, under combined stress.

Under certain circumstances of load the rib will be under negative bending stress, *i.e.*, there will be tension in the lower and compression in the upper fibres at a given section. Under other circumstances of load the same section will be under positive bending stress.

Other classes of arched ribs. This is the general method of ascertaining the stresses in a 3-hinged arched rib. This is the best form of metal arch for cases where the moving load coming on the arch is small compared to the dead load. Where the span is comparatively small, and the live load bears a large proportion to the dead load, the 3 hinged arch is unusual. There are two other classes of arch which are used, either (1) without hinges altogether, or (2) with hinges at the abutments. Into the full investigation of these it is not intended here to enter, because the subject is not only complex (as pointed out above) but because it is at best approximate. It may be sufficient to point out that in (1) the reactions do not pass through the springings of the arch, and at those points there will generally be some bending moment. In the case of (2) the reactions pass through the abutments, and the locus of the point K (the intersection of the direction of reaction for any load at any point) is found from the equation

$$y_0 = \frac{32c^2d}{25c^2 + 5b^2},$$

b, c, and d having the same values as before (*Fig.* 140), and y_0 being the distance of K from AB.*

At the centre

$$b = 0, \quad y_0 = \frac{32}{25}d.$$

At the ends,

$$b = c, \quad y_0 = \frac{16}{15}d.$$

The curve produced is shown on *Fig.* 144.

Taking moments about C, and then about A,

$$V_1 = P\frac{c+b}{2c}, \quad V_2 = P\frac{c-b}{2c}.$$

and

$$H : V_1 = c - b : y_0.$$

$$H = P\frac{c^2 - b^2}{2cy_0}$$

Hence the components of the reactions due to any load can be computed, and the funicular polygon drawn and compared with the actual form of the proposed rib.

The value of the moment, mentioned above, viz., $-H(CD' - CD)$ is a perfectly general one, and applies equally to arches which are hinged, and those which are not hinged. The difficulty is to find the value of H. Although this is easy enough in the case of uniform loads, it is not so simple in the case of passing loads; it must in such cases be considered for various positions of such a load, and the effects produced under such varied circumstances considered for various parts of the rib. Without drawing polygons for

* The proof of this is given in Professor Johnson's *Framed Structures*. It is somewhat lengthy, and is omitted here for that reason.

each case the stress produced at any part of the actual rib can be ascertained by the method of moments.

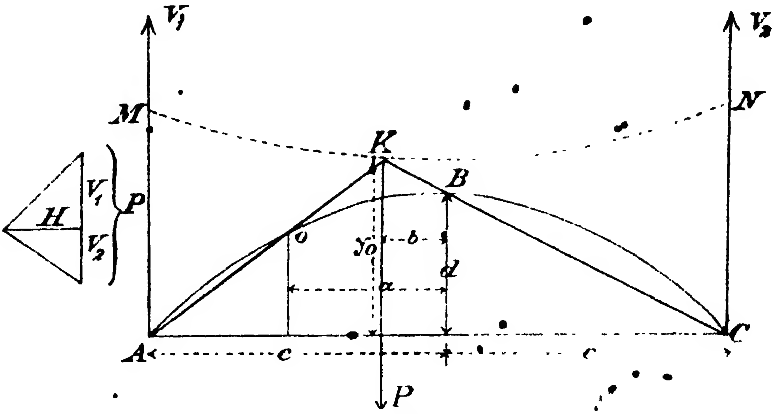


Fig. 144.

Mr. Claxton Fidler has patented an arched rib, hinged at the springing and crown, of the form shown in *Fig.* 145. From the springing to crown, each half rib is a parabolic girder, of which the central depth is about $\frac{d}{2}$. The straight line AB is a tangent at B to the parabolic arc BC, and the neutral axis of the two ribs is a parabolic curve.

Mr. Fidler's arched rib.

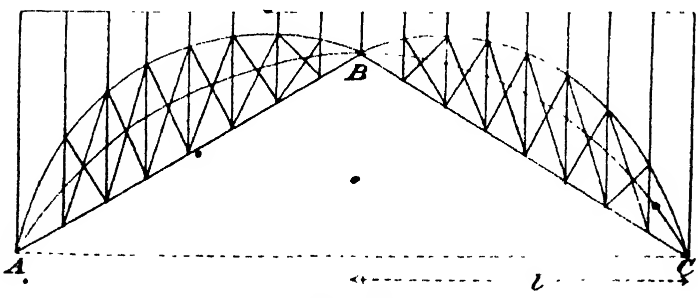


Fig. 145.

This is a most economical form of 3 hinged arch. Inverted it becomes an equally economical suspension bridge.

The horizontal stress on either rib for a uniformly distributed load of w per unit of length over the whole span is

$$H = \frac{wl^2}{4d}$$

For live load wl distributed over the half span l,

$$H_{ml} = \frac{w_1 l^2}{4d}.$$

There is no stress in the diagonal bracing for either of these cases.

Braced Iron Arches.

Braced iron arches.

"Braced iron arches are those in which the arch rib and horizontal rib are connected together with diagonal bracing. In order to determine the stresses on such an arch it is assumed to be pivotted at its crown and springings, so that each half arch with its bracings forms an independent frame or girder. Arches of this kind are frequently made without these pivots or hinges, but with small abutting surfaces instead; the smallness of these surfaces as compared with the other dimensions of the arch practically constitutes the arch a hinged one."*

In the braced arch in *Fig.* 146, with a weight W in the centre, the directions of reactions pass along the dotted lines AB and BC. Draw Bb on any scale representing W, and bb_1 and bb_2 parallel to the reaction lines. This completes the parallelogram of forces for the apex and gives us the magnitude of the reactions. The vertical and horizontal components of the reactions can at once be obtained by drawing a horizontal line b_1, b_2.

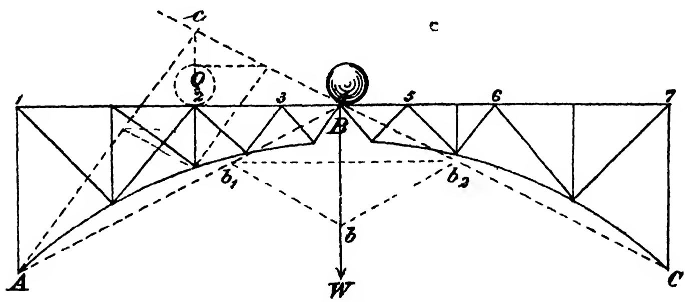


Fig. 146.

If the weight were not at the centre, but at some point such as Q, the reaction at the right abutment must pass through the hinge at B, so if we join BC and produce it to the vertical through Q at the

* Anglin, *Design of Structures.*

point C, we get the right abutment reaction, and joining cA we have the left abutment reaction. Drawing the parallelogram of forces as before we find the magnitude of these reactions, and the values of the vertical and horizontal components. The latter, it will be observed, are equal.

Having found the reactions (due to all weights on the arch), it is easy to draw a stress diagram for the braced arch by the ordinary system, or to ascertain the stress in any member by the method of sections.

APPENDIX.

Proof that Mr. Claxton Fidler's Graphic Method of representing the Stresses in a Bowstring Girder, subject to a Uniform Rolling Load, is correct.

The point to be proved is, that if the verticals from the ends of the girder, AG, CK, represent the reactions $\frac{1}{2}wl$ (where w = the load per unit and l = length in the same units), then RS represents on the same scale the compressive stress on ED, and the length of any bar such as FD represents on the same scale the greatest compressive stress that will come on that bar in the passage of the load in either direction (see *Fig.* 6, *Plate* X.).

It is evident, from the principle of the triangle of forces, that if AG, CK represent the end reactions, then CG and AK must represent the thrusts of the ends of the curved rib. AC must represent, similarly, the horizontal tension in the string. AC must also represent the horizontal component H of the flange stress, since that is at all points equal and opposite to the tension in the string. Hence AC also represents the thrust in the rib at the highest part.

The vertical lines AG, CK therefore are the loci of the ends of the lines indicating the rib stresses. Hence the stress at any intermediate point can be found by drawing a tangent at that point, and measuring the length intercepted between the two verticals.

So far the truth of the proposition is evident, but it is not so clear why the length of each diagonal bar represents the maximum compressive stress on it. It is, however, clear that if we can prove that the horizontal component of the stress in such a bar is equal to the horizontal tension in the string, divided by the number of panels, then the actual stress in the bar will be represented by the length of the bar itself, since that length = panel length × sec θ, θ being the inclination of the bar to the horizontal.

The proof of this is based upon two theorems, which have been previously shown to be true :—

1. The horizontal stress produced in *the flange or boom* of any girder by any load = the $M_f \div$ depth of the girder.

2. The horizontal component of the stress in any panel of the *web bracing* in any girder is = the difference between the horizontal flange stresses in the two sections of the flanges or booms joined by that bracing.

Thus we may tabulate (as in Lathom's method, see page 118) the flange stresses in any girder subject to any given rolling load by considering the

calculated value of the M_f, and dividing by the measured depths of the girder at successive points.

Having obtained thus the flange stresses, we can next write the horizontal components of the web stresses by taking the difference of the amounts shown in the first table.

In the case of the parabolic girder, subject to a uniform load as in *Plate* X., it will be found that the horizontal component of the web stress in any bar is greatest when the whole of the panels on one side of it are loaded, and when the other panel points are unloaded. The amount of such stress will be found to be equal to the total H (horizontal flange stress) ÷ number of panels.

In *Fig.* 146*a* let the greatest depth = 1, the panel breadth = 1, and the panel load = 1. Then from measurement of the four verticals in the 5-paneled girder the lengths are 0·64, 0·96, 0·96, and 0·64 successively.

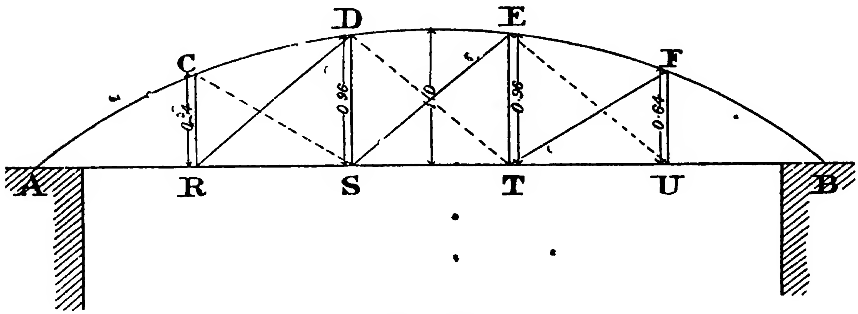


Fig. 146*a*.

To find the horizontal flange stresses, we have to find the M_f when each panel point is loaded by the load = 1, and divide by the depth at each panel point.

(1). *When the first panel point R is loaded with load* = 1. The reaction at A = 0·8, and at B = 0·2. The M_f at R = 0·8 × 1 = ·8, and the length RC = 0·64.

Hence horizontal flange stress at C = 0·8 ÷ 0·64 = 1·250.
,, ,, ,, D = (0·8 × 2 − 1 × 1) ÷ 0·96 = 0·625.
,, ,, ,, E = (0·8 × 3 − 1 × 2) ÷ 0·96 = 0·417.
,, ,, ,, F = (0·8 × 4 − 1 × 3) ÷ 0·64 = 0·312.

(2). *When the panel point S is loaded, the other points being unloaded*— The reaction at A is now 0·6, and the M_f at R = 0·6 × 1.

Hence horizontal flange stress at C = 0·6 × 1 ÷ 0·64 = 0·938.
,, ,, D = 0·6 × 2 ÷ 0·96 = 1·250.
,, ,, E = (0·6 × 3 − 1 × 1) ÷ 0·96 = 0·834.
,, ,, F = (0·6 × 4 − 2 × 1) ÷ 0·64 = 0·625.

In like manner, by considering the load at T, and then at U, calculating the M_f at each panel point, and dividing by the depth, we get the following table :—

Table showing the Horizontal Flange Stresses in a Parabolic Girder of 5 panels, each of length = 1, and loaded with panel loads each = 1.

Position of Load.	Stress at C.	Stress at D.	Stress at E.	Stress at F.
At R	1·250	0·625	0·417	0·312
,, S	0·938	1·250	0·833	0·625
,, T	0·625	0·833	1·250	0·938
,, U	0·312	0·417	0·625	1·250
Totals . .	3·125	3·125	3 125	3·125

The greatest horizontal stress produced by the passage of the load is a constant throughout the girder, and is equal to 3·125, which is the same as the tension in the string, and $= \frac{Wl}{8d}$.

To find the web stresses we take the difference of the flange stresses in the above table. Thus, to find the horizontal component of the web stress in the panel CRSD, we take the difference between the horizontal flange stress in AC and that in CD, *i.e.*, + 1·250 and + 0 625 = − 0 625, when the load is at R. Similarly, for that position of load; the horizontal component of web stress in the panel DSTE is the difference between the horizontal flange stress in CD and that in DE, *i e.*, − 0·625 and +0·417 = − 0·208.

In like manner we can deduce from the former tabular statement the following :—

Table showing the Horizontal Stress in each Panel of the Web in a Parabolic Girder of 5 panels, under conditions as stated above.

Position of Load.	Panel CS	Panel DT	Panel EU.
At R	− 0 625	− 0 208	− 0·105
,, S	+ 0·312	− 0 417	− 0 208
,, T	+ 0·208	+ 0 417	− 0·312
,, U	+ 0·105	+ 0 208	+ 0·625
Totals	− 0·625 + 0·625	− 0 625 + 0 625	− 0·625 + 0·625

From this table we see that the maximum horizontal web stress, either positive or negative, is a constant, and is equal to 0 625. But, since the tension in the string is 3·125, the horizontal component of the web stress bears to it the ratio of 0·625:3·12, or 1:5, which is the ratio of any of the panel lengths AR, RS, etc., to the whole length AB. Since AB represents on the graphic diagram the total tension in the string, AR, RS, etc., will represent, on the same scale, the horizontal components of the web stresses, and RD, SE, etc., will represent, on the same scale, the actual stress on the inclined bars.

By this graphic method, the stresses on a parabolic girder can be ascertained more easily than in any other class of bridge.

CHAPTER VII.

SUSPENSION BRIDGES.

Advantages for Military Work.—Disadvantages.—Stress in Cable for Uniform Load.—Position of Maximum Shear and Bending Moments.—Direction and Pull on Anchorage.—Methods of Stiffening Roadway.—Method of Stiffening Cables.—Details of Construction.—Appendices

SUSPENSION BRIDGES.
Advantages.

THE advantages of suspension bridges in purely military operations need not here be dwelt upon. Many examples in recent campaigns have proved the value of this form of structure for the passage of ravines and rivers, especially those with swift currents and in mountainous countries. The advantages of adaptability to wide spans, facility of erection, cheapness and portability of the materials, which have made this form of bridge so important in military operations, hold good also where permanent bridges have to be erected for ordinary roads. In the hill roads in India this form of bridge has been very largely used.

The reason of the usefulness of this class of bridge will be at once apparent when we consider the following facts stated by the late Professor Fleming Jenkin, of Edinburgh University :—

"A man might cross a chasm of 100′ hanging to a steel wire 0·21″ in diameter, dipping 10 feet; the weight of the wire would be 12·75 lbs. A wrought-iron beam of rectangular section, 3 times as deep as it is broad, would have to be about 27″ deep and 9″ broad to carry him *and its own weight*. It would weigh 87,500 lbs. The enormous difference would not exist if the beam and wire had only the man to carry, although even then there would be a great difference in favour of the wire; the main difference arises from the fact that the bridge has to carry its own weight. The chief merit of the suspension bridge does not, therefore, come into play until the weight of the rope or beam is considerable when compared with the platform or rolling load; for although the chain will, for any given load, be lighter than the beam, the saving in this respect will, for small spans,

In h
M_f at
table :

be more than compensated by the expense of the anchorages. In large spans the advantages of the suspension bridges is so great that we find bridges on this principle of 800′ or 900′ span constructed at much less cost per foot run than girder bridges of half the span."

In mountain gorges the suspension bridge principle is sometimes the *only* one that can be adopted, owing to the great expense that would be entailed by the construction of piers for girder spans of moderate length.

The disadvantages of this form of structure are :— **Disadvantages.**

(1). The suspension bridge is peculiarly liable to the destructive effect of wind from beneath, lifting the structure suddenly and then dropping it. As the wind in mountain gorges is often of peculiar force, structures exposed to its action should be designed with special reference to such uplifting effect. In addition to this vertical effect, lateral oscillation is produced by the wind and must be guarded against.

(2). The want of rigidity in a suspension bridge is another very serious disadvantage. This is the reason why it has not been applied to long span railway bridges to a greater extent than has actually been the case. Any heavy loads passing over the bridge tend to cause deformation, loosening of joints, and excessive load stresses; and this tendency is increased with increase in the velocity of the passing loads, and with synchronous impact.

In the design, therefore, of such bridges attention must be specially devoted—(i.), to the arrangements for stiffening the entire structure, and (ii.), to the resisting of wind stresses. **Points to be attended to in design.**

Materials.

The materials for the cables may be either—(1), steel wire rope; (2), wire lashed together *in situ*; (3), iron or steel links **Wire rope.**

Steel wire rope is, in civilized countries, very suitable, as it is both cheap, strong, and easily applied. But in out-of-the-way regions it is difficult to transport it.

Table VI. gives the weights and prices of steel rope manufactured by Messrs. Bullivant & Co., Mark Lane, London.

TABLE VI.

Table of weights and prices of steel wire hawsers.

Steel wire hawsers sold by Messrs. Bullivant & Co., 72, Mark Lane, E.C.

Size, circumference, inches	Weight per fathom, lbs	Guaranteed breaking strain, tons	Price, 90 fathoms £	s	Price, 120 fathoms £	s	Equal to tarred hemp hawser Inches	Diameter of barrel or sheave round which it may be worked	Remarks.
6	33	88	52	10	70	0	19	36	Messrs. Bullivant sell patent reels and automatic nippers suitable for each size.
5½	28	74	48	0	64	0	17	33	
5	23½	64	39	10	52	10	15	30	
4½	15	39	23	15	31	10	13	27	
4	11	33	19	10	26	0	12	12	
3½	9	26	16	5	21	10	11	21	
3¼	8	22	14	5	19	0	10	19½	
3	7	18	12	10	16	10	9	18	
2¾	5½	15	11	0	14	10	8½	16½	
2½	4½	12	9	5	12	5	7½	15	
2¼	3¾	9	7	5	9	10	6½	13½	
2	2¾	7	6	10	8	10	5¾	12	
1¾	2	5½	5	5	7	0	5	10½	
1½	1¾	4	5	0	6	10	4	9	
1¼	1	2½	4	10	6	0	3½	7½	
1	¾	1¼	3	15	5	10	2½	6	

Wire in bundles.

Telegraph wire stretched and lashed together in bundles has been most successfully used in the Himalayan Bridges by Capt. and Brevet Lieut.-Col. F. J. Aylmer, V.C., R.E. The wire can be made up in coils of no greater weight than a man can carry. The advantage of this in remote districts is obvious.

The Fribourg Bridge (*Plates* XII., XIII., XIV.), erected about 1835,* over a span of 807 feet, is a notable instance of this method of construction. Each main cable was formed of two bundles, each $5\frac{1}{2}''$ in diameter and composed of 1,056 threads of wire 0·12″ in diameter. The ultimate strength of all the cables was 2,500 tons, with a gross area of about 48 square inches. At every two feet the bundle was firmly bound round with wire. In approaching the piers the two cables on each side of the bridge gradually spread out and unite into one flat band of parallel wires, which in this form passes over the three friction rollers on the top of the pier. The coils of wire as delivered on the works weighed from 18 to 20 lbs. After delivery, if found to be without defect, each coil was immersed during two hours two or three times in a cauldron of boiling linseed oil mixed with a small quantity of litharge and soot. The coils, after being dried, were then rolled on wheels about 16′ in diameter, the ends of one coil being spliced to another until a drum was filled up. The forming of the wires into cables was then carried out. Each of the four suspending cables was composed of 20 strands, viz., 12 with 56 wires, and 8 with 48. A block of oak *b* (*Fig.* 6, *Plate* XIV.) was fixed upon a low frame, and firmly retained in its place, the curved part being protected with sheet iron. At a distance of 614′ from it (equal to half the entire length of the cable), two other blocks *a* and *c* were placed, from each of which projected hooks to receive the cruppers which terminated the strands. Throughout the distance of 614′ transverse cylinders were placed to support the wires.

Fribourg Bridge. Manufacture of main cables.

To stretch the wire, one end was fastened at *a* and being passed through one of the cruppers, was carried off in a cart bearing the reel. It was thus unwound, then bent round *b*, and there given a tension of 220 lbs., by means of a cord passing over a horizontal cylinder *e* and *d* at each end of the walk and having a weight of 220 lbs. attached. After passing round *b* the wire was taken back to *c*, passed round the crupper there, and subjected to the same tension as the other length. This operation being continued till the end of the wire was reached, that end was then united to the first thread temporarily fastened to the block *a* in commencing the

* A more modern instance of a suspension bridge, whose cables are entirely of wires bound up, is the magnificent bridge connecting New York and Brooklyn; span 1,600′, with 4 cables each with 5,600 wires, and with a total diameter of 15·75″ to each cable. The details of the construction of these cables could not be ascertained by the writer, and it is considered that possibly the information about the Fribourg Bridge, old fashioned though it is, may perhaps be equally useful to those for whom this book is intended.

operation. The two parts of the strand near the crupper were then bound for a length of 18″ by a spiral ligature. In addition to this, temporary ligatures were bound round the strand at 3′ or 4′ intervals, and these were not removed till the strands were about to be united into one great cable. The strand thus bound together was payed over with a coat of the same oil varnish through which the wire had already passed.*

This procedure is here given in detail as it may possibly be of use to officers who have to carry out similar work in inaccessible regions. The detail of the construction of the mooring chains and suspension rods will be described later.

The following is the specification for a suspension bridge of 850′ span recently constructed over the Ohio:—

Specification for Ohio Bridge.

The wire used in this bridge for the cables to be No. 6 B.W.G. The ultimate strength of the wire shall be 190,000 to 205,000 lbs. per square inch, its elastic limit 95,000 to 100,000 lbs. per square inch. A variation of wire will be allowed of 0·006″ in the length of one wire. All wire as soon as made must be passed through an approved preparation of linseed oil to protect it from oxidation. If the wire is tested to destruction, an elongation of not less than 3 per cent. in 12″ shall be allowed. The wire shall also be capable of being wound round ½″ rod without showing signs of cracks. The wrapping wire shall be No. 10 ordinary soft wire of 80,000 lbs. ultimate strength, and the cable to be properly wrapped by a wrapping machine. The cables shall be adjusted to a height as directed by the engineer, and to be made of seven strands of parallel wire of 55 square inches net section. One wire out of a stock of 40 may be tested for the acceptance or rejection of the lot. The strands to be laid on a wooden platform and one coat of paint applied before its erection, and kept as clean as possible. All strands must be made under strain, well adjusted, and have a fixed length given by the Engineer and checked by the guide wire. After the cable is wrapped it shall get three good coats of paint.

Iron or steel rods in links.

Iron or steel rods or bars linked at intervals are frequently used as cables, and the suspension rods are hung from the same pins that join the links in the main cables. In this construction the joints are obviously of the greatest importance.

Temporary suspension bridges may be formed of hemp ropes,

* This description of the Fribourg Bridge, and the plates illustrating it, are obtained from an old book on bridges, now out of print, published by Weale in 1843.

canes, twisted cables of twigs, etc., but the use of such expedients does not come within the limits of this work.

The arrangements of the materials above mentioned is such that the cables are flexible and adjust themselves to the load. Mr. Claxton Fidler has, however, pointed out that instead of employing a flexible chain with a rigid roadway, it is possible, and in every way more advantageous to make the curved chain or rib rigid in itself. When the chain is flexible and adjusts itself to inequalities of load, the flexibility prevents any moment of flexure being produced in the chain at any point, and the stress on the chain is pure tension. But when the chain is rigid the form given to it must be such as to resist the moments produced by all inequalities of load. How this is to be done will be demonstrated presently. Meantime it may suffice to say that in connection with this, the material to be used will be steel, or iron, in plates, angles, bars, etc., just in the same way as in a girder bridge.

Theoretical Considerations.

Theoretical considerations

That the curve of the suspension chains is a parabola* readily follows from the consideration of its being the inverted form of the bowstring bridge as demonstrated in the last Chapter. A separate proof is given in Appendix I. to this Chapter. The horizontal component H of the stress on each cable at all parts is

$$H=\left(\frac{w+p}{2}\right)\frac{a^2}{8d} \qquad \ldots \quad (\text{i.}),$$

where w is the dead weight, and p the live load, per unit of length of the whole bridge uniformly distributed all over, a is the span and d the greatest dip. The length of any suspension rod y from a point on the curve distant x from one support to the (horizontal) tangent at the lowest point is

$$y=\frac{4dx^2}{a^2} \qquad (\text{ii.}).$$

The tension in cable at the towers is H sec θ, or

$$T=\left(\frac{w+p}{4}\right)a\sqrt{1+\left(\frac{a}{4d}\right)^2} \qquad \ldots (\text{iii.}).$$

Length of cable between piers—

$$l=a+\frac{8d^2}{3a} \qquad \ldots (\text{iv.}),$$

Trautwine gives an approximate rule $l=a+\cdot 23d$, when d is not greater than $\frac{1}{12}a$.

* Actually it is not exactly a parabola because the extra weight of the suspension bars at each end of the bridge prevents the load from being equally distributed all along.

Design of stiffening girder.

The tendency to deformation in the cables caused by passing loads, which is one of the chief objections to the suspension bridge, can be obviated by the use of a stiffening girder to carry the roadway, the function of which will be to distribute the passing load equally on the cables. The M_{ff} of such a girder will be $\frac{pa^2}{108}$ and the positions of this M_{ff} will be at points $\frac{1}{3}$ of the distance of the span from the piers. The maximum shearing stress will be $\frac{pa}{16}$, and will be at both ends and at the centre.

Here p = weight on *both* cables per unit of length of span.

a = span in same unit of length as taken for p.

For proof of the above values, see Appendix II.

It is usual to make the booms and bracing equally strong throughout. It must be remembered that as the M_{ff} may be either positive or negative, the stiffening girder should be designed so that the booms may take either tension or compression, and as the shearing stress may be positive or negative (the head of the load being always a point of contraflexure), the girder should be counter-braced throughout.

Stiffening girder made of uniform section and bracing throughout.

Since the maximum shear occurs at both ends and at the centre, and the M_{ff} at the $\frac{1}{3}$ points, it is necessary to make all the dimensions to carry these stresses, thus making uniform sizes throughout the entire truss.

The dead load being entirely suspended from the cable, the amount of the shear in the girder is not affected by it.

The design of the stiffening girder is thus reduced to simple proportions, the M_{ff} being $\frac{pa^2}{108}$, and the maximum shear $\frac{pa}{16}$. The usual design is a Howe truss, which may also form a convenient parapet to the bridge.

The roadway girders must be anchored down at the abutments, because the ends would otherwise rise under certain positions of the load.

As an illustration, we may take the following:—

Example of design of stiffening girder.

EXAMPLE 15.—*Design a stiffening girder for a suspension bridge with a span of* 100′, *weight per inch run,* 55 *lbs., width of roadway being* 5 *feet. Timber deodar.* (*Fig.* 147).

This would be a bridge for a road for pedestrians and pack

animals, such as is common in the Himalayas. The side railings may be 4′ high.

Here $\quad p=35, \quad a=100\times 12=1200$ inches.

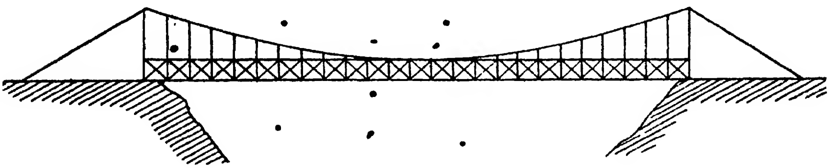
$$M_{ff}=\frac{pa^2}{108}=\frac{1440000\times 35}{108}=466666 \text{ inch-lbs.}$$

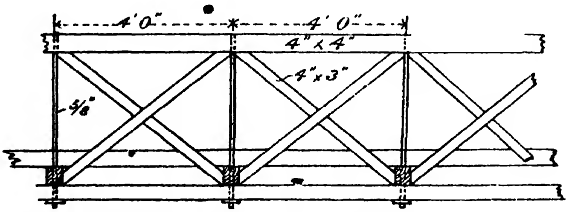
Using the side railing as a stiffening girder with a height of 4′, and taking the *safe* resistance to crushing of deodar at 800 lbs. per square inch, we have

$$M_{ff}=M_r=rad=800\times a\times 4\times 12$$

or
$$a=\frac{466666}{48\times 800}=11\cdot 6 \text{ inches.}$$

Make 4″ × 4″, which would do for both booms.*





Enlarged elevation of stiffening girder

Fig. 147.

* This section being 16 square inches and the length between any two verticals being 4′ (see *Fig.* 147) we have ratio $l:d=12$ and Equation 43), p. 160, Part I., for timber columns becomes

$$P=\tfrac{5}{8}\,r_c\,A=666\times 16=10656 \text{ lbs.}$$

The actual stress is $\qquad \frac{466666}{48}=9720$ lbs.

Hence 4″ × 4″ will do quite well.

The braces have to take maximum shear, *i.e.*—

$$\frac{pa}{16} = \frac{35 \times 1200}{16} = 2625 \text{ lbs.},$$

which, for a bar at 45°, becomes $2625 \times 1{\cdot}414$.

The inclined bars will be $4 \times 1{\cdot}414 = 5{\cdot}6'$, say 67″ long, and the least dimension will be about 3″. Hence $\frac{l}{h} = 22$ and $P = \frac{1}{2} r_c A$ (see p. 160, Part I.). r_c being 800, $A = \frac{26{\cdot}25 \times 1{\cdot}414}{400} = 9{\cdot}3$ square inches.

Make 4″ × 3″ and use $\frac{5}{8}$″ iron rods* as verticals (see *Fig.* 117).

If the span and load be such that timber is not suitable for the stiffening girder, and steel has to be substituted, the flanges or booms should be designed as compression bars, of **T** or **L** iron, and a high factor of safety should be taken on account of the reversal of stress.

Plate XV. shows the suspension bridge on the Chakrata Road over the Jumna, designed by the late Major-General Sir James Browne, R.E., in which the principle of the stiffening girder is very clearly shown.

Fribourg Bridge deficient in rigidity.

If we apply these rules to the investigation of the stiffening girder in the Fribourg Bridge we shall see that it is very deficient in rigidity. The roadway is 21′ wide, the balustrade truss is about 5′ from centre to centre of booms or chords, and the upper boom or chord has a section of 0·32 square feet. The timber having a safe resistance to crushing (r_c) of, say, 1,200 lbs., this gives a value of

$$M_1 = rad = 1{,}200 \times 0{\cdot}32 \times 5 = 1{,}900 \text{ foot-lbs.}$$

Now $$M = \frac{pa^2}{108}, \text{ and } a = 807 \text{ feet.}$$

Hence $$p = \frac{1900 \times 108}{807 \times 807} = 0{\cdot}31 \text{ lbs. per foot run,}$$

which is very small. Hence *any* live load practically will cause some oscillation.

This led to the strengthening of the bridge in 1880.

* Wrought iron can bear 5 tons per square inch safe tension. Hence $\frac{1}{2}$″ bar takes $\cdot 7854 \times \cdot 25 \times 2240 \times 5 = 2080$, which is hardly enough. Use $\frac{5}{8}$″ bars.

Dredge's System of Sloping Rods.

In this system the platform is hung from the main chains by a series of sloping rods, either parallel, making an angle ϕ with the vertical, or at varying angles. By this system a horizontal component of stress is introduced into the sloping rods, which is counteracted by longitudinal stresses in the roadway. It is, in fact, the same principle that we have seen applied to braced girders, where the inclined braces transmit part of the shearing stress to the booms. Sloping suspension rods

The stresses on the various members of such a bridge may be readily determined by a stress diagram. Let *Fig.* 148 represent such a bridge, the bottom chord being free at the ends, and consequently in tension. Draw the load line $x \ldots\ldots y$ representing the weight suspended from one main cable, and divided into xp, po, etc., representing the vertical weight from each tension rod. From x and y draw lines xh, ya parallel to the sloping rods at the ends, and ph, og, etc., horizontal, representing the horizontal stresses at various panels. The horizontal tension at the centre $\left(\frac{Wl}{8d}\right)$ may be drawn qr, found by drawing from x and y lines parallel to the tangents to the curve at the piers. Then join qh, qg, etc. These lines will represent the stresses in various parts of the chain.

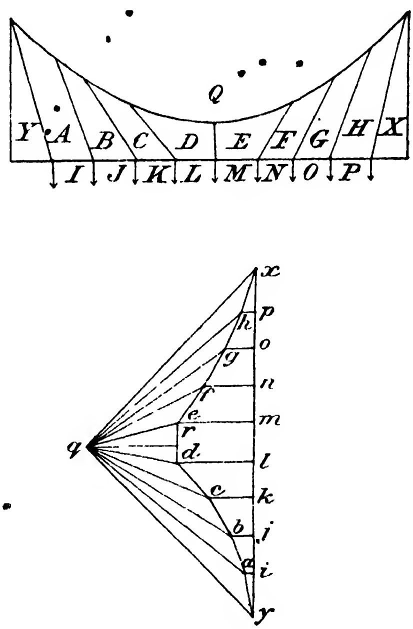


Fig. 148.

This form of suspension bridge, proposed about 1830 by Mr. Dredge, of Bath, was considered at the time to be a great improvement over the system of vertical rods, because by varying the inclination of the bars, making them more nearly approach the vertical at the abutments than at the centre, it was considered that Disadvantages.

the greatest economy of material could be obtained both in the chain and in the suspension rods, on much the same principle as that proposed by 2nd Lieut. Graham Fraser, R.E., for roofs (see p. 283, Part I.). Unfortunately, the effect of oscillation produced by transverse loads, and the uplifting action of the wind, seems to have been lost sight of in this construction, and it is unquestionably less adapted to resist these external influences than the other forms of this structure.

Method of Dual Cables and Alternate Suspension Rods.

Alternate suspension rods from a system of dual cables.

To neutralize oscillation, the suspending rods are often hung alternately from one and another of the several cables forming the main chain. A recent instance of this, applicable to a bridge in a remote region, is the Olvusâa bridge over the Southern Hvita in Iceland, one span 246′, and a half span of 143′ (described by Major Appleton, R.E., in the *R.E. Professional Papers*, 1895). Each main cable is composed of three separate cables, the centre one of which has a strength equal to the other two combined. The suspension rods are hung from the centre cable and from the two outer cables alternately. These rods are at 6′ intervals, and the roadway is formed transversely of rolled steel joists $5'' \times 3'' \times \frac{7}{16}''$, supporting channel irons $9'' \times 3'' \times \frac{7}{16}''$. This construction gives great stiffness to the structure, which is only intended for pedestrians and pack animals. The position of the point of contra-flexure is no longer exactly at the head of a moving load, and any tendency that there is to bend is met by the resistance to bending of the channel irons. The bridge is further stiffened laterally by wind bracing, and by the splay of the bracing near the towers. The details of this bridge are shown in Major Appleton's paper above referred to, and are well worth studying.

American System of Inclined Tension Bars from the Main Towers.

American system of inclined stays.

Another expedient for stiffening the roadway is to use inclined bars from the summit of the towers to various points in the stiffening truss, as far as $\frac{1}{4}$ the span (see dotted lines on left of *Fig.* 149). The tensions in such bars can only be determined by the consideration of the relative rigidity of the bars and the various members of the stiffening truss. This investigation is not very satisfactory because of variations in workmanship and elasticity.

Influence of Temperature.

Influence of temperature.

The effect of changes of temperature in suspension bridges is most marked. In girder bridges these changes are provided for by roller bearings at the ends—a device which is not applicable to the suspension bridge.

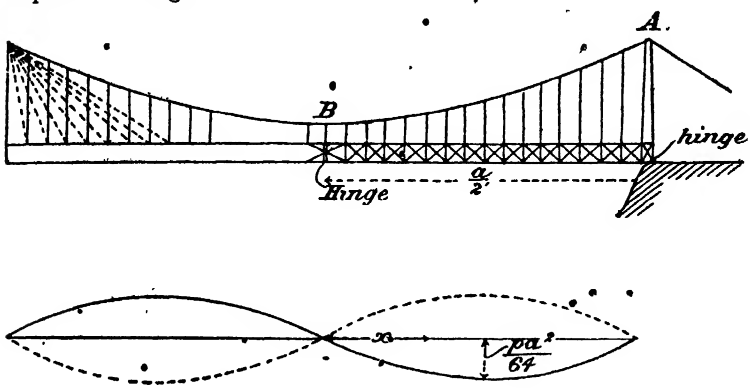


Fig. 149.

"If the temperature of a bar of iron is raised from the freezing to the boiling point, *i.e.*, through 180° F. or 100° C., its length is thereby increased by $\frac{1}{8}$ per cent. or by $\frac{1}{800}$ of its original length l. The elongation Δl due to any given range of temperature can therefore be easily calculated. If the chain of a suspension bridge is thus elongated by any rise of temperature, while the span remains unaltered, the dip of the chain will, of course, be increased, and the consequent deflection or depression of the roadway in the centre of the span may easily be found as follows:—

Let $\frac{a}{2}$ = half span.

d = central dip of the chain.

$\frac{l}{2}$ = length of the semi-parabola AB measured along the curve. (*Fig.* 149).

Then, by the properties of the parabola—

$$\frac{l}{2} = \sqrt{\frac{a^2}{4} + \frac{4}{3}d^2},$$

and

$$d = 0{\cdot}866\sqrt{\frac{l^2}{2} - \frac{a^2}{4}}.$$

Now, if the length of the half chain is increased by Δl, we shall have the increased dip of the chain—

$$d + \Delta d = 0{\cdot}866\sqrt{\left(\frac{l}{2} + \Delta l\right)^2 - a^2}$$

Variations allowed for in England, and abroad.

In England the observed expansion of iron structures from their coldest length appears to be $\Delta l = \frac{l}{2 \times 2400}$, corresponding to a range of temperature of about 60° F. But in America, and, indeed, in most foreign countries, the range of temperature is considerably greater, and a range of 100° to 150° is commonly provided for. The cables of the Cincinnati Suspension Bridge have a span of 1,057 feet and a dip of 89 feet at mean temperature, but the dip varies by as much as 2 feet between the summer and winter. This variation, however, is partly due to the expansion of the side spans, which is, of course, attended with a horizontal motion of the saddles upon the summit of the pier, so that the central span is reduced, while the length of the central cables is at the same time increased.

Whatever the length or form of the side spans, let Δa represent the horizontal motions of the saddles due to the expansion of the side chains; then, the central span being reduced by $2\Delta a$, the augmented dip of the chains at the centre of the span will be

$$d + \Delta d = 0{\cdot}866\sqrt{\left(\frac{l}{2} + \Delta l\right)^2 - \left(\frac{s}{2} - \Delta a\right)^2}.*$$

From the principles of deflection, using the formula $V = \frac{n'rl^2}{Ey}$, when we know the actual deflection of a girder, we can ascertain the corresponding intensity of stress in the flanges. Thus it is possible that there may be an initial intensity of stress in the stiffening truss, amounting to a very considerable quantity, arising solely from accommodation to the temperature, without any live load at all. The available strength may thus be seriously reduced, and this will be enhanced by the elongation of the mooring cables, due also to increase of temperature.

Effect of a central hinge.

We thus see that if a central hinge be introduced, as in *Fig.* 149, so that the girder can rise or fall with changes of temperature, we get rid of the difficulties of temperature, but

* Mr. Claxton Fidler, p. 354.

the calculations for the stiffening girder given above are no longer applicable in their entirety. The maximum shears occur when the bridge is half loaded. (*See* Appendix II.). In *Fig.* 149 let the whole span be covered with the dead load, which, per unit of length $= w$, and live load over half the span per unit $= p$. Then the total load on the bridge $= a\left(w + \frac{p}{2}\right)$. The two stiffening girders, each of a length $= \frac{a}{2}$, hinged at their point of meeting, must each be designed to bear a load of $\frac{p}{2} \times \frac{a}{2}$ uniformly distributed, each subject to alternate upward and downward bending stresses.

The M_f diagrams are shown in *Fig.* 149 (lower part). If $x =$ horizontal distance of any point from the centre, the M_f on each girder

$$= \pm \frac{px\left(\frac{a}{2} - x\right)}{4}$$

At the centre of each girder $x = \frac{a}{4}$ and

$$M_{ff} = \frac{pa^2}{64}.$$

Hence in a hinged suspension bridge the M_{ff} is about $\frac{1}{4}$ of the M_{ff} due to the same unit load acting on a span of the same total length, unsupported by the cable. In this case also the roadway girders must be held down at the abutments as well as supported.

The dotted curve in *Fig* 149 shows the M_f diagram when the left half of the bridge is loaded.

Mr. Claxton Fidler, however, has pointed out that the duty of carrying the rolling load is to a great extent performed twice over, once by the chain, and once by the stiffening girders. This duplex system is, of course, extravagant, and it is proposed to effect economy by using a rigid rib, consisting of an upper and lower member united by diagonal bracing, and of a depth proportioned to the M_f as in *Fig.* 150. Mr. Fidler's stiffened rib.

This form has not only the advantage of stiffness, but "the united sectional areas of the upper and lower members need be no greater, or very little greater than that of a single flexible chain or arch designed to carry the same total load; and having this sectional area they will resist the bending action of the unequal load without suffering any further or greater stress."*

* Claxton Fidler, *Bridge Construction*, p. 362.

The form thus given is illustrated in *Fig.* 150. If w be the dead load, and p the live load, per unit of length, a the span, and d the dip—

$$H \text{ (the horizontal stress running through the whole bridge)} = (p+w)\frac{a^2}{8d}.$$

The outline of each rib is a straight line AB, and a parabolic curve AA′B, and the neutral line of each rib is a parabola (shown dotted), which bisects everywhere the vertical depth of the rib and passes through A, B, C. The stress H is divided equally between the upper and lower ribs; it is therefore

$$= -(p+w)\frac{a^2}{16d}.$$

The M_f due to the live load covering half the span AB will be represented by the parabolic diagrams in *Fig.* 149. The horizontal flange stress H′ due to these moments will be uniform throughout, and if d' represents the greatest central depth OB′—

$$\pm H' = \frac{p}{8d'} \times \frac{a^2}{4} = \frac{pa^2}{16d}* \text{ (since } d=2d').$$

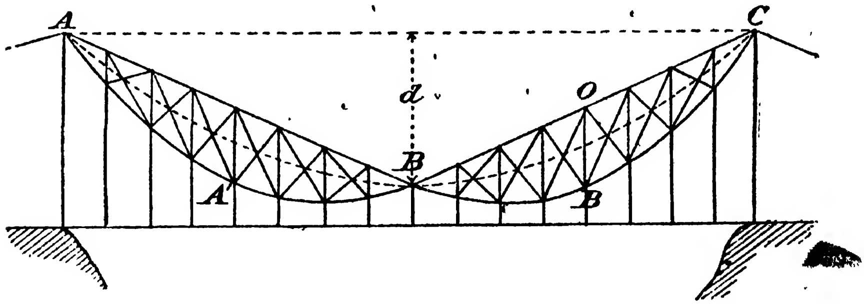


Fig. 150.

"But the tensile stress running through the whole suspension bridge will now be reduced to

$$H = -\left(w+\frac{p}{2}\right)\frac{a^2}{8d},$$

and in each member the total horizontal stress will be

$$H = \frac{H}{2} + H'.$$

* For each cable $\pm H' = \frac{pa^2}{32d}$.

Hence in the upper member of AB, $H = -\dfrac{wa^2}{16d}$.

,, lower ,, ,, $H_2 = -(w+p)\dfrac{a^2}{16d}$.

,, upper ,, BC, $H_2 = -(w+p)\dfrac{a^2}{16d}$.

,, lower ,, ,, $H_2 = -\dfrac{wa^2}{16d}$.

"Therefore, if the members are designed, as they must be, to carry the entire load $w+p$, they will be strong enough to carry the unequal load, for in no part is the tensile stress greater than that due to the entire load. At the same time, it may be observed that the tensile stress is nowhere less than that due to the dead load w, so that the stresses will not alternate between tension and compression like those in the auxiliary roadway girder."*

Stiffened rib in virtually two festoons.

It will be observed that this form has the appearance of two festoons, and they evidently include all the distorted curves that can be produced by a passing load. Although elaborated by Mr. Claxton Fidler, it was proposed by other scientists as long ago as 1861. The recent application of it to the shore spans of the Tower Bridge in London will be familiar to everyone.

This form also gets rid of the necessity of having a very small dip, and, therefore, great initial tension in the main cables.

Stresses on Mooring Cables or Backstays.

Mooring cables or backstays.

When the main cables are continuous over the piers, and pass over rollers with little frictional resistance, the tension in the chains at either side will be the same whether the angles of inclination with the vertical be the same or not. If these angles are the same, the pressure on the piers will be vertical (A, *Fig.* 151). If they are not the same, an inclined thrust will be brought to bear on the pier, the amount of which can be readily ascertained by the parallelogram of forces, as in B, *Fig.* 151. If T be the tension on either chain, θ the angle of inclination of the tangent to the horizontal, R the vertical pressure on the pier—

$$R = 2T \sin\theta \ (\textit{Fig. } 151);$$

and as

$$T = H \sec\theta = \frac{wa^2}{8d}\sec\theta,$$

we get

$$R = \frac{2wa^2}{8d}\tan\theta = \frac{2wa^2}{8d} \times \frac{4d}{a} = wa.$$

* Claxton Fidler, *Bridge Construction*, p. 563.

That is, *the vertical pressure on each pier is equal to the whole distributed load on the bridge.*

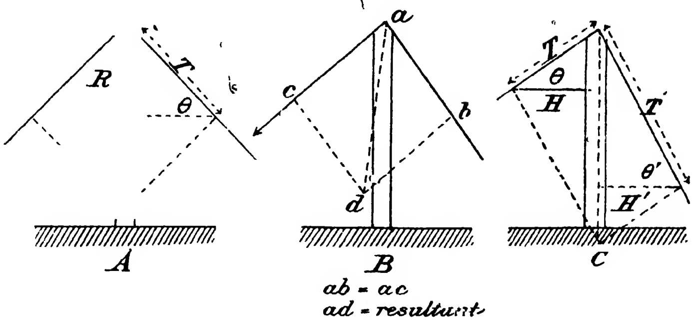


Fig. 151.

When the ends of the main cable and the mooring cables are fastened to a saddle free to move, the resultant pressure on the pier will be vertical, the horizontal components of the tension on the two cables will be the same, but the total stress will vary according to the inclination. If θ be the angle which the main chain tangent makes with the horizontal, and θ' that of the mooring chain, then

$$T = H \sec \theta, \text{ and } T' \text{ (tension on mooring cable)} = H \sec \theta',$$

H being the horizontal component $= \dfrac{wa^2}{8d}$ (*Fig.* 151, C).

Stress on pier should be vertical.

It is very important that the stress on the piers should be vertical, so as to avoid all bending stress and inequality of pressure. Hence the cables should be free to slide over the piers.

Anchorages.

Anchorage

The details of the anchorage are very important.

It is desirable to have a narrow passage along the mooring chains, so that they may be periodically inspected.

If the main cables are of wire bound together, or of steel wire rope, it is advisable to secure them above ground to solid anchor bolts or chains of iron or steel well protected from oxidation. Even if decay does begin it can hardly affect the interior of the bolts, if solid, and will be evident on examination, whereas with wire cables it may be affecting the interior, seriously diminishing its working area, without giving any indication on the surface. Hence it is advisable to exclude wire cables entirely from the anchorage.

Two instances may here be cited of failures of suspension bridges owing to the anchorages giving way, and in both cases the passage of troops was the immediate cause.

Failure of anchorage at Broughton.

The first was at Broughton, in the north of England, in 1831. The span was 143′, and the anchorage was an iron bolt 2″ in diameter at right angles to the suspension link. The dead weight of the bridge was calculated at 43 tons. A party of soldiers marching across in step caused serious undulations, and the bridge gave way. The weight of the men on the bridge was calculated at 1·8 tons, which, combined with the dead weight of the bridge and the dip of the cables (1 in 11·64), gave 37·2 tons on each* anchor bar. The bearing of the bolt was $3\frac{1}{2}$″. Hence the intensity of shear on the bolt was $\frac{37 \cdot 2}{2 \times \cdot 7854 \times 4} = 5 \cdot 9$ tons per square inch, which gives a factor of safety for *good* iron of 3. For bending, the maximum intensity of stress is found by equating the M_B on the bolt, *i.e.*—

$$\frac{Wl}{8} = \frac{37 \cdot 2 \times 3 \cdot 5}{8} = 16 \cdot 25 \text{ inch-tons}$$

with the M_r of the bolt, *i.e.*

$$\frac{rI}{y} = r \times 1^3 \times \cdot 7854.$$

Hence $r = 20 \cdot 7$ tons per square inch.

The bolts were, therefore, originally just sufficiently large, if of good sound iron, to stand the dead weight, without any factor of safety. But the effect of the troops marching across in step, the diminished area of metal from rust, the probable deterioration of the metal—a fact which was proved on examination—led to the disaster. For safety, the bolts should have been about $3\frac{1}{2}$″ diameter.

Failure at Ostra Witza.

The other case occurred on the 15th September, 1886, at Ostra Witza, in Austria. The span of the bridge was 216′, and the anchor stays consisted of 12 links. The chamber enclosing the anchor chains was open to the surface drainage of the road, and the effect on the links had been that all were more or less affected by rust, one of them being completely eaten through. Failure was caused by a

* The pull on each chain being half of

$$\frac{wa}{2}\sqrt{1+\left(\frac{a}{4d}\right)^2},$$

where $wa = 48$, $a = 143$, and $d = 12 \cdot 3'$.

squadron of Uhlans trotting across the bridge. The whole of them were precipitated into the river below, and 6 were killed.

The official report made of this bridge 14 months before the disaster said that it was in good and safe condition.

Independent anchor bolts open to periodical inspection are now usually adopted. The pins and links should be designed like those in machines, with a liberal factor of safety.

Masonry in anchorage.

As regards the masonry of the anchorage, the simplest rule is to make the weight of the masonry = 4 times the pull at the vertex. If the anchorage is in solid rock, careful examination must be made for faults, etc. Blasting should be sparingly resorted to in excavating the anchorage chambers, as the blast may cause fissures. In rock it is difficult to say at what depth the anchorage should be, except that the mass above the anchorage may be considered built work. If the stratification of the rocks is at right angles to the direction of the pull, a comparatively shallow anchorage may suffice; if the stratification is in the line of the pull, it will be necessary to go deep.

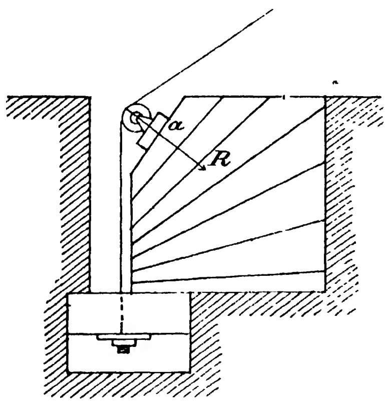


Fig. 152.

Fig. 1, *Plate* XIII., shows the anchorage of the Fribourg Bridge. The arrangements for transferring the pull to the rock on either side are noteworthy.

In masonry, by changin the direction of the mooring chains, as in *Fig.* 152, the overturning moment of the pull is reduced. It should be remembered that the layers of masonry at *a* should be at right angles to the resultant thrust R. A suspension bridge cross the Seine at Paris failed in 1826 from neglect of this precaution.

Mooring cables.

As regards mooring cables, the following description of those used at the Fribourg Bridge may be useful:—

MOORING CABLES.

These were made under shelter in covered walk about 85 feet in length. In this walk was dug a longitudinal trench, 20 inches in width, and 3 feet

4 inches in depth. In this trench was placed a line of beams 82 feet in length, and 12 inches square; 16 inches above this line of beams was placed another line of similar dimensions, supported by uprights at about every 10 feet, and strongly strapped to these uprights.

The extremities of each of the beams abutted against uprights of oak, the tops of which stood a few inches higher than the surface of the upper beam. In the head of each of these uprights was bored a hole $2\frac{1}{2}$ inches diameter, the lower part of which was exactly on a level with the surface of the beam. In this hole works the screw bolt, *aa*, $2\frac{1}{4}$ inches diameter, and 31 inches in length (see *Figs.* 1 and 2, *Plate* XIV.). This bolt carries a screw for about half its length, and the other end is attached to the crupper of the cable. By means of a small lever about 3 feet in length, a nut, through which the screw passes, is worked round so as to adjust the end of the bolt, *aa*, to any required position during the fabrication of the cable. The small crane *b* (*Fig.* 2, *Plate* XIV.) was employed to move the weight which each wire was required to support at the instant of its inflection upon the crupper. The vertical post of this crane swung round as a pivot, and the end of the horizontal arm carried a small pulley.

In commencing the manufacture of a cable, the extremity of one of the wires wound upon a reel was attached to one of the beams, and passed through the neck of the crupper. The wire was then carried to the other extremity of the walk, and supported against the neck of the other crupper; about 2 feet beyond this the wire was clasped by a pair of pincers attached to a cord which passed over the pulley on the horizontal arm of the crane. To the other end of this cord was attached a shell weighing 220 lbs., which ordinarily rested on the ground. When the strain was to be applied to the wire the crane was swung round on its pivot, so as to tighten successively the cord, and then the wire; it is evident that at the moment when the arm of the crane stood out at such an angle as to force the whole length of wire and cord into a perfectly taut condition, the weight was raised from the ground and supported by the wire and cord. By the same process, and with a similar apparatus, the weight was raised by the wire at the other end of the walk. Whenever the least variation was found in the distance between the two cruppers, whether arising from alternations of temperature or other causes, this distance could be exactly re-adjusted by means of the screws, already described, at the end of the walk. When all the threads of the cable had been thus stretched and placed together, the end of the wire was united to the first end which had been temporarily attached to the beam. In this state each cable was well payed over with a coating of oil varnish, prepared as before, and this varnish was forced as much as possible into all the vacuities between the wires.

The cable was then strongly bound with annealed wire (No. 14), at the extremities of the cruppers. This wire ligature was continued for about 2 feet in length. The threads of the cable were then bound into one solid bundle by a close spiral envelope of wire for the whole of that length which was destined to be placed in the mooring shafts. The remainder of the cable was bound at every 2 feet with a ligature for the length of 8 inches; and in order to press these ligatures more tightly, and to render the cable more cylindrical, a kind of circular vice was contrived to fit the figure of the cable, and the two

parts of this vice, being forced together by means of a screw, a great pressure was applied to every ligature.

When the first cable had been thus fabricated, it presented a most satisfactory appearance; but it was no sooner taken from the frame on which it had been stretched than the elasticity of the great mass of wire which composed it came into play, and caused it to assume a series of double curves like those of a corkscrew. It would have been impossible to force it in this state into the opening of the mooring shaft. After many ineffectual efforts to maintain the cable in a straight line when at perfect liberty, in order that it might be placed in its position in the mooring shaft, M. Chaley adopted the following expedient. He caused to be prepared a number of small deal laths cut from green wood; these laths were ⅞ of an inch thick and 2 inches wide, and the whole length of the cable being enveloped in four thicknesses of these, the whole mass was firmly tied round with ligatures of annealed wire at every 9 or 10 inches. Thus enveloped in a case of wood, the cable was left to itself, and experienced no change except a very small amount of torsion. This kind of packing round the cables had the further advantage of protecting them from injury during the operation of fixing them in their places.

Eight cables of the same length, intended for the two sides of the valley, were successively fabricated and packed round in this manner. *Fig.* 1 is a section of one end of the frame, showing the long beams, the upright, the tightening screws, and crupper. *Figs.* 2 and 3 are an elevation and plan of one end of the frame, showing the long beams, the upright, the crupper, and part of the cable in course of manufacture; also the crane, with the weight suspended over the pulley. The crane is here shown in the position it takes when the weight is being supported by the wire. The dotted lines on the plan and section show the position of the arm of the crane when the weight is at rest on the ground.

Wind Stiffening.

Wind guys. To counteract the lifting action of winds it is customary to take separate wind ties from either bank below the roadway to certain parts of the roadway.

A method proposed by Mr. Brunel for a suspension bridge in a country subject to hurricanes was to have an inverted suspension chain in parabolic form beneath the roadway. This chain may be splayed laterally at the ends of the span.

For lateral wind pressure the main chains may be similarly splayed, and the whole of the roadway should be stiffened by diagonal bracing. (See the plan of the Jumna Bridge, *Plate* XV.).

Captain Capper, R.E., in a valuable paper on suspension bridges in the Himalayas,* has pointed out that where a bridge is made too close to highest water level, wind ties may be caught by some floating obstruction and wreck the bridge. This must be borne in

* *R.E. Professional Papers*, 1896.

mind in design. Captain Capper proposes that where head room is possible the dip of the inverted cables (on the parabolic form proposed by Brunel) should be about half that of the main span. Those used at the Bunji bridge over the Indus were two $1\frac{1}{2}''$ steel wire ropes made fast to anchorages in the rocks about 40′ up and down stream, working on rollers attached to the cross beams, and overlapping at the centre of the bridge as shown in *Fig.* 153, which shows the ties in *plan.* These ties could not be taken much below the level of the roadway as the bridge was too close to the highest water level.

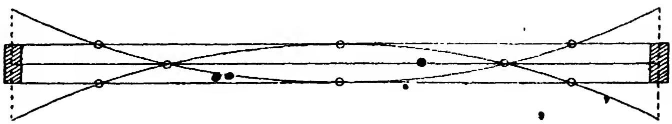


Fig. 153.

Erection of Suspension Bridges.

A few notes of a general nature may here be given on the subject of erection. Erection.

In comparing relative methods, speed and economy are of less importance than safety.

Where it is possible to do so, scaffolding or temporary staging of some sort is undoubtedly the easiest method, but the fact that suspension bridges are usually made over a gorge or some other place where an intermediate support is impracticable, renders this class of erection rarely possible. Scaffolding.

Even if not practicable over the whole span, scaffolding may be very useful over some part of it.

In rivers where the height of the roadway is at no great distance above the water, and where the force of the current admits, such scaffolding may either be supported on piles or on floating supports anchored. In any such case, the framing should be so designed in a self-contained manner, of separate braced portions so lashed together that in the event of any sudden rise of water, or approach of a storm, the lashings may be cut in a few minutes, the frames float off and be secured lower down stream. An anchor can also be attached to each frame so as to catch it when cut adrift.

In any case the erection of the abutment towers, piers, and construction of the anchorages would have to precede other work.

Erection at Fribourg.

In the case of the Fribourg bridge, the ground directly under the bridge was available for operations, though the height was too great for scaffolding. In this case the cables were conveyed on a large drum $6\frac{1}{2}'$ in diameter to a spot below the bridge. Meantime scaffolding had been erected round each tower, and on the top of the scaffolding at each side were fixed two windlasses, and a third windlass was erected in the line of the bridge towards the anchorage. On the axle of this a 1″ hempen rope some 760′ long was wound then passed with one or two turns over the windlasses on the pier, and then secured to a lighter rope, the other extremity of which was fixed to a fourth windlass in the valley below at the position of the $6\frac{1}{2}'$ drum. The same was done to the other end of the bridge. The position of the windlasses is indicated on *Fig.* 1, *Plate* XII.

The two extremities of the suspension cable were then strongly secured at either side to the hempen cables, the windlasses set to work and gradually the whole of the cable was wound off the drum. When this was done, the workmen at one side ceased operations, the one end was hauled over the pier, made fast temporarily and then the other end hauled over the other pier, and made fast temporarily. The main cables were afterwards made fast permanently to the mooring cables.

Temporary bridge across the chasm.

When the work has entirely to be carried out from one side or both, a temporary suspension bridge is necessary. This should be attached to scaffolding or staging resting on the main piers. To get it across in the first instance various expedients have been adopted (bow and arrow with string, small mortar, etc.) which need not here be given in detail. A light traveller which can be hauled backwards and forwards, should be slung to the temporary wire rope, and by means of it the main cables can be got over and fastened.

After the main cables are over and adjusted there is less difficulty about the remainder of the structure, as the main cables form a base of operations for the rest of the work. It was by this means that the cables for the bridges over the Indus, erected by Captain Capper, were got into position, and it was by this means that the Clifton bridge was built up.

Olvusâa bridge.

At the Olvusâa bridge a $\frac{5}{8}''$ steel rope was stretched from the top of one pier to the bottom of the next, and a derrick of suitable length was erected over the centre of one pier. The cable has one end attached to the anchorage, and a tackle was fastened to it some 30′ back. It was then hoisted to position A (*Fig.* 154) and placed

in an S hook riding on the $\frac{5}{8}''$ rope. The length was hauled out by the winch D, and the operation repeated till the cable was across, when the end was made fast to the anchorage on the far bank. A sling chain was then attached to the cable and it was lifted into position on the quadrant. To enable the roadway to be built out, a platform, hung from the cables by the four corners, and provided with tackle to raise or lower it to the required level, was used.

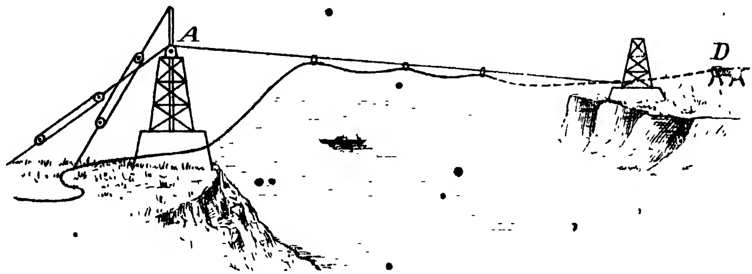


Fig. 154.

Captain Capper mentions, *inter alia*, the following points about erection :—

Get some temporary bridge over if possible.
Always put up an overhead wire.
Have plenty of pulley's and light rope.

Make up everything you can on shore and fit it there. It is difficult to get men to work well and quickly on a shaking platform.

APPENDIX I.

Proof that the curve of the suspension chains is a parabola.

PROOF that the curve of the chains in a suspension bridge is a parabola. Let the curve OPA (*Fig.* 155) represent half the span of a suspension cable loaded with a load w per foot run. Let a = the total span, and d = central dip or versed sine. If half the bridge be removed and the tension on the chain at its lowest point be represented by a force H, this force will be horizontal.

Take the point O where it acts, as the origin of co-ordinates, and let the ordinates of any point P on the curve be PD = y, and OD = x. Then taking moments at P

$$H \times y =$$

$$x^2 = \frac{2H}{w} y$$

which is the equation to a parabola referred to its vertex.

Taking moments at A the top of the tower,

$$H \times d = \frac{wa^2}{8} \text{ or } H = \frac{wa^2}{8d} \quad \ldots (2).$$

This represents the uniform horizontal stress. Substituting this value of H in (1), we get

$$x^2 = \frac{a^2}{4d}y \text{ or } y = \frac{4dx^2}{a^2} \quad \ldots\ldots\ldots (3).$$

This gives us the length of the suspension rods to the tangent at the lowest point.

The tension in the cable at any point

$$= H \sec\theta = H\frac{EP}{ED},$$

and when

$$x = \frac{a}{2} \text{ and } y = d,$$

$$\sec^2\theta = \left(\frac{a}{4}\right)^2 + d^2 \div \left(\frac{a}{4}\right)^2,$$

and tension

$$= \frac{wa}{2}\sqrt{1 + \left(\frac{a}{4d}\right)^2} \quad \ldots (4).$$

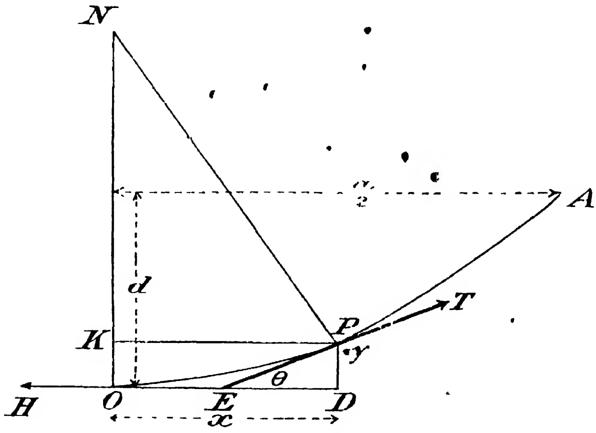


Fig. 155.

Another proof is as follows:—

Taking the same notation as before, we see that the arc OP is kept in equilibrium by three forces—(1), the tension H at O; (2), the weight of the roadway along OD, *i.e.*, wx; and (3) the pull T on the chain at P, which is tangential to the curve at that point. These three forces are parallel to the sides of the triangle PED, hence

$$\frac{wx}{H} = \frac{PD}{ED}.$$

But $PD = y$ and $ED = \frac{1}{2}x$, since the weight on OD is uniformly distributed, and therefore the three forces H, wx and T must pass through the same point E.

Hence
$$\frac{wx}{H} = \frac{y}{\frac{1}{2}x};$$
$$x^2 = \frac{2Hy}{w},$$

which is the equation to a parabola, as above.

From P draw PN (*Fig.* 155) at right angle to PE, and ON at right angles to DO. Then PK being drawn parallel to OD, the triangle PKN is similar to PDE, and the three forces acting on OP are proportional to the sides of the triangle PKN. Then

$$T : xw :: PE : PD :: PN : PK :: x \operatorname{cosec} \theta : x.$$

Hence
$$T = wx \operatorname{cosec} \theta,$$

where θ = angle of inclination of the tangent to the horizontal.

When $x = \frac{a}{2}$ and $y = d$

$$T = \frac{wa}{2}\sqrt{1 + \left(\frac{a}{4d}\right)^2}$$

Since
$$T : xw :: PN : PK$$
$$:: PN : x,$$
$$T = w \times PN,$$

or the tension of the chain at any point is equal to the line normal to the tangent at that point and intercepted by the axis of the curve, multiplied by the unit of weight on the chain. It is easy, therefore, to find graphically the tension at any point on the chain.

APPENDIX II.

To Find the M_β and Shear in Stiffening Girders.

When the load is not uniform, the cable will tend to swing into a new curve, a tendency which will be increased as the dip d increases, and the tension H diminishes, for we see from Equation (2), Appendix I., that H varies inversely as d. Investigation of stiffening girder

The object of the stiffening girder, upon which the load primarily rests, is to distribute the load uniformly upon, and therefore to prevent any deformation of the cable. Under such circumstances the stress in the suspension rods is constant from end to end. If the load is not uniformly distributed on the cable the girder is not doing its work.

Another assumption which will be true when the stiffening girder does its work properly is that its deflection, if acting alone, would be considerable as compared with the deflection of the cable when the load is distributed uniformly all over it. If, therefore, the cable does not appreciably increase its dip for the assumed distribution of the concentrated load, then the cable may

be assumed to carry all the live load, while the girder merely serves to distribute it.

The condition of equilibrium of the external forces acting upon the girder requires that the algebraic sum of the vertical components, and of their moments shall = 0. These external forces are, on each girder :—

(1). A uniform downward load of $\frac{1}{2}p$ lbs. per foot over the distance x (*Fig.* 156), that distance being supposed to be under the live load px, half of which is borne by each girder.

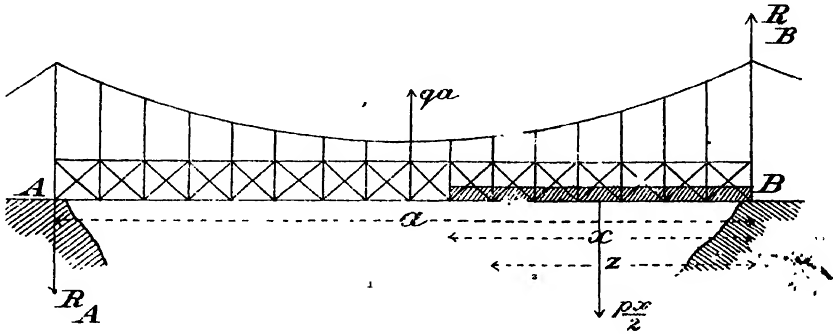


Fig. 156.

(2). A uniform upward pull of q lbs. per foot from *each* cable over the entire span.

(3). The two end reactions R_A and R_B.

We have assumed by hypothesis that $\frac{1}{2}px = qa$, or $q = \frac{px}{2a}$.

The c.g. of the downward forces $\frac{px}{2a}$ is $\frac{x}{2}$ from B, while the c.g. of the suspension chain and rods is at the centre of the span, *i.e.*, $\frac{a}{2}$ from B. Hence as the upward forces = downward ones they form a couple whose arm is $\frac{a-x}{2}$, and moment $\frac{px}{2}\left(\frac{a-x}{2}\right)$. This couple can only be balanced by another couple of the same moment, *i.e.*, the end reactions, whose arm is a.

Hence $$+R_A a = -R_B a = \frac{px}{2}\left(\frac{a-x}{2}\right).$$

Hence $$+R_A = -R_B = \frac{px}{4a}(a-x) \quad \ldots\ldots \quad (5).$$

Having found thus the external forces which act upon the girder, we can investigate the maxima bending moments and shearing stresses. Since the end reactions are both positive and negative for different loadings, the girder must rest upon the piers and *be anchored to them* sufficiently to resist the maximum negative end shear or reaction.

Investigation for Maximum Shear.

Let the uniform passing load cover a distance from $B = x$, and let z be any distance from B, either greater or less than x.

Then when z is less than x the shearing stress at z, S

$$= -R_A + q(a - z) - \frac{p}{2}(x - z).$$

Substituting

$$\frac{px}{2a} \text{ for } q \text{ and } \frac{px}{2a}\left(\frac{a - x}{2}\right) \text{ for } R_A,$$

$$S = \frac{px}{4a}(x - a - 2z) + \frac{pz}{2} \quad \ldots \quad (\alpha).$$

Similarly, when z is greater than x

$$S = \frac{px}{4a}(x + a - 2z) \quad \ldots\ldots (\beta).$$

For a given load $\frac{px}{2}$, we see that S is a maximum when $z = x$, and a minimum when $z = a$ or $= 0$.

Thus when $z = x$ we have, from (β)—

$$S = \frac{px}{4a}(a - x) \quad \ldots\ldots (\gamma).$$

and when $z = a$ or $= 0$ -

$$S = \frac{px}{4a}(x - a) \quad \ldots\ldots (\delta).$$

Now $a - x$ is greater, positively, than $x - a$.

Hence we see that the shear at the head of the load is always numerically equal to, but of opposite sign from, the shears at the ends of the span. In the centre when $x = \frac{a}{2}$ we have the greatest value of the shear.

Substituting in (γ) and (δ) the value of $x = \frac{a}{2}$,

$$S = \tfrac{1}{16} pa \quad \ldots \quad (\eta).$$

Maximum Bending Moments.

When z is less than x—

$$M_f = -R_A(a - z) + \frac{p(a - z)^2}{2} - \frac{p(x - z)^2}{4}.$$

Substituting value of R_A—

$$M_f = -\frac{px}{4a}(a - x)(a - z) + \frac{px}{4a}(a - z)^2 - \frac{p(x - z)^2}{4} \quad \ldots\ldots\ldots\ldots (a).$$

Similarly, when z is greater than x—

$$M_f = -\frac{px}{4a}(a-x)(a-z) + \frac{px}{4a}(a-z)^2 \quad \ldots\ldots\ldots\ldots (b).$$

When $z=x$, we see that $M_f=0$, and in (a) M_f is positive, and in (b) it is negative. Hence *the head of the load is always a point of contraflexure.* We have already learnt that the shear here is always equal and opposite to that at the two ends, hence we see that both the loaded and unloaded portions of the girder can be treated as a simple beam uniformly loaded. This leads us to conclude that the M_{fl} occurs at the middle part of the unloaded and of the loaded portions, the former negative and the latter positive.

When $$z=\frac{x}{2},\ M_{fl}=\frac{px^2}{16}\left(1-\frac{x}{a}\right) \quad \ldots\ldots (c),$$

and when $$z=\frac{a+x}{2},\ M_{fl}=-\frac{px}{16a}(a-x)^2 \quad \ldots\ldots (d).$$

To find for what position of the load M_{ff} is a maximum we differentiate equation (c)—

$$\frac{dM_f}{dx}=\frac{px}{8}-\frac{3px^2}{16a}=0,$$

whence $$x=\tfrac{2}{3}a \quad \ldots\ldots (e).$$

Similarly differentiating equation (d) we have

$$\frac{dM_f}{dx}=a^2-4ax+3x^2=0,$$

whence $$x=\tfrac{1}{3}a \quad \ldots\ldots (f).$$

Therefore the maximum downward moment occurs at the middle of the load when it extends over $\frac{2}{3}$ of the span, and the maximum upward moment occurs at the middle of the unloaded portion when the load extends over $\frac{1}{3}$ of the span.

Inserting the maximum values of x in equations (c) and (d) we find

$$M_{ffl}=\frac{pa^2}{108} \quad \ldots\ldots (g)$$

for both upward and downward moments.

For a load $\frac{p}{2}$ per foot acting on a simple truss a in length the M_{ff} would be $\frac{pa^2}{16}$.

Hence in the suspension bridge the M_{ff} is about $\frac{1}{4}$ of the M_{ff} due to the same unit load acting on the same span unsupported by the cable.

The maximum shears were found to be $\frac{1}{16} pa$ or just $\frac{1}{4}$ what they would be on simple trusses of the same length. The M_{ff} and shearing stress diagrams are as shown in *Fig.* 157 from calculations worked out by Col. A. Cunningham, R.E. Practically they may be taken as uniform throughout, as stated on page 168.

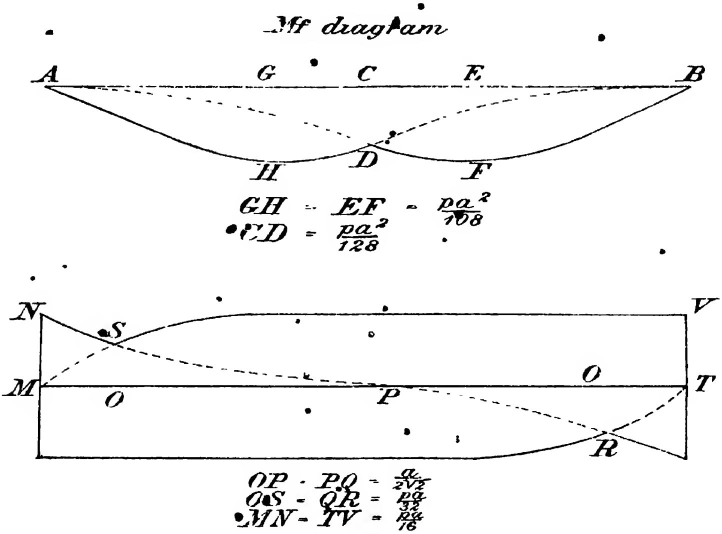


Fig. 157.

CHAPTER VIII.

Cantilever Bridges and Movable Bridges.

Cantilevers. Use in Military Work.—Example.—Bascules.—Swing Bridges.—Drawbridges.—Floating Bridges.

Cantilever Bridges.

In this chapter it is intended to describe briefly the principles of design of cantilever bridges, illustrated by an example, and then shortly to mention without going into details various classes of movable bridges.

Cantilever Bridges.

The practical disadvantages of the shifting of the points of contra-flexure in bridges of continuous beams subject to passing loads has already been pointed out. This may be met by introducing a break in the continuity of the girder by a hinge or other form of connection, and thus mechanically fixing the points of contra-flexure.

Thus in *Fig.* 158, which represents a 3-span bridge, the ends A and C are supported, or anchored, on hinged bearings, and there may be hinges at BB′, which cause the portion BB′ to be an independent girder supported on the cantilevers AE and CB′.

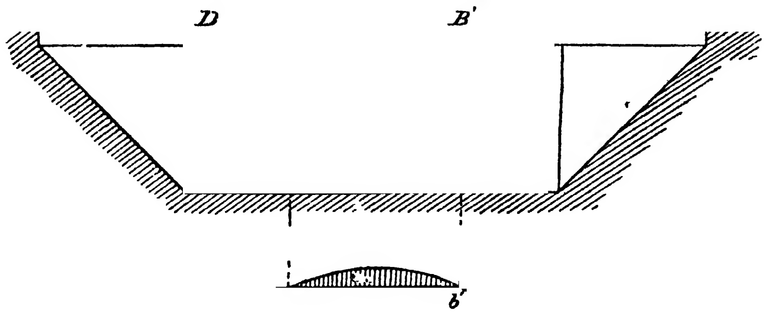


Fig 158.

To find the M_f diagram when the points BB′ are settled, all that we need do is to draw the M_f diagram for the span DE as if it were

supported only, mark the positions of BB′ at *bb′*, join *bb′* and produce it both ways. The ordinates above and below that line give the M_f values at all points. The values of the M_f for various loads, etc., will be found from the principles given in Chapter IV., Part I., Cases 11 and 12.

This principle of cantilever design has been adopted for many large bridges, *e.g.*, the Forth Bridge, the Lansdowne Bridge at Sukkur, etc.

Apart, however, from these colossal structures, which are of a class beyond the ordinary designs likely to be carried out by those for whom this work is intended, the cantilever principle can be applied under certain circumstances with great advantage.

In the Himalayas the inhabitants have long recognized the advantage of this principle, and many of the native bridges are formed of logs of timber securely weighted at one end and carrying at the other the ends of a beam (*Fig.* 159). The span is thus subdivided into three parts, two cantilevers and a beam. Himalayan Bridges.

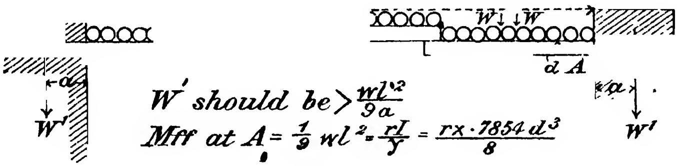


Fig. 159.

Bridges on this principle have been frequently constructed in mountain warfare, notably in the recent Chitral Expedition. (*R.E. Professional Papers*, 1896, Paper VI.). The limiting span in this expedition was 120 feet (*Fig.* 160).

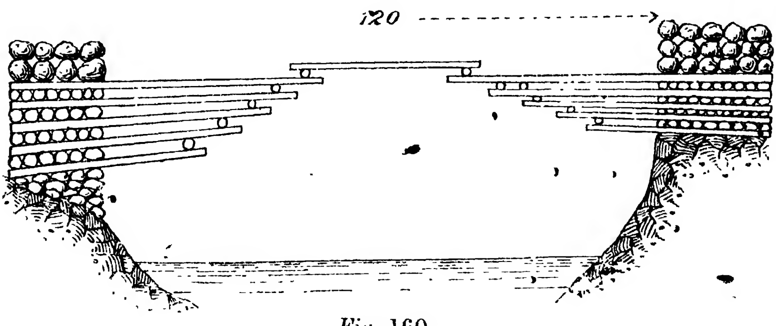


Fig. 160.

Use of ties.

The advantages of this form of bridge are that it can be rapidly made without skilled labour, and with the simplest materials and tools. The deflection and "spring" of the bridge is the chief disadvantage. This can be obviated by having wire ties, as in *Fig.* 161.

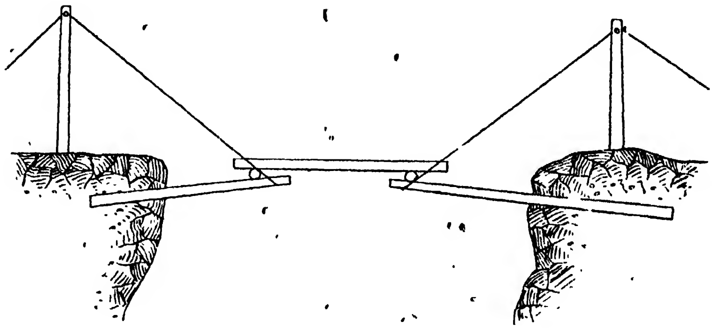


Fig. 161.

Wangtu Bridge

The Wangtu Bridge over the Sutlej (*Plate* XVI.), designed and built by Captain (now Colonel)* Lang, R.E., is a good illustration of this class of structure applied to permanent roads. The span is 120′, the central portion between tips of longest cantilevers being 30′. A Howe truss $5\frac{1}{2}'$ deep is carried across this opening, but with a bearing of 45′, and continued to either abutment. The cantilevers marked *d* are four in number, 15′ projection. Those marked *e* are five in number, 10′ projection.

To find the stresses on this bridge we may assume that the weight per square foot of roadway was 40 lbs. (French rules give 41 lbs. for highway bridges remote from towns. In this case, the region is in the heart of the Himalayas). With a roadway of 10′ wide and 45′ span this gives for each truss 9,000 lbs. = W.

$$M_R = \frac{Wl}{8} = \frac{9000 \times 45 \times 12}{8} = 607{,}500 \text{ inch-lbs.}$$

$$M_1 = rad = r \times 9'' \times 4''\dagger \times 66'' = 2376r.$$

$$2376r = 607{,}500.$$

* *Roorkee Treatise*, Vol. II., p. 296.

† a = area of upper or compression member of the truss, *i.e.*, $9'' \times 4''$.

$r = 256$ lbs. as the value of the compressive stress per unit. This is well within the limits of strength for deodar (3760), and, in addition, it may be noted that the M_{ft} would, owing to the continuity of the girder, be rather less than $\frac{Wl}{8}$, probably $\frac{Wl}{10}$, and the load would seldom amount to 40 lbs. per square foot.

Braces.—The end reaction of each girder (15′ span) being 4,500 lbs., the stress on each brace is $4{,}500 \times 1{\cdot}414$ (*i.e.*, cosec 45°) = 6,350 lbs. The scantling is 5″ × 4″, and length 6′. Hence ratio of length to least dimension is 72 : 4 or 18 to 1. Hence $P = \frac{1}{2} r_c A$ (see p. 160, Part I.). $8350 = \frac{1}{2}\, 20 \times r_c$. $r_c = 635$, which is amply safe.

Cantilevers d.—Four of these carry 9,000 lbs., or each 2,250 lbs., and the length is 15′ or 180″. Hence $M_{ft} = 180 \times 2{,}250 = 405{,}000$ inch-lbs., $= M_r = \frac{rI}{y}$, which for round logs $= {\cdot}7854\, R^3 \times r$. Taking a value of r at 1,200, $R^3 = 430$. Hence $R = 7{\cdot}55$, and diameter $= 15{\cdot}1''$, say 16″.

Cantilevers e.—Five of these carry the weight borne by the cantilevers d + about 9′ of roadway, *i.e.*, $9{,}000 + 9 \times 10 \times 40 = 12{,}600$ lbs., or each carries 2,520 lbs. The length is 10′. Hence $M_{ft} = 120 \times 2{,}520 = 302{,}400$ inch-lbs., whence R (calculated in the same way as before) $= 6{\cdot}84$. Make 14″ diameter.

In like manner the cantilevers b and a would be calculated.

The ends of the cantilevers are not only weighted, but anchored to the solid rock.

Movable Bridges.

These may be subdivided into :— Movable bridges.

1. Bascule bridges.
2. Swing bridges.
3. Drawbridges.
4. Floating bridges.

1. *Bascule bridges* are those which revolve round a horizontal axis, Bascules. and may either be lifted by a chain passing over a pulley, or constructed with counterpoised weights, and working by a rack and pinion. The most notable instance of this class of bridge is the recently finished Tower Bridge in London, where there are two bascules spanning an opening of 200 feet.

The lower member in such bridges may be arched so that the bridge when closed forms a complete braced arch, hinged at the

crown. Many drawbridges in fortifications are on the bascule principle with counter-weights. (See Colonel Lewis' *Fortification*).

Swing bridges, various types. 2. *Swing bridges* are of several classes.*

(*a*). Those which turn entirely on rollers or wheels.

(*b*). Those in which the weight is proportioned so as to be in part borne by the rollers and in part by a centre pivot.

(*c*). Those entirely swung on a centre pivot.

(*d*). Those which are lifted on a water centre by hydraulic power

(*e*). Those that rest and turn on a water centre, having a constant upward pressure, but not sufficient to lift the whole weight.

(*f*). Those where the weight is almost entirely buoyed up, having only a small portion resting on rollers.

Whatever be the pivoting arrangements of a swing bridge as indicated above, there are certain features common to all classes.

When not swinging, the movable part is evidently a continuous girder over two equal spans, or two large and one small centre span. The M_f diagram will then be somewhat as shown in *Fig.* 162 for the dead load, or for a live load all over. When the bridge is swinging, the M_f diagram will be as shown in dotted lines. Some of the parts of the bridge near the circumference of the swinging circle will be under reversed stress, and will, therefore, require counter-bracing.

The details of the machinery do not come within the scope of this work.

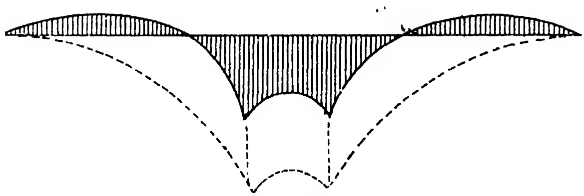


Fig. 162.

In some cases swing bridges may be constructed with a short arm counter-weighted instead of two arms of equal lengths. An instance of such a structure is described in Colonel Lewis' *Fortification, Plate* XXI.

Drawbridges. 3. *Drawbridges.*—These may be of the bascule class, as above mentioned, or may be *traversing*, *i.e.*, moving backwards and forwards horizontally, a principle which has also been applied to large bridges in some recent instances, *e.g.*, over the Dee at Hawarden in Cheshire.

* *Minutes of Proceedings, Institution of Civil Engineers*, Vol. VII.

There are many different types of drawbridges used in connection with permanent fortification described in Colonel Lewis' *Permanent Fortification.* For the most part these are suited chiefly to the requirements of forts only, and are not of general utility. There is, however, one exception to this rule, and that is Lieut. (now Major-General Sir John) Ardagh's bridge. (See *Plate* XVII.).

This revolves round a horizontal pivot, yet it is not of the ordinary bascule type, as there is no counter-weight. The following description is taken from a paper by the inventor in *R.E. Professional Papers*, 1868 :—

"The drawbridge now proposed requires no counterpoises, and may be made twice the length of the old drawbridges with the same convenience. The bridge OX, when down, is supported at the inner extremity by two keys OO, which may be made self-acting, but which it is better (for military purposes) to work by hand, and at the outer extremity it rests as usual on the standing part. At the points B it is supported by the tension bars YB, which have axes at each extremity. Sir John Ardagh's bridge.

"When the bridge is raised it follows the motion indicated in the diagram, where it is shown in an intermediate position, xy, $x'y'$.

"The point A represents the centre of gravity of the bridge moving on the horizontal line XX', a condition which ensures equilibrium in every position.

"The point of suspension B moves on the circumference of the circle, of which Y is the centre, and YB the radius.

"The points xy and $x'y'$ will then be constrained to describe the curves shown in the diagram.

"In order to investigate the general problem, let it be assumed that the bridge (at a certain intermediate position) makes an angle of $\phi°$ with the horizontal, and that the given quantities are represented as follows :—

$$AB = a,\ Bx'y' = b,\ Axy = c,\ YB = r, \text{ and } YO = p.$$

Then $\qquad y = c \sin \phi.$

$$\therefore \quad \sin \phi = \frac{y}{c},$$

$$x = (a + c) \cos \phi + \sqrt{r^2 - (p + a \sin \phi)^2},$$

$$x = (a + c)\sqrt{1 - \frac{y^2}{c^2}} + \sqrt{r^2 - \left(p + \frac{ay}{c}\right)^2} \ldots \ldots\ldots(1).$$

And $$y' = -(a+b)\sin\phi,$$

$$\therefore \qquad \sin\phi = -\frac{y'}{a+b}.$$

$$x' = -b\cos\phi + \sqrt{r^2 - (p + a\sin\phi)^2},$$

$$x' = b\sqrt{1 - \left(\frac{y'}{a+b}\right)^2} + \sqrt{r^2 - \left(p - \frac{ay'}{a+b}\right)^2} \ldots\ldots (2).$$

"The equations (1) and (2) are in a convenient form for calculation, and represent the curves shown. As the end xy moves in the air, we are only concerned to ascertain the path on which it is necessary to guide the inner extremity $x'y'$.

"In the simplest form of bridges of this description, where the centre of gravity bisects the span, and the point of suspension meets the outer extremity of the bridge when raised—

$$a + b = c = p, \quad a + c = r = \sqrt{c^2 + b^2}.$$

Hence $$(c-a)^2 = b^2 = (a+c)^2 - c^2,$$

or reducing $$c^2 - 2ac + a^2 = a^2 + 2ac.$$

$$\therefore \qquad c^2 = 4ac \text{ and } c = 4a = p,$$

$$b = 3a \text{ and } r = 5a.$$

$$a : b : (c-p) : r :: 1 : 3 : 4 : 5.$$

"This makes the calculations exceedingly simple, for substituting in equation (2) the values of the other constants in terms of a we have

$$x' = -b\sqrt{1 - \frac{y'^2}{(a+b)^2}} + \sqrt{r^2 - \left(p - \frac{ay'}{a+b}\right)}$$

$$= 3a\sqrt{1 - \left(\frac{y'}{4a}\right)^2} + \sqrt{(5a)^2 - \left(4a - \frac{y'}{4}\right)^2}.$$

"In a span of 20 feet, a will be $= 2 \cdot 5$, and the equation takes the determinate form of

$$x' = -7 \cdot 5\sqrt{1 - \frac{y'^2}{100}} + \sqrt{156 \cdot 25 - \left(10 - \frac{y'}{4}\right)}$$

which, for example, placing the value of y' at -6, will give

$$x' = -7 \cdot 5\sqrt{1 - \tfrac{36}{100}} + \sqrt{156 \cdot 25 - 132 \cdot 25}$$

$$= \mp 6 \pm 4 \cdot 8 = \pm 1 \cdot 2 \text{ and } \pm 10 \cdot 8.$$

The co-ordinates of the four points on the curve to $y = \pm 6$.

"All the calculations are made in the same manner, and the curve may be set out either from the values thus obtained, or geometrically, or mechanically. The inner extremity of the bridge bears, through the intervention of trucks, on rails adjusted to the curve thus determined.

"When it is altogether closed, its upper part occupies the exact position which would be required for a common drawbridge of half its span.

"The force required to move it will be merely that which is needed to overcome the friction of the axes at Y, B and O.

"The mode in which it is proposed to apply the force is an integral part of the scheme.

"A rail, DC, runs through a ring at D, and works on an axis at C attached to the standard and strut fixed on the bridge at this point.

"A moment's consideration will show that when the bridge is up the strut AC will lie in the position OD: and that the axis, in travelling from C to D, will describe a very flat curve, so that the rail may run through the ring at D with ease.

"It is proposed to apply the force for moving the bridge to this rail, the power in small bridges being direct manual labour, and in very large ones the rail being drawn in by a winch and friction-wheel.

"The absolute weight required to move a very roughly constructed model of the bridge, the platform of which weighed $6\frac{1}{4}$ lbs., was on an average 8 ounces, being $\frac{1}{12}$th of the weight put in motion. Any-one who has seen the model will admit that the friction of the full-sized bridge is not likely to be proportionately so great; and in the experiment related the force was applied to one rail only.

"The best experiments on frictional resistance will give a smaller result. Morin's co-efficient for cast iron upon wrought is ·07 and ·08. Taking the larger, and bearing in mind that the rail moves through 16 feet while the bridge moves 10, the necessary power would be—

$$P = W\frac{10 \times \cdot 08}{16} = \cdot 05, \text{ or } \tfrac{1}{20}\text{th.}$$

"Let the bridge be assumed to weigh 5,000 lbs. The force required to move it will be $\frac{5000}{12} = 416$ lbs., or $\frac{5000}{20} = 250$ lbs., according to the different hypotheses, or from six to four men—a number which would certainly be available to guard against contingencies. Gun-tackles may be provided to hook on to the eyes of the rails (or friction gear, as previously mentioned). One man on each side would then be sufficient to raise or lower the bridge.

"The middle rail will move in a similar manner, but need not be employed for the working of the bridge.

"The rails CE, joining the fixed with the moving parts of the bridge, are of iron pipes, fitting on studs, at either end, the long studs being at E and the short ones at C, so that if the bridge be closed, however suddenly, these connecting rails will merely fall off their studs without interfering with its motion.

Uses of this bridge in fortification.

"No particular dimensions are proposed for this kind of bridge, but it is suggested that 14 and 21 feet would be convenient limits, the former for sally-ports, the latter for large fortresses. They would then cover gateways, respectively 7 and 12 feet high. Of such small span are many of the existing drawbridges, that it would seem desirable, when any of the old platforms are renewed, to substitute bridges of this construction, which might be done at a very slight expense.

"In applying this principle to dock, river, and canal bridges, it will most frequently be convenient to make the bridge in two parts, cambered in the centre, as shown in the smaller section on the plate, where half the bridge is shown in an intermediate position.

"The standards in these cases will require to be but half the height necessary for the ordinary lifting bridge, and no counter-balancing weights are required—decided advantages when the opening is in the middle of a long lightly-built bridge.

"Here, perhaps, it would be better to raise the bridge by chains instead of by the rails CD, and to employ a second set of chains attached at OO for lowering it.

"It is important to observe that C is the best point to apply a force to move the bridge either in or out, better even than the centre of gravity A; and that a force applied in the direction XY would be not so effective and much more troublesome."

As regards the tension on suspending rod, it is evident that if the end X of the bridge be resting on a support, and the end O be supported on keys, the re-action R at B can be ascertained from the consideration of the beam being continuous over two spans OB, BX. If the beam be not bearing on a support at X, the case will be that of a beam strained over a pier, as in *Fig.* 60, p. 79, Part I. In either case, having found the re-action, the stress on the bar YB is $\frac{1}{2}$R sec ϕ, where ϕ is the angle of inclination of the bar with the vertical.

When the beam is in the position shown in *Fig.* 3, ϕ being the angle of BY with the vertical, or of AB with the horizontal, and W

the weight of the bridge, then the tension on $BY = \frac{1}{2}W \cos \phi$, and the horizontal pressure against the wall at $x'y' = \frac{1}{2}w \sin \phi \cos \phi$.

The pressures on the piers for such a bridge as that shown in *Fig.* 5 are easily ascertained by principles already considered under the heading of suspension bridges.

This ingenious principle has not, so far as the writer is aware, been applied to bridges for docks, etc., and yet there can be no doubt about its economy, both as regards its construction and power required to open it.

4. *Floating Bridges.*—These call for little comment, as the subject, which is so important from a military point of view, is exhaustively treated in text-books on military bridges. From a permanent point of view these bridges have the advantage of cheapness; but they are liable to be damaged by floods, are dependent either on bottom anchorage, or on the attachment to a horizontal mooring cable secured on both banks, and they impede navigation.

CHAPTER IX.

Bridge Piers of Timber and Iron.

Various Forms of Piers for Bridges.—External Forces.—General Form of Design.—Stresses in Various Members.—Details of Construction.

Bridge Piers. In previous chapters we have discussed various types of bridge *spans* in timber and iron, but the subject is not complete without some consideration of the design of *piers* of those materials.

Masonry generally used in England. Most of the information in this chapter has been derived from two standard works, viz:—Professor Warren's *Engineering Construction in Iron, Steel, and Timber*, and *Modern Framed Structures*, by Professors Johnson, Bryan, and Turneaure. In Australia and America (for which countries those books are primarily written) timber piers are far more extensively used for railway bridges than they are in England, where, indeed, they are now but sparingly employed. Even piers of iron and steel are less used in England than some other countries, although there are some notable exceptions, *e.g.*, the main span piers of the Forth Bridge, which are 360 feet high. Even where the piers are of masonry, they are built, in one case at least (the Tower Bridge), on a skeleton of steel. For the most part, however, we have to go abroad to find the braced pier largely used in bridge construction.

Importance for military engineers. For military engineers the subject is of great importance, because the temporary or semi-permanent bridges which have to be erected for military purposes are more expeditiously made with braced piers than solid piers of masonry or concrete.

Foundations. Such piers as those which we are about to consider may either be borne on a foundation of piles—the piles themselves being continued, in some instances, above ground to form the standards of the trestles—or they may be based on foundations of masonry, to which they are anchored by means of bolts. Into the question of the design of such foundations it is not necessary to enter, as the subject has been already considered in Chapter XIII., Part I.

When the trestle is resting upon a ground sill carried by a foundation of masonry, the sill should be above ground, and not covered with earth, so that decay may be delayed as long as possible (see *Fig.* 168).

Pile trestles rarely exceed 20′ to 30′ in height. If less than 10′ high they do not require bracing.

The principles of design of braced piers are the same whether the material used be timber or iron, but the practice slightly differs—in that with timber the trestles are generally single, and are placed at intervals not usually exceeding 30 feet, whereas with iron or steel trestles, it is usual (though more frequently so in America than in Great Britain) to have them formed in towers of two trestles braced together, corresponding to the "four-legged trestle" used in military bridging. The diagonals in timber trestles are designed to act as struts, whereas with iron and steel trestles the diagonals should be calculated as ties. The reason for this difference is on account of the material being better adapted for each class of stress respectively. Timber trestles.

It may here be mentioned that cast iron is not a suitable material for the construction of the piers in such structures. It is true that it is well adapted for the bearing of compression, but it is not well suited for the transverse stress that may be induced by lateral forces; it is subject to hidden flaws, and the method of fastening the diagonals affords opportunity to unscrupulous manufacturers to put in bad work, and is, at best, a source of weakness. The failure of the Tay Bridge in 1879 is an object lesson which will not be forgotten. There the columns were sufficiently strong to bear the vertical pressure of roadway and passing trains, but were full of flaws, especially at the joints, and thus, when subject to lateral wind pressure, gave way. Cast-iron piers, disadvantages.

Steel columns may nowadays be built up in many forms, with combinations of H, channel, T, and angle irons, Lindsay troughs, etc. Into these and the calculations involved it is unnecessary here to enter. The broad principles of the design of columns have already been considered in Chapter IX., Part I. Steel columns.

In all cases the external forces which bridge piers have to resist are:—(*a*). The vertical weight of the roadway and passing loads. This weight will, of course, be greater in the lower than in the upper tiers of the trestle, and in tall trestles (those over 20 feet) the increment of weight should be taken into account. (*b*). The lateral force of the wind. This is assumed to act horizontally. Probably it will blow upwards at a slight angle, and its vertical component External forces involved. Wind pressure.

will tend to neutralize the weight of the structure and to endanger its stability. "The wind surface of a train is assumed to be 10′ high and the centre of pressure $7\frac{1}{2}$ feet above the rails. It is taken at 330 lbs. per foot of track for the train. The wind pressure on the girders is taken at 50 lbs. per square foot of surface exposed when the span is unloaded, and 30 lbs. when the span is loaded. (*c*). Centrifugal force, acting when the bridge is on a curve. The line of action is taken at 5 feet above rail level. The amount depends on the weight of the train, the curvature, and the velocity with which the train is moving. For a speed of 30 miles per hour the centrifugal force is approximately 1 per cent. of the weight of the train for every degree of curvature. For speeds of 40, 50, and 60 miles per hour it is approximately 2, 3, and 4 per cent. Thus for a train weighing 4,000 lbs. per lineal foot of track moving at a rate of 40 miles per hour on a 6 degree curve (955′ radius) the centrifugal force would be $4{,}000 \times 0{\cdot}02 \times 6 = 480$ lbs. per linear foot of track." (Professor Johnson).

Given these external forces, it is a matter of no great difficulty to investigate the stability of any form of braced structure which we may devise to resist them.

Simplest fo[rm] of trest[le]

In its simplest form, such a trestle, viewed in elevation transverse to the line of the railway, will consist of two uprights, generally constructed with a slight batter, with a groundsill connecting the feet, and a capsill uniting the heads. It will be laterally braced by diagonals, and by horizontals at vertical intervals which diminish towards the lower parts of the trestle, and reduce the unsupported length of the main struts, or columns.

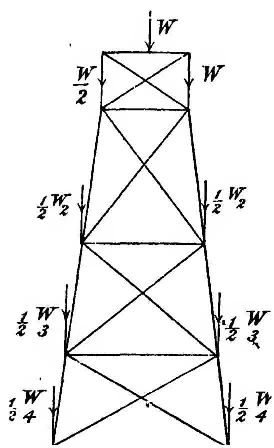


Fig. 163.

Distribution of external forces

If the weight of the roadway and rolling load be represented by W (*Fig.* 163), then $\frac{1}{2}$ W will be borne by each column, and if W_1, W_2 be the weights of the trestle above each joint, and θ be the angle which the inclined columns make with the vertical, the pressure which is brought to bear upon any column, apart from the effect of any lateral load, is $(\frac{1}{2}W + \frac{1}{2}W_1 + \frac{1}{2}W_2 \ldots\ldots) \sec\theta$. The stress

brought to bear upon any of the horizontals by the action of the vertical forces alone, apart from the effect of the wind or the centrifugal force, is expressed by $\frac{1}{2}(W + W_1 + W_2 + \ldots\ldots) \tan \theta$. The diagonals are under no stress from the symmetrical vertical load.

To ascertain the stresses produced by the lateral forces, the simplest plan perhaps is to consider the effect of the horizontal forces alone on the structure, and to add these to the stresses produced by the vertical forces, thus obtaining the maximum pressure from the consideration of the algebraical sum.

If P_1 be the horizontal wind pressure upon the carriages or trucks (*Fig.* 164), P_2 be the horizontal wind pressure upon the girder, and $p_1, p_2 \ldots\ldots$ be the wind pressure upon the trestle at the various joints, then the stability of the structure *as a whole* may be ascertained by taking moments of all the vertical and horizontal forces about the toe on the leeward side. This will not, however, give us the stresses on any particular member, hence we must first find the horizontal pressure produced in the capsill AB by the action of the lateral forces. This will evidently be $= P_1 + P_2 +$ the resolved increment of the extra pressure brought upon the inclined columns by the action of these lateral forces $= x \tan \theta$, where $x =$ the vertical increment brought upon the leeward side and taken from the windward side. Let a be the vertical distance of the point of application of P_1 above AB, b the vertical height of the point of application of P_2, then Investigation of stresses.

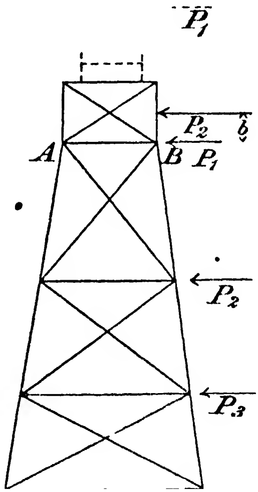


Fig. 164.

$$x \times AB = P_1 \times a + P_2 \times b,$$

or

$$x = \frac{P_1 a + P_2 b}{AB}.$$

Let the stress thus found for AB, *i.e.*, $P_1 + P_2 + x \tan \theta$, be denoted by the symbol H. Then laying off H on any scale of tons

we may draw a stress diagram in the usual way, as in *Fig.* 166, which will give us the stresses in any of the members. These, added to those already found as due to the vertical loads, give us the stresses due to the combined effect of vertical and horizontal forces.

It is noteworthy that in *Fig.* 165 we have the external forces P_1, P_2, etc., acting on *both* of the uprights. This is because the structure is open to wind action throughout. No account is taken of the shelter afforded by the members on the windward side. This can, however, be allowed for in *Fig.* 166 by making *ef* and *fg* less than *ir* and *rs*. There is no reaction on the windward side of the abutment when the lateral wind force only is considered. The total reaction on either side is *gt* (*Fig.* 166) $+ \frac{1}{2}(W + W_1 + \ldots\ldots)$. The legs should be tied together at the base, the tension being $= mt$ (*Fig.* 166).

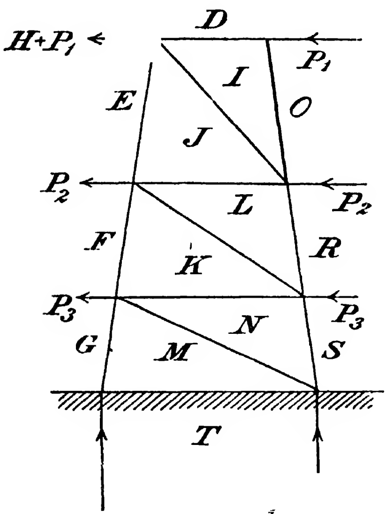


Fig. 165.

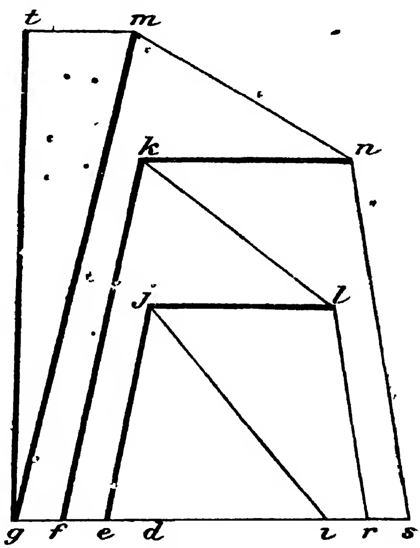


Fig. 166.

From the stress diagram we see that the total compression in GM is greater than the tension in NS, the latter being = compression in FK. So also compression in EJ = tension in LR.

With the same system of bracing and wind on *left* the positive stress in LR now = negative stress in GM. In the diagonals and horizontal members the stresses are of equal amount, but reversed in nature.

If each bay were counterbraced as in *Fig.* 164, and the six diagonals designed to resist tension only, the horizontal members would always be in compression. If, however, only one diagonal be in each bay, as in *Fig.* 165, all the members would have to be designed to bear either class of stress.

Details of Construction.

We must now consider the details of construction in various materials. In timber, the simplest form is that shown in *Fig.* 167, where there are two inclined struts, united at the upper end by a capsill, with a horizontal piece tying them together near the ground, and braced diagonally. Such a form as this is suited when the Details of trestles.

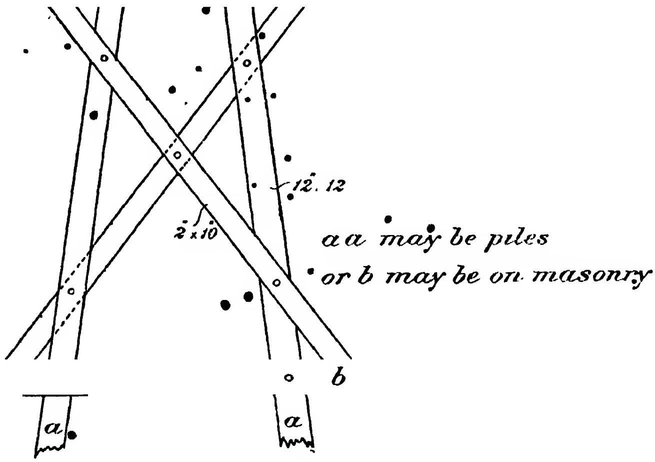


Fig. 167.

height is not great, and where there is only a narrow roadway above. If used with a wide platform there would be considerable tendency to deformation, owing to the eccentric loading of the struts and to the tendency to bend in the increased length of the diagonals, as well as a bending moment in the capsill. Hence, for wider roadways, two, or at least one, vertical is given intermediate to the outer struts, see *Fig.* 168. The actual loading of these struts is indeterminate, but it is probably sufficient if it is considered that the weight at each side is divided equally between two (if there are two) or uniformly among all if there are an odd number.

The struts may either be of squared logs or of round spars, adzed off where the diagonals cross.

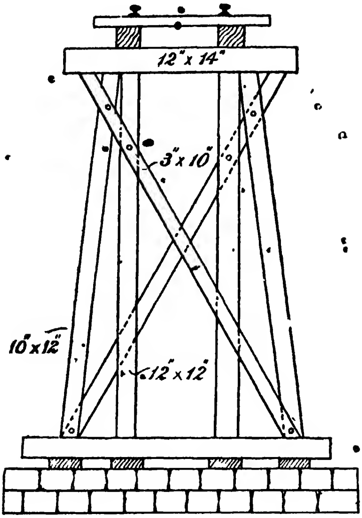


Fig. 168.

Where the weight is likely to come always at definite points on the capsill, as in the case of a railway, it is evidently best to bring the ends of the columns directly beneath those points, so as to avoid bending moments in the capsill and eccentricity of stress in the struts. *Figs.* 169, 170, 171 show some of the standard trestles used in American railways.

Joints.

The question of the best form of joint between struts and capsill deserves consideration. Mortise and tenon in all cases is not satisfactory for permanent work, though it is good enough for temporary bridges. A form which is much used in America is shown in *Fig.* 172, where the horizontal piece *bb* is made double and the strut notched on both sides, the whole being secured by bolts. This form facilitates repairs. The longitudinals (see *aa*, *Fig.* 172), which correspond to the bridging joists in an ordinary floor, should be calculated as fixed beams uniformly loaded. They should not be notched, therefore, to the capsills, and preferably they should be formed of two or three deep baulks, separated from each other by washers or tubing and bolted together. By having two or more pieces in each joist they can be made to reach over two or more spans and break joint. Where they terminate they should be spliced by large packing blocks *cc* (*Fig.* 172), notched down over the capsill.

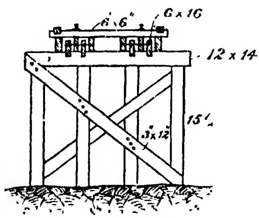


Fig. 169.

Iron castings, or wrought plates, stamped or forged to fit the top of the verticals and the sides of the horizontals, and then spiked

on, are also used, and are very good, for by using them all framing is dispensed with.

Corbels of wood are frequently used at the top of the trestles to give greater bearing for the longitudinals, and to reduce the span. Corbels or capping pieces.

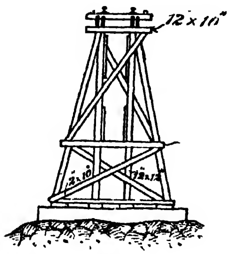


Fig. 170.

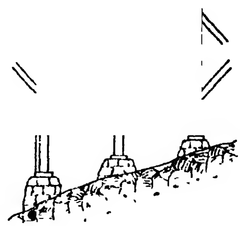


Fig. 171.

Where trussed beams are not used, or even where the longitudinals are strutted from below, corbels are not necessary. The shortening of the span effected by their use is inconsiderable, they furnish large surfaces for the penetration of water and consequent decay, and they are apt to rock on their bearings and loosen the joints. The bearing area of the capsill and the strength of the joists are easily calculated, and the advantages of any corbel arrangement seldom counterbalance their disadvantages.

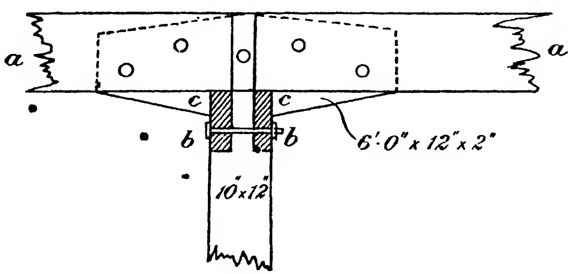


Fig. 172.

Where the bridge is high, economy demands that the trestles should be as far apart as possible. In American railways, where timber trestles are used, 25 feet centres seem to be about the maximum span. In road bridges this distance may be considerably increased. The Hurroo Bridge (*Plate* V.) has the piers 40 feet 6 inches apart.

Transverse diagonals.

Transverse Diagonals.—These are very important parts of the structure, as their function is the resistance of possible deformation caused by lateral forces of whatever kind. On low trestles, *i.e.*, those less than 20 feet high, these lateral forces would be the lateral vibration of passing loads, as wind could have but little effect so close to the ground; but with trestles of greater height than the above, there is no doubt that the wind pressure will be the most important of the lateral forces involved. The diagonals are frequently planks spiked on to the outside of the trestle, but Professor Johnson has pointed out that this method of fastening is both unscientific and inefficient, because of the small bearing area of the spikes and their tendency to bend under transverse stress. It would be well, if it were possible, to put the diagonals between the uprights in the plane of the trestle, but it is seldom possible to do so, because of other timbers which come in the way. Hence the diagonals must be fastened at the sides in most cases, but, in order to give as large a bearing as possible, it is advisable to fasten them with large wooden trenails, say $1\frac{1}{2}$ inches in diameter, turned to $\frac{1}{32}$ inch larger than the intended hole.

High trestles should be divided into panels or tiers, and diagonals introduced in each.

Longitudinal bracing.

Longitudinal Braces.—In timber construction it is necessary to provide longitudinal bracing. These should be arranged in the case of iron or steel piers to unite the pairs of trestles forming a tower or pier. Horizontal waling pieces alone are not sufficient.

In road bridges these are not of quite so much importance as they are in the case of railway bridges, because in the latter there is an enormous longitudinal pull caused by the re-action of the rails against passing trains, and a still greater contrary pull when the train has the brakes on. These must be resisted by longitudinal bracing, and this can be introduced in the direct line of the bridge, alternately from capsill to groundsill. They should, therefore, not be spiked on outside, as in *Fig.* 173, but be as in *Figs.* 174, 175. Professor Johnson remarks, "to omit them altogether, as is so often done, even in high trestle work, is simply criminal."

Where the trestle is more than 40 or 50 feet high there are various ways of effecting the vertical joints. They may either be constructed in tiers set one on the top of another, with an air space between the sills, except under the posts, where seasoned oak bearing planks are carefully framed in, or they may abut; or cast-iron shoes may be made to fit both above and below.

As regards the *decking* of wooden railway bridges, the American custom is to use 8″ × 8″ cross sleepers, spaced 16″ centre to centre, thus leaving 8″ space, which admits air to all sides of the sleeper. This is an obvious advantage, and there is also the advantage that the spacing allows ashes from the engine to fall through. For railway work barrels of water should be placed at intervals to extinguish fires from sparks falling on sleepers, which are usually very inflammable owing to the creosote treatment. Decking of wooden bridges.

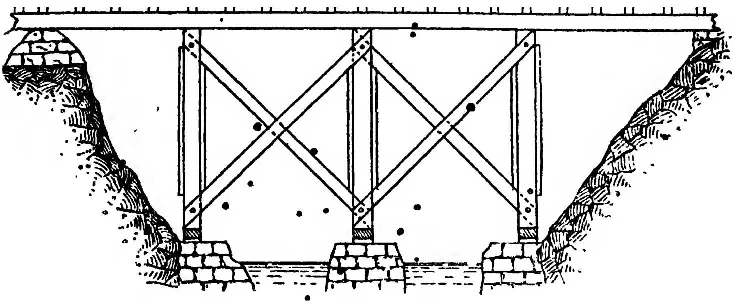


Fig. 173.

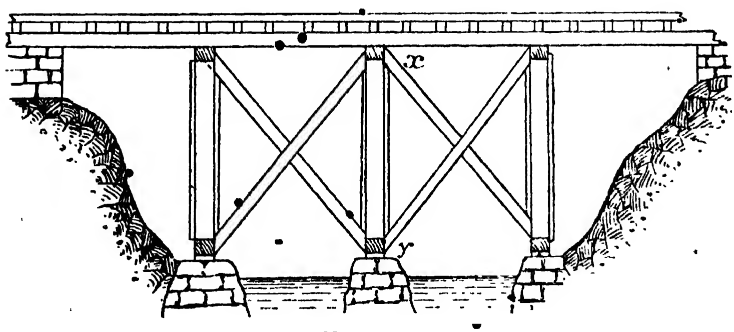


Fig. 174.

Bolts, Spikes, etc.—Experiments made in America show that smooth bolts have a greater grip than jagged ones. The holding power is greater after the bolts have been in for some time than it is immediately after driving, a fact which corresponds with the strength of piles (see p. 297, Part I.). The ultimate resistance of a bolt in "soft" timber may be taken at 10,000 lbs. per lineal foot for a 1″ bolt. Fastenings.

Screw-bolts have a resistance 50 per cent. greater than plain round bolts.

Where two surfaces of timber touch they should be painted with white lead or tar, and the bolts should be coated with white lead and linseed oil, hot or cold tar.

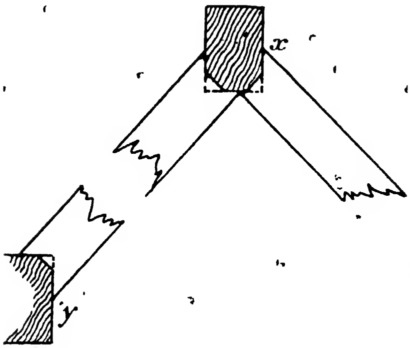


Fig. 175.

Iron piers, fastenings.

With iron piers, the diagonals should be tension bars made fast by pin and link connections to gusset plates (or some such device), on the verticals of the trestles Means should be provided for tightening these up if necessary.

As the cost of the piers increases rapidly with the height, it is necessary to use longer spans with high piers than would be required with low ones.

The trestles should be designed to be stable against lateral pressure without the necessity of anchorages. This obviates the necessity of much masonry at the bases of the piers and long anchor bolts connecting the piers with the masonry.

Construction at the base of the columns.

The batter of the columns varies from $1\frac{1}{2}$ to 3 inches to the foot. If the bridge be on a curve, the outer columns should be given greater batter than the inner ones. At the foot of the column there should be a sole-plate, generally a horizontal plate planed to a smooth surface, and fastened by angles to the columns. It rests on a bed-plate bolted to the masonry, the area of which is sufficient to distribute the pressure without exceeding safe limits of the available strength of the masonry.

The holes for the bolts should be slotted in the sole-plate, so as to allow it to slide upon the bed plate.

CHAPTER X.

ARCHES AND DOMES.

Difficulties attending Investigation.—Definition of Terms.—Parabolic Theory.—Principles of Wedge.—Line of Resistance.—Scheffler's Theory.—Example. Abutments.—Domes.

ALTHOUGH masonry arches have been successfully built for centuries—indeed, the origin of this form of construction is lost in antiquity—the principles of their design are still very imperfectly known. ARCHES. Origin of masonry arches not known.

At first it would appear that the same principles which have been applied to any other form of structure spanning an opening might be applied to the case of this particular method of solving the general problem. Where the span, the method of fixing, and the loads are known, we have no difficulty in ascertaining the bending moments produced in the span as a whole, and it might at first sight be supposed that to arrange the construction of the masonry arch in such a way as to resist exactly the variations in the ascertained Moments of Flexure ought not to be a matter of any difficulty. Difficulties of subject.

It has been pointed out in Chapter VI., when treating of arched ribs, that if an arched rib of any given form be subject to any given loads, the M_f curve for those loads can be drawn with ordinates corresponding in proportion to the rise of the given arch, and that the M_f at any point produced by the loads can be measured from the difference of the two curves. It would at first sight appear that this procedure ought to be applicable to the investigation of arches of any given material and form. Whatever the given form of the arch may be, there must be some arrangement of the loads which would give a M_f diagram closely corresponding to that form, and it would apparently not be difficult to arrange that the actual moments produced in the curved arch ring should be met by the design of the ring and the materials employed, much in the same way as the Rules for metal ribs not quite applicable to masonry.

bending moment in a wall produced by the action of lateral forces is met by the arrangement of the material (see pp. 298—321, Part I.).

External forces and method of fixing usually uncertain.

Unfortunately, there is an initial flaw in this reasoning, which in most cases renders accuracy impossible. We have begun by assuming (i.) that the external forces acting on the arch ring are known; and (ii.) that the method of fixing is known. In most cases neither of these assumptions is justified.

As regards (i.), if the arch were supporting a fluid, there would be no difficulty. If it were supporting weights that were transmitted to it by vertical walls, there would be comparatively little uncertainty. But as it usually supports a covering of earth, with a roadway above, subject to the weights of passing traffic which are transmitted to the arch ring with a greater or less distribution according to the nature of the intervening material, the actual loading must be largely a matter of conjecture.

As regards (ii.), it is evident that in ordinary cases the fixing must produce at least as much complication as we have seen it does in the case of arched ribs of metal fixed at the ends.

Concrete bridge at Munderkingen.

It is true that both the limitations just mentioned have been overcome in practice. For example, in a concrete arch 164′ span, 16′ 5″ rise, constructed recently over the Danube at Munderkingen, the arch is hinged both at the abutments and at the crown, whereby the difficulty about fixing is overcome, and the weight is transmitted by means of vertical spandril walls running parallel to the line of the bridge.* This arch was designed to bear a distributed load of 82 lbs. per foot super + the dead weight of arch ring and roadway, etc. The spandril walls brought the weight on known points.

Principle of inverted suspension bridge.

Arches have been designed in the United Kingdom on the principle of the inverted suspension bridge. The procedure is, to lay out on a vertical board, to scale, the span and rise of a given arch, then suspend from hooks, which denote on the board the ends of the span, a chain whose links correspond in weight to the weights of the proposed arch ring and its superincumbent load, with extra links, capable of being attached at any part, whose weights similarly correspond to the weights of passing loads. The curves assumed by the chain in the various positions, due to the application of the passing load at various points, are carefully marked on the vertical board, and the arch ring is then designed so as to include within the centre third of its thickness all possible positions of the load.

* See *Minutes of Proceedings, Institution of Civil Engineers*, Vol. CXIX.

This method is practical, but it could only be strictly applicable to a case such as that of the Munderkingen bridge, where the ends were hinged, the weights vertical, and (an important point) where the arch ring was thoroughly bonded together.

Such an arch would, in elevation, be of a curve nearly corresponding to a parabola. The horizontal stress H in it would be nearly uniform, and the thrust on the abutments would be H sec θ, or $\frac{Wl}{8d}$ sec θ, as in the suspension bridge. This thrust may be approximately taken as acting in most arches at a point on the joint $\frac{1}{3}$ of the thickness from the springing. Value of abutment thrust.

This method, however, would only be approximately correct, and some other method of investigation is desirable.

Definition of Terms.

Before going further into the general theory on the subject it is necessary to define the terms used in the description of arches. Definition of terms.

The wedge-shaped stones or bricks of which the arch is built are called *voussoirs*. Voussoirs.

The interior or concave surface of the arch as a whole is called the *soffit*. Soffit.

In the section of an arch, as shown on *Fig.* 176, the inner line is called the *intrados*, the outer line the *extrados*. Intrados and extrados.

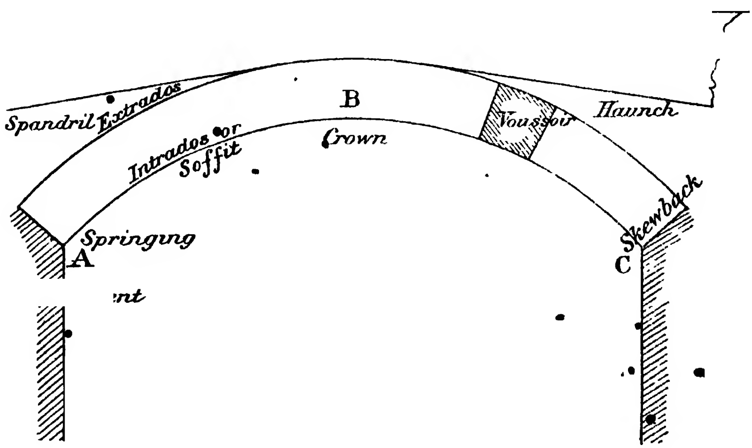


Fig. 176.

Where the arch begins (A, C, *Fig.* 176), the point is called the *springing*, the highest point (B) being the *crown.* These may be referred either to the intrados or extrados.

Haunch. At the sides there is an indefinite part of the arch called the *haunch.*

Skewback. The inclined surface of the abutment at the springing against which the arch rests is called the *skewback.*

Spandrils. *Spandrils* are the parts of the structure between the extrados and the roadway.

Other terms, such as abutment, skew arch, groined arch, etc., are probably already familiar.

Wedge Theory.

Wedge theory It may assist in the understanding of the arch principle if we consider first the action of the wedge.*

Let ABC (*Fig.* 177) represent a wedge of hard material driven between two solids by a force P. Let the angle BAC $= 2\theta$. The wedge will be kept in equilibrium by the reactions R and R′, and by the force P (neglecting for the present the friction exerted by the sides). The direction of R and R′ will be at right angles to BA, AC. Produce R and R′ to meet at a, and complete the parallelogram of forces, drawing ad on any scale $=$ P.

Then
$$
\begin{aligned}
R : P &:: ab : ad,\\
&:: AB : BC,\\
&:: AB : 2BD.
\end{aligned}
$$

$$
R = \frac{P}{2} \times \frac{AB}{BD} = \frac{P}{2 \sin \theta} \qquad \text{(i.)}.
$$

Now if we include the effect of friction along the planes AB and AC, we know from the principles of statics that F the total friction $=$ R tan α where α is the angle of repose of the material (see p. 306, Part I.). In the case of stone without mortar $\alpha = 33°$, hence tan $\alpha = \cdot 65$. With fresh mortar $\alpha = 37°$ and tan $\alpha = \cdot 75$. (This proves that when an arch is built of stone it will require no support from the centering until the joints make an angle of 37° with the horizontal). Call tan $\alpha = \mu$, then $F = \mu R$. On *Fig.* 177 draw $be = F$ on the same scale that $ad = P$, and draw bf parallel to AD. The bf represents the resolved part of F that is directly opposed to the force P on *each* side of the wedge.

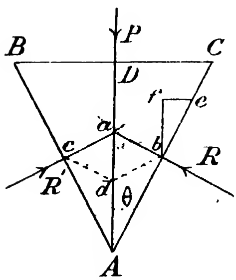


Fig. 177.

The total force Q that friction opposes to P is measured by

$$
2bf = 2 \times be \times \frac{bf}{be} = 2be \cos \theta = 2F \cos \theta = 2\mu R \cos \theta,
$$

which from equation (i.)

$$
= \mu P \cot \theta \quad \dots\dots\dots\dots\dots \text{(ii.)}
$$

* Partly taken from Tarn's *Mechanics of Architectu*

The forces acting on the wedge now become P + Q, the reactions R and R′, and the effect of friction on the sides, μR and μR′. The friction on the sides opposes the force P in pushing in the wedge, but it opposes any attempt in the opposite direction to withdraw it. So long as Q is greater than P, the wedge will not be pushed backwards by the reactions R and R′ after the pressure P is removed.

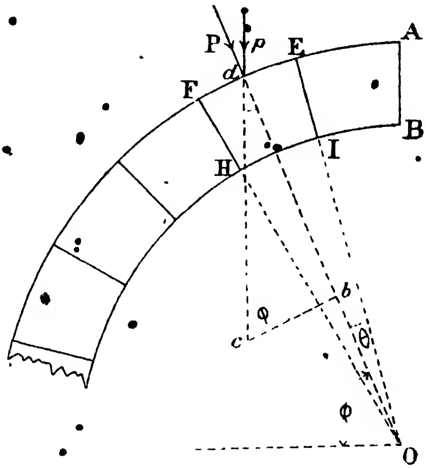


Fig. 17

Circular segmental arch.

Applying this to the case of an arch built of voussoirs, which radiate towards a common centre:—Take any one of these EIFH (*Fig.* 178). Now, whatever be the magnitude and direction of the load coming on the extrados FE it is certain that there will be some part of it P which, resolved in the direction of the radius dO, tends to drive the wedge in. If we know the vertical force p acting on the surface FE, we can find P for P $= p \sin\phi$, since

$$\sin\phi \quad \frac{db}{a} \quad P$$

where ϕ is the angle which dO makes with the horizontal.

Now from equation (ii.)—

$$Q = \mu P \cot\theta = \mu p \sin\phi \cot\theta \qquad \text{(iii.)},$$

and the voussoir will remain in its place as long as the vertical components of R + R′ are less than P + Q.

When a horizontal roadway is borne by an arch which is either a semi-circle or a segment of a circle, and therefore has a constant value of θ, as ϕ approaches 90° $\sin\phi$ gets greater. When ϕ becomes very small, *i.e.*, from the haunches downwards, the value of Q rapidly diminishes unless we increase p. Hence in such arches it is necessary to add to the weight on the haunches, so as to prevent failure by the increased reaction.

Parabolic segmental arch.

If instead of adding external weight we make the curve flatter at the haunches, we increase the value of ϕ and decrease that of θ at those parts, and in so doing increase both $\sin\phi$ and $\cot\theta$. Thus in *Fig.* 179 the arch is made on three centres (two of which are shown), so as to approximate the parabolic form. The two curves touch at the joint BB′. The value of ϕ for the voussoir 4 is 48°, whereas if the curvature from the crown had been continued it would have been 40°. The value of ϕ for voussoir 3 is 54°, and θ is 6½°, whereas θ for voussoirs 4 is 2°.

Taking the value of p as the same in both cases (really in practice it would be greater in voussoir 4 than in 3), and substituting in equation (iii.) values of μ (for stone and fresh mortar) $\sin\phi$, and $\cot\theta$, we have—

For voussoir 3— $P+Q=p\times \cdot 809+\cdot 75p\times \cdot 809\times 8\cdot 7=6\cdot 079\,p$.

For voussoir 4— $P+Q=p\times \cdot 743+\cdot 75p\times \cdot 74\times 28\cdot 6=16\cdot 66\,p$.

If the circular form had been continued ϕ would have been $=41^\circ$, and $P+Q$ would then have been $=4\cdot 88\,p$.

Advantage of flattening the arch at haunches.

So that by flattening the arch at the haunches we have considerably increased its stability, without going to the expense of adding external weight.

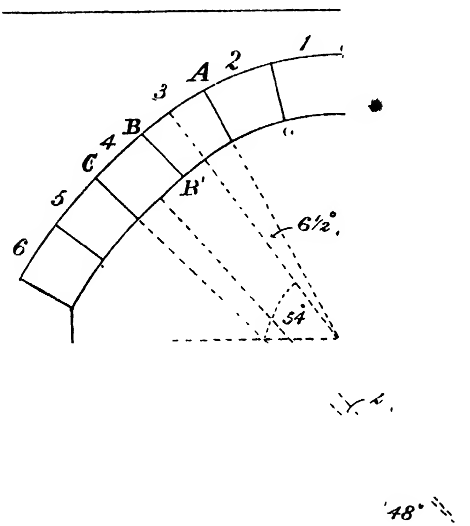


Fig. 179.

This is a corroboration of the view alluded to above, that the economic form of an arch under equal, vertical loads approximates the parabola.

The question, however, will naturally follow: Even assuming we do know the vertical loads coming on each voussoir, how can one tell at what point the flattening of the curve should begin, and what degree of curvature should be given?

Joint of rupture.

This question is one to which no direct answer can be given. It is evident that, unless the arch be so designed, either purposely or fortuitously, so that the pressure of external loads produces uniformly distributed compression at every joint (an ideal state of affairs, the converse of a suspension bridge), in most cases there will be a joint where the bending moment produced in the arch ring is a maximum. This joint is called the "joint of rupture." It is also defined as the joint at which there is the greatest tendency to open at the extrados, since the effect of the maximum bending moment will be to crush the masonry at one side (the intrados) and open it at the other side, as in *Fig.* 274, p. 303, Part I.

Professor Rankine (*Applied Mechanics*, pp. 199—204) has investigated this subject from a mathematical standpoint with the thoroughness of research of which he was master. Into these investigations it is not intended here to enter, especially as the determination of the point can only be found by approximation.

Joint of rupture in segmental arch.

Practically we see that a segmental arch tends to fail by sinking at the crown and opening out at the haunches, as shown in *Fig.* 180, and that to prevent this, masonry backing or loading is generally introduced as far as the joint of rupture. This joint lies where the radius makes an angle (ϕ) of from 35° to 45° with the horizontal.

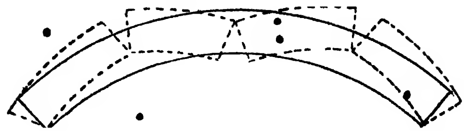


Fig. 180.

To find point and angle of rupture.

Professor Rankine gives the following graphic investigation for finding the point of rupture and angle of rupture, which has already been alluded to on p. 267, Part 1.:—

Let C be any point in a loaded arch (*Fig.* 181), and let the vertical line ob (*Fig.* 182) represent a scale of loads, taking $oh = \mathrm{P}$ to represent the vertical load supported on the arc AC. Draw oc parallel to the tangent at C, and through h draw hc horizontal, cutting oc in c. Draw several similar lines oc', oc'', etc., parallel to the tangents to the rib at various points, and the corresponding horizontal lines $h'c'$, etc., representing the horizontal thrusts. Through the points a, c, c', etc., thus found draw a curve. The point d in that curve which is furthest from the load line ob will indicate the point where the horizontal thrust is a maximum. Join od, and find the point in the arc where the tangent is parallel to od; this is the "point of rupture," and the "angle of rupture" is the angle doa.

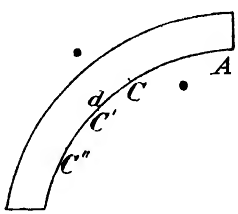


Fig. 181.

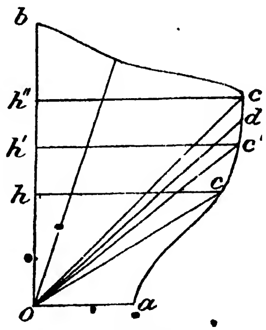


Fig. 182.

Line of Resistance of the Arch.

Line of Resistance.

Although arches of a parabolic form are theoretically more stable than those of a circular form, yet, inasmuch as the latter are more

common and are easier to construct, we must devote attention to their stability. Our investigation, so far, has only brought us to recognise that in an arch, either a semi-circle or approaching a semi-circle, pressure from without on the haunches, is necessary to preserve equilibrium, and that at some point between 35° and 45° from the horizontal there will be the greatest tendency for the arch to break.

Arch considered as an inverted chain

We might have arrived at a somewhat similar result if we had considered the arch as an inverted chain. If we have a chain loaded with weights corresponding to the vertical weights on voussoirs, their ends terminating on a horizontal line, corresponding to the roadway in a bridge, we have seen, in Chapter VII., that the natural curve the chain would assume would be very nearly a parabola (*Fig.* 183). To make that curve circular we must apply extra weights at C and D (*Fig.* 183). Conversely in the circular arch we have to apply weights at the haunches.

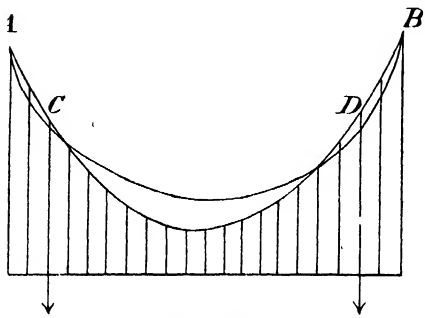


Fig. 183.

If we consider the voussoirs to be made with curved surfaces which will assume various positions under various loads, we can see that the line joining the points of contact of the stones will correspond exactly to the inverted curve of a suspension bridge chain (unstiffened) (*Figs.* 184 and 185). This curve is the Line of Resistance.

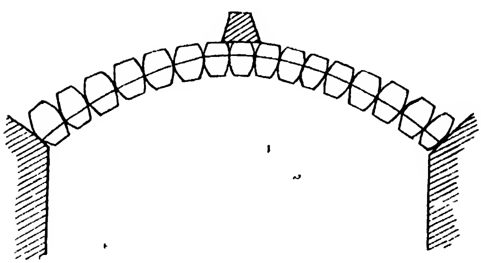


Fig. 184.

The Line of Resistance in any masonry structure has been defined on p. 299, Part I. It is the line which joins the points

where, in a succession of joints, the line of the resultant of all the forces acting on the masonry above each of the joints in succession cuts the joints (*Fig.* 270, p. 300, Part I.). In an arch the investigation is generally carried out with reference to a series of voussoirs. The greater the number of the voussoirs, and the consequent number of joints, the more nearly will the Line of Resistance approach a curve, and the greater the accuracy.

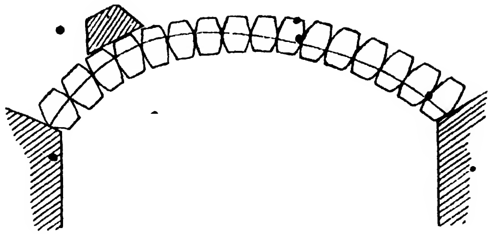
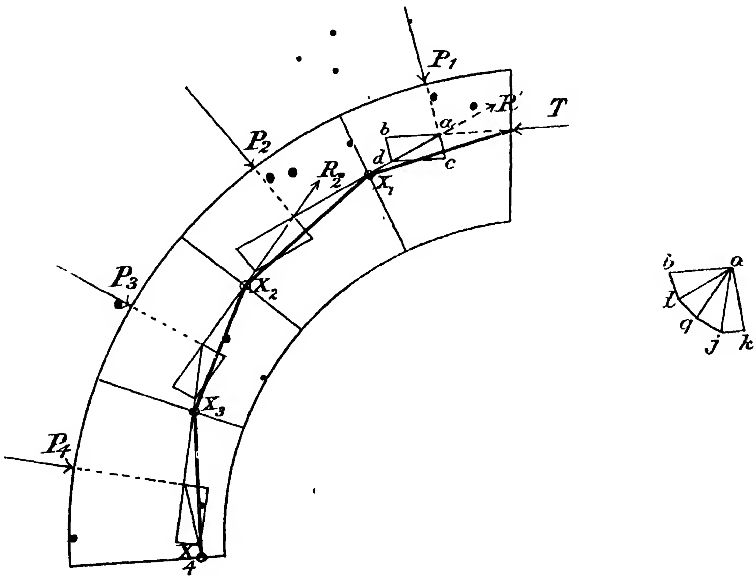


Fig. 185.

To obtain a clear idea of what is meant by the Line of Resistance, let us suppose that the half arch in *Fig.* 186 is constructed of four voussoirs, and that the resistance of the right-hand portion of the arch is replaced by the force T, whose magnitude, direction, and

Example of Line of Resistance in an Arch.



point of application are assumed to be known. Let the resultant pressures acting on the voussoirs (including the weight of the stone) be P_1, P_2, P_3, P_4. The voussoir 1 is held in equilibrium by the forces T and P_1 and a resultant R_1 acting at the point X_1 in the joint between 1 and 2. X_1 is found by drawing *ab* and *ac* to represent T and P_1 and by completing the parallelogram forces, so that *ad* represents R_1, and the position of its action is found by producing *ad*. Similarly voussoir 2 is held in equilibrium by R_1, P_2, and R_2, the last of which is found similarly to act at X_2. In like manner the points X_3 and X_4 are found. The line joining X_4 X_3, X_2, X_1, etc., is the Line of Resistance.

It is clear, however, that the accurate determination of the above depends on the accuracy with which we can fix the external forces P_1, P_2, etc., and also the value, direction, and position of the horizontal force T. These matters will be subsequently investigated. At present we assume that the external forces at least are known in magnitude, direction, and point of application.

Safety against overturning, crushing and sliding.

If the arch ring be so designed that the Line of Resistance lies everywhere within the limits of the ring, every voussoir will be *safe against overturning*.

If the position of the Line of Resistance with reference to any joint, and the amount of the normal resultant pressure on that joint, be known, it is possible to ascertain (on the principles explained on pp. 301—305, Part I.) whether *the arch is safe against crushing* and opening at the joints.

If the direction of the resultant pressure be known at every joint and compared with the direction of the joint, it is possible to ascertain whether *the arch is safe against sliding* at that joint.

Thus if we can accurately fix the Line of Resistance in an arch ring of any given form, we can investigate without difficulty, with reference to each joint, the three conditions of stability, viz., against (i.), overturning; (ii.), crushing; (iii.), sliding (see pp. 298—306, Part I.).

Stability against sliding depends on direction of joints.

It will be observed that while the stability of the arch against overturning and against crushing are dependent on the same conditions, the stability against sliding is dependent only on the direction given to the joints, and any possibility that there may be of one voussoir sliding on another may be met by alteration in that direction.

We have, therefore, in order to investigate the conditions of stability, first to *assume the form* of our arch, either copying some

existing arch, or working by formulæ such as those of Trautwine (see pp. 323—325, Part I.), or guided by such investigation as we have considered above in connection with the wedge theory and joint of rupture. Our next step is to apply to the given arch the rules for ascertaining the Line of Resistance, and if this does not lie within the limits of the arch ring, the depth of the voussoirs must either be altered, or the form of the curvature changed.

As regards the limits of the Line of Resistance, Professor Rankine has suggested that it should not lie anywhere outside the centre third of the ring, and that although "arches have stood, and still stand, in which the centres of resistance of joints fall beyond the middle third of the depth of the arch ring; but the stability of such arches is either now precarious, or must have been precarious when the mortar was fresh."* Sir Benjamin Baker, however, has stated,† with reference to this, that "Considering that at least 90 per cent. of all the arches in the kingdom were necessarily in that condition, it was very important that some protest should be made against such an inference from a theory which even Professor Rankine himself did not accept in other masonry works." We may, therefore, be justified if we extend the limit to the central *half* of the arch ring. This is Dr. Scheffler's limit. Limits should be within central half of assumed ring thickness.

As regards the limit of the maximum pressure it is customary to allow a factor of safety of 10, *i.e.*, the maximum intensity of pressure on any joint shall not exceed $\frac{1}{10}$ the ultimate crushing strength of the stone masonry, concrete, or brickwork of which the arch ring is built.

Open joints in the extrados of an arch are especially to be guarded against, as they may admit water, which may freeze, causing increased internal stresses. For this reason the extrados should be covered with a layer of asphalte or cement. Open joints in extrados to be guarded against.

Principle of Least Resistance.

We see from *Fig.* 186 that if the position of the crown thrust T be altered, the Line of Resistance will no longer be that shown, but may differ from it considerably and yet lie within the limits of the arch ring. Thus if we take different values and positions for T (the horizontal thrust) we may draw an infinite number of Lines of Resistance, each consistent with equilibrium. The question then Principle of Least Resistance.

* *Civil Engineering*, p. 417.
† *Minutes of Proceedings, Institution of Civil Engineers*, Vol. CXV., p. 108.

arises, which of all these is correct. The theory which is most usually adopted by engineers is that which is known as the Principle of Least Resistance, or least horizontal thrust; it is based on the following principle, first proposed by Canon Moseley, and afterwards elaborated by Dr. Scheffler:—

"If the forces which balance each other in or upon a given body or structure be distinguished into two systems, called respectively active and passive, then will the passive forces be the least which are capable of balancing the active forces consistently with the physicial conditions of the body or structure." Assuming, therefore, that the thrust at the crown is a passive force called into action by the external forces, it will be the *least possible* that is consistent with equilibrium.

This theory furnishes a condition by which, if the external forces and the direction of the thrust are known, the Line of Resistance can be found in the given form of arch ring.

Joint of horizontal thrust, in symmetrically loaded arch.

It is evident that at some joint of the arch ring, imaginary or real, the thrust will be horizontal, just as at some point in the chain of a suspension bridge the tangent to the flexible curve will be horizontal. In an arch of symmetrical form and with a symmetrical load this *joint of horizontal thrust* will evidently be at the crown. In other cases its position will be found by methods which will be hereafter explained. In the theoretical investigation of stability the joint of horizontal thrust is always the starting point of calculation, although it must be clearly understood that such a joint is an *imaginary* plane drawn in a radial direction through the arch ring. It will not coincide with any actual joint of the masonry, except in Gothic arches, because there is always at the crown of an arch (at or near which the thrust is horizontal) a keystone, or voussoir, with both sides inclined at the same angle to the horizontal.

Value of horizontal thrust.

To find the *value* of the horizontal thrust, when we know the position of the joint:—Let W (*Fig.* 187) be the sum of the known external loads (including the weight of the voussoir) acting at their common c.g. Let a be any arbitrary point in the skewback, and let CD be the length of the joint of horizontal thrust. For equilibrium, taking moments of T and W about a, Ty must $= W \times x$. For the same position of a, Wx is constant, and $T \times y$ is also constant, therefore for T to be a minimum, y must be a maximum.

In this investigation, however, it has been assumed that the common c.g. of all the loads on the arch has been found. This will not usually be the case at first. Generally, the problem to be solved

is how to find the value and point of application of the horizontal thrust where there are a series of voussoirs loaded with known weights. *Fig.* 188 represents such a case.

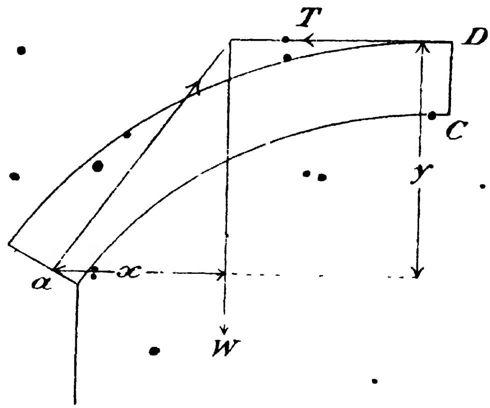


Fig. 187.

Let ABCD (*Fig.* 188) represent a portion of an arch ring of any form between the joint of horizontal thrust CD and the springing AB, and let the vertical and horizontal components of the weights on the voussoirs be w_1, w_2, w_3 .... and h_1, h_2, h_3 ..... respectively, and let T = the horizontal thrust act at some point in CD, such as *a*. If the area within which the Line of Resistance must lie is limited

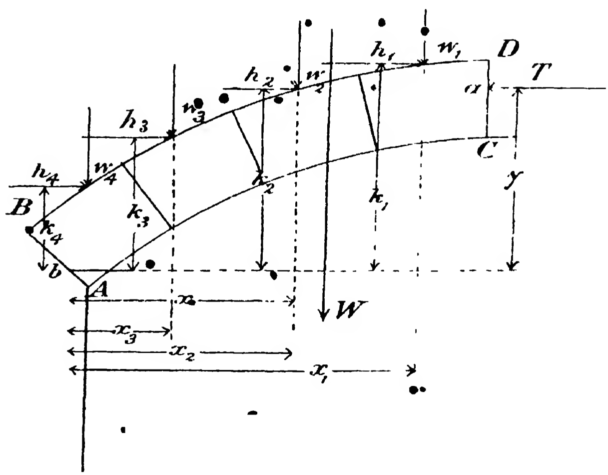


Fig. 188.

by the central half of the arch ring we may take A*b* as ¼ AB, and C*a* as ¾ CD. Take moments round *b*, the lower limit in the joint

at the skewback, let the arms of the moments of w_1, w_2......be x_1, x_2......, of h_1, h_2......be k_1, k_2......, and of T be y.

Then
$$Ty = w_1x_1 + w_2x_2...... + h_1k_1 + h_2k_2......$$
$$= \Sigma wx + \Sigma hk,$$

hence
$$T = \frac{\Sigma wx + \Sigma hk}{y}.$$

If then with this value we draw a Line of Resistance starting at a, and ending at b, and if we find that when drawn it lies within the prescribed limits throughout the *whole* of the central strip, that will be the true Line of Resistance.

The method of ascertaining the vertical and horizontal components of external pressure will be examined later.

Horizontal thrust when loads are not symmetrical.

When the arch is not symmetrically loaded, the joint of horizontal thrust will occur where an imaginary radial plane divides the arch into two parts, such that the weight of *each part and its vertical load* is *equal to the vertical reaction at the adjacent abutment.* Thus, in *Fig.* 189,

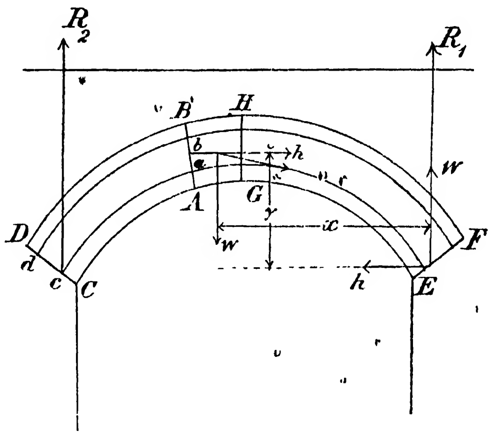


Fig. 189.

let the vertical loads on the arch ring be all known, so that the vertical components R_1, R_2 of the reactions at the abutments are known from the ordinary method of obtaining reactions in a beam of given length under given loads at fixed points. Then, from the known vertical forces, we can ascertain how much of the arch and its load is equal to R_1, and how much to R_2. If ABEF be that portion of the ring and load, the vertical component of which is equal to R_1,

and ABCD that portion equal to R_2, AB is the joint of horizontal thrust. For if we take the weight of the portion ABGH between AB and the crown, and know, by hypothesis, that its weight w is included in the reaction R_1, we see that these equal weights tend to produce a negative couple whose moment is wx. To maintain equilibrium, there must be a positive couple furnished by the horizontal component h' of the thrust on $BbaA$, and the horizontal component of the reaction h. This couple has a moment hy. The resultant of h_1 and w acting on $BbaA$ must therefore act towards the right abutment; hence the Line of Resistance must be inclined towards that abutment, and the resultant thrust cannot be horizontal until the Line of Resistance reaches AB.

Hence we can find the joint of horizontal thrust if we know the vertical external forces.

Another rule for finding the joint of horizontal thrust is that it lies vertically below the common centre of gravity of all the external loads.

The steps which must be taken in the investigation of the true position of the Line of Resistance therefore are:— Steps to be taken to find true Line of Resistance.

(1). Find the joint of horizontal thrust. If the arch and load are symmetrical, this will be at the crown, otherwise it will be that joint which divides the ring in the manner just indicated.

(2). Mark on the arch ring the position of the central half $abcd$ (*Fig.* 189).

(3). Ascertain the external loads on a portion of the arch ring ABCD between the joint of horizontal thrust and the springing at C. Draw a vertical line through the centre of gravity of these loads. (How to find this will be presently described).

(4). Then begin by considering the horizontal thrust at AB to be a minimum consistent with stability, and therefore acting as high up as possible within the limits ab. In other words, take moments of the weight W_2 and the horizontal thrust T round the lowest limit admissible in CD, *i.e.*, the point c. Then

$$W_2 \times gc = T \times ck;$$

or

$$T = \frac{W_2 \times gc}{ck} \quad (\textit{Fig } 190).$$

(5). With this value of T start drawing the *trial* Line of Resistance. If this falls within the limits prescribed, it will, according to the theory of Moseley and Scheffler, be the true Line of Resistance.

(6). But if this trial line falls outside the prescribed limits,

another line must be drawn, taking the point of action of T a little below b, and as ck will now be less, T will be greater than before.

(7). The limiting values of T are :—A minimum when it is taken as acting at b and moments are taken about c, and a maximum when it acts at a and moments are taken about d.

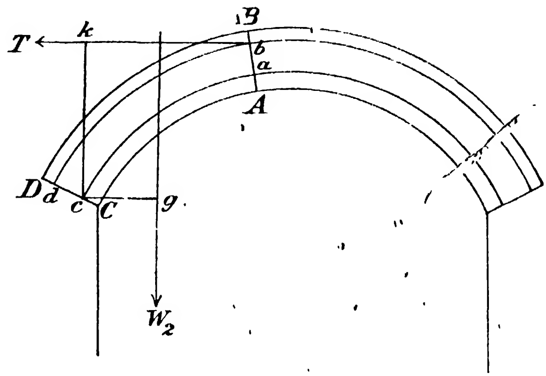


Fig. 190.

(8). If after repeated trials it is found that no value of T will give a Line of Resistance falling anywhere within the prescribed limits the arch is unstable, and the form of it will require alteration. Either a new *thickness* of ring should be adopted, or the *loads* must be rearranged, or the *curvature* of the arch must be reconsidered, and the process, as above described, repeated.

Graphic Diagram of Loads.

Graphic diagram of loads. In the Line of Resistance, instead of drawing on the arch ring itself all the parallelograms of forces as in *Fig.* 186, it is evident that we can represent all the forces acting on the figure in a separate polygon as ab .....k, where ab represents the horizontal thrust T, and bd, dg, gj, jk represent the external forces. This saves a considerable amount of drawing and confusion. It is the same as drawing a polar diagram and funicular polygon, as in the case of arched ribs.

Therefore (if we know the magnitude, direction and point of application of the external forces) we may construct a diagram of the forces acting on the arch ring by first drawing to scale OB

(*Fig.* 192) the calculated value of T, the horizontal thrust, then drawing BA from B, in magnitude $= P_4$ and parallel to its direction. Similarly, draw BC on the other side of OB in magnitude $= P_5$ and parallel to its direction. In like manner all the other external forces can be drawn $p_1, p_2, p_3 \ldots\ldots p_{10}$ equal and parallel to $P_1, P_2 \ldots\ldots P_{10}$. Then join their extremities with O. On *Fig.* 191 draw *ob* at the point of application of the horizontal thrust horizontal until it cuts the directions of P_4 and P_7 at *a* and *b*. From *a* draw *ad* parallel to OA, meeting the direction P_3 at *d*, and from *b* draw *bc* parallel to OC, meeting the direction of P_6 at *c*. In like manner complete the funicular polygon. The points (marked $\times$) where this polygon cuts the joint on *Fig.* 191 give points on the Line of Resistance. If this falls within the prescribed limits, well and good; the line thus found is (on Moseley's principle) the true Line of Resistance. If the line is beyond the limits anywhere, a fresh and greater value of OB must be taken, corresponding to a lower position of *o*, and a new polygon of forces drawn (the values of P_1, P_2, etc., being as before). If this does not satisfy equilibrium, another value must be tried, and if no position and value of OB will give satisfactory results, either the form of the arch ring must be altered, or the external loading in some way re-arranged.

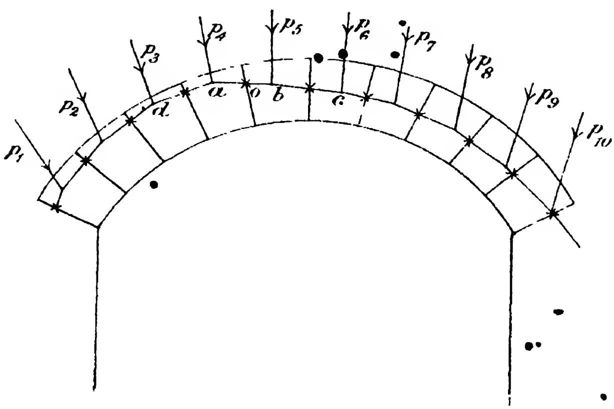


Fig. 191.

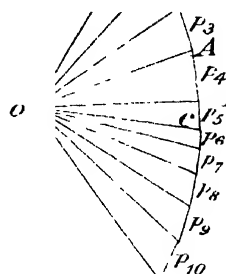


Fig. 192.

This method of working by trial and error is tedious, and it is obviously of advantage to reduce the labour involved by any expedient which will indicate, after the first trial Line of Resistance

has been drawn, the position of the line which is likely to give the most favourable results. This is ascertained as follows:—

In *Fig.* 193 let the first or trial Line of Resistance be that indicated which passes with its maximum deviation above the limits of the centre half at the joint FF′, and its maximum deviation below the limits at EE′, and let *f* and *e* be the two points on those joints through which it is desired the curve should pass. The form of the true Line of Resistance will be as indicated in dotted lines.

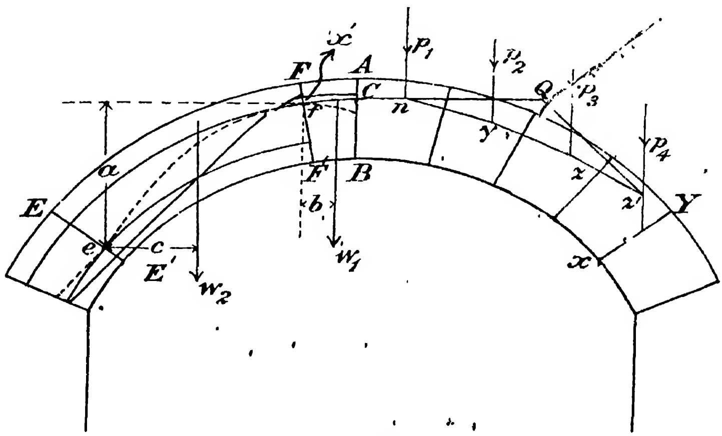


Fig. 193.

Let W_1 be the line drawn through the known c.g. of FF′AB, and W_2 be the line drawn through the known c.g. of EE′AB. Let T be the new *unknown* value of the horizontal thrust, and let x be the *unknown* vertical distance above or below f of its point of application. If we take a as the known vertical distance of f above e, we can find the values of the two unknown quantities T and x as follows:—

Taking moments about f—

$$T \times x = \pm W_1 b \qquad .(\alpha),$$

and taken moments about e—

$$T(a + x) = \pm W_2 c \ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta).$$

From (α) and (β) we ascertain T and x, and can at once draw the Line of Resistance shown dotted on *Fig.* 193, which will pass through e and f.

Method of finding Line of Resistance expeditiously in segmental arches.

A purely graphic method of ascertaining the c.g. may be conveniently used in connection with this. Let it be required to find the position of the vertical through ABXY (*Fig.* 193), a portion of the arch ring on the right of the crown, divided into four voussoirs, the weights on which are known (and for simplicity in this case are assumed to be vertical). Draw the line *abcde* (*Fig.* 194) vertical, in which $ab = P_1$, $bc = P_2$, etc., and take any point O joining O*a*, O*b*......O*e*. Then draw from C C*x* parallel to O*a*, meeting the direction of P_1 at *x* and from *x* draw *xy* parallel to O*b*, meeting the direction of P_2 at *y*. Draw *yz* parallel to O*c*, *zz'* parallel to O*d*. From the point *z'* on the line of P_4 draw *z'*Q parallel to *e*O, meeting the C*x* produced at Q. Then Q is evidently on the line of the resultant of all the forces, *i.e.*, is on the line of their common c.g.

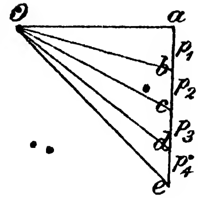


Fig. 194.

On this principle it is possible to determine the Line of Resistance of an arch where it is evident from the small rise that the Joint of Rupture must be at the springing. Let *Fig.* 195 represent an arch

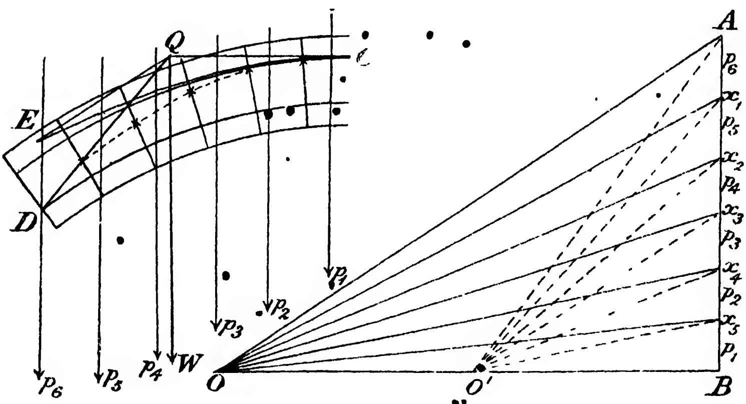


Fig. 195.

with a span = 5 times the rise. Draw AB to represent the loads $P_6 + P_5 + P_4 + P_3 + P_2 + P_1$ and take any point O, joining OA, OB, Ox_1, Ox_2, etc. Draw the trial Line of Resistance CE, and, as above, find the position Q of the line through the c.g. Now, in order to draw the true Line of Resistance CD, we must bear in mind that the segment of the arch is kept in equilibrium by three forces, the horizontal

thrust at C, the weight acting through Q, and the reaction at D. As the first two intersect at Q, the line of action of the third must be DQ. Draw AO′ parallel to QD, and join $O'x_1$, etc., as shown in dotted lines. These represent the true pressures on the joints, and from them the true Line of Resistance can be drawn as shown in dotted lines.

This is the most expeditious method of obtaining the true Line of Resistance in a segmental arch. By this means the troublesome calculations for the position of the common c.g., and for the value of the horizontal thrust, are avoided.

External Loads, their Magnitude and Direction.

External loads.

In the investigation which we have considered above we have assumed that we know the magnitude and direction of all the external loads, but, as pointed out at the beginning of this Chapter, in most cases there is much uncertainty about the actual loading.

In an arch acted upon by a fluid, such as the case of a shaft of any material standing vertically in water, the external pressure will everywhere be normal to the tangent at the extrados, the Line of Resistance will assume the form of a circle, and the problem is simple.

Pressure in the case of a road bridge.

With the ordinary case of a road bridge having earth filling above the arch, the pressure will not be normal to the tangent, nor will it be all vertical. Frequently in books treating of the arch the load on any voussoir is assumed to be all vertical, and to be that contained between two imaginary vertical lines drawn from either end of the extrados of that voussoir to the level of the roadway above, plus the weight of the voussoir itself. This can only be true when the depth of earth filling is inconsiderable. It is not generally considered, for instance, that the arch of a culvert under a high embankment bears the whole weight of the earth vertically above it, although undoubtedly the weight in the centre of the culvert longitudinally is much greater than at the sides.

Pressure of earth.

Professor Rankine* has shown that the horizontal pressure of earth at any point cannot be greater than $\frac{1+\sin\phi}{1-\sin\phi}$, nor less than $\frac{1-\sin\phi}{1+\sin\phi}$ times the vertical pressure, ϕ being the angle of repose. Professor Baker has pointed out that if $\phi = 30°$, the minimum value

* *Civil Engineering*, p. 320.

of the horizontal stress will be $\frac{1}{3}$rd the vertical pressure, and that although the method of ramming the earth filling will have an appreciable effect on the amount of horizontal thrust exerted, it is improbable that however carelessly that ramming is carried out it will reduce the thrust below the minimum just quoted. Hence the horizontal intensity may be taken as $= \frac{1}{3} wh$, where $w =$ weight per cubic foot, and $h =$ height or depth of any vertical plane under earth pressure.

An arch, however, may have (one may say it always has) spandril filling of masonry above its extrados of a different specific gravity to the earth filling. Or it may itself be built of concrete, it may have masonry spandril filling, and then earth, or metalling, or some other weight of a totally different nature, such as water in a canal aqueduct, or rails and sleepers, as in a railway viaduct. How, then, can we reduce the load on a diagram to a common factor which will enable us to apply the rules for the investigation of the Line of Resistance? This may be found either analytically or graphically.

(*a*). Analytically we may take moments of the entire mass and its component parts round any point, as b (*Fig.* 188). Let x be the horizontal distance from that point of the vertical through the common c.g., x_1, x_2 ... the horizontal (known) distances from the same point to the verticals through the c.g. of the separate masses, w_1, w_2 ... (*Fig.* 188).

Then
$$x = \frac{w_1 x_1 + w_2 x_2 + \ldots}{w_1 + w_2 + \ldots}.$$

(*b*). Graphically. 'The first step is to reduce the actual load upon an arch, including the weight of the arch ring itself, to an equivalent homogeneous load of the same density as the arch ring. The upper limit of this imaginary loading is called the *reduced load contour*. For example, suppose it is required to find the reduced load contour for the arch loaded as in *Fig.* 196. Assume that the weight of the arch ring is 160 lbs. per cubic foot. Reduced load contour.

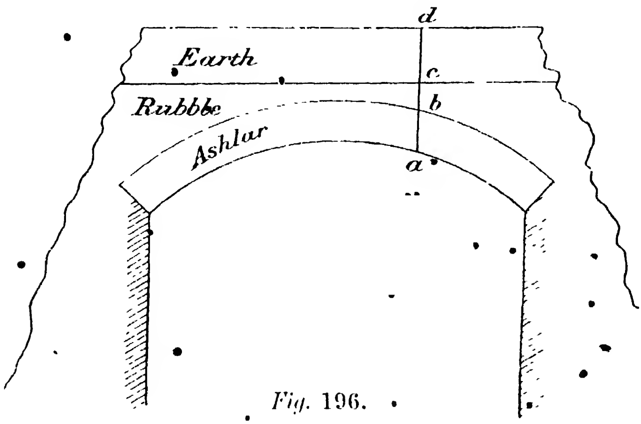


Fig. 196.

that of the rubble backing 140, and of the earth 100. Then the ordinate at a to the load contour of an equivalent load of the density of the arch ring is equal to $ab+bc\frac{140}{160}+cd\frac{100}{160}=$ say gf (*Fig.* 197). The value of gf is laid off in *Fig.* 197. Computing the ordinates for other points in the load contour gives the line EF (*Fig.* 197). The area between the intrados and the reduced load contour is proportional to the load on the arch. In a similar manner a live load (as for example a train)* can be reduced to an equivalent load of masonry, in which case the reduced load contour would consist of a line GH above and parallel to EI for that part of the span covered by the train, while for the remainder of the span the line IF is the reduced load contour. The second step is to draw the arch ring and its reduced load contour on thick paper to a large scale, and then with a sharp knife carefully cut out the area representing the load on each arch stone. The c.g. of each piece can be found by balancing it on a knife edge, and then the position of the c.g. is to be transferred to the drawing of the arch."†

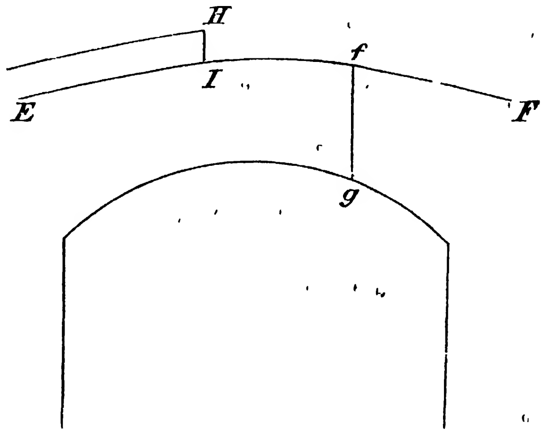


Fig. 197.

Distribution of Passing Loads.

Distribution from earth filling.

There are still two more points to be discussed in connection with the external forces acting on an arch, and the resistance produced.

The first of these questions is:—In the case of loads passing over the surface of an ordinary road crossing an arch, to what extent does the metalling and earth filling distribute the load?

To this there can be no absolute answer, as the distribution will

* This assumes that the weight of the train acts directly on the vertical portion below it. This is not quite the case, as will be presently pointed out.

† Professor Baker, ***Masonry Construction***, p. 459.

evidently vary with the nature of the road surface and the earth. The practice among Continental Engineers* is to consider the distribution at an angle of 45° on either side of the vertical axis through the centre of the load, or in other words, the load is equally distributed over a cone of earth whose apex is the surface directly below the weight and the angle = 90°. It is evident that the greater the depth between the roadway and the extrados of an arch, the less effect will be produced by passing loads; and the probability is that the distribution is not less than that indicated by an angle of 45° with the vertical (see *Fig.* 73, p. 80).

Another question is how far any spandril filling or backing may be taken as adding to the thickness of the arch ring? We may find, for instance, that the Line of Resistance comes perilously near the extrados of the actual arch *ring* at a point where above that extrados there is masonry. If that masonry is included in the arch limits, there will be an ample margin of safety, but if not, it will be unsafe, and the question is how far are we justified in taking the masonry filling into consideration? Whether spandril should be considered as part of the ring.

If the arch is built of ashlar, with a through bond between the spandrils and the arch ring, or if formed of carefully made and laid concrete, it seems reasonable to include the spandrils.

If, on the other hand, the arch ring is of brickwork or rubble, and the spandril of brickwork laid in the usual horizontal courses or rubble, either random or coursed, it would be hardly justifiable to consider the spandril as having any other effect than merely adding additional distributed weight.

Stability of Abutments and Piers.

When the magnitude, direction, and point of application of the resultant thrust produced on the skewback has been ascertained by the investigation given above, the stability of the abutments is a matter of no difficulty. Abutment acted on by at least three forces.

The thrust (T, *Fig.* 198) of the arch thus ascertained is one of several external forces acting on the abutment, and the resultant of these forces must be such as to fulfil the three conditions of equilibrium in masonry.

The other external forces vary with circumstances. In the case of abutments with earth backing (see *Fig.* 198) the earth thrust (P) is a most important external force, and for small arches under a

* Pascal, *Ponts Métalliques*.

heavy surcharge it necessitates a heavy abutment. In the case of service reservoirs arched but not backed with earth (*Fig.* 237, p. 294) the water pressure inside the reservoir adds to the thrust of the arch in tending to push the abutment outwards and endanger its stability. The weight W of the abutment itself is always one of the external forces.

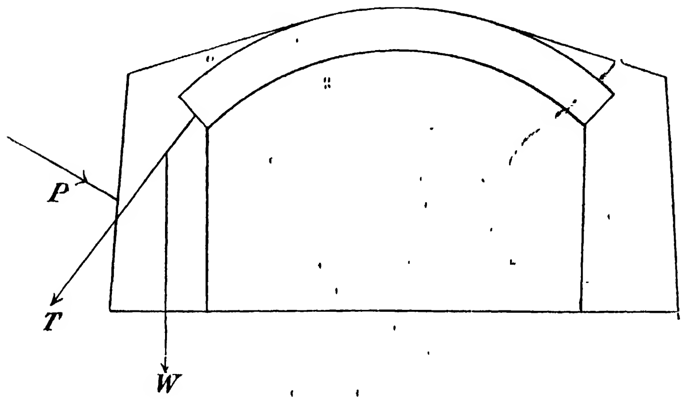


Fig. 198.

Joints may inclined.

The Line of Resistance of the abutment may be so inclined to the direction of the joints as to endanger its stability against sliding. To obviate this it is sometimes necessary to have the joints inclined, as in *Fig.* 279, p. 307, Part I. Such abutments are "in truth a continuation of the arch" (Rankine).

Thrust on piers.

Where there are two arches in a bridge, as in *Fig.* 199, the horizontal components of the thrust neutralize each other, leaving the vertical components only to be borne by the pier. When one only of the arches which spring from the pier is loaded with a travelling load, there will be an excess of horizontal thrust on the pier. The amount of this thrust may be roughly computed by multiplying the travelling load per lineal foot by the radius of curvature of the intrados at the crown in feet.

The pier should for practical reasons have sufficient breadth for the whole of the two arch rings which spring from it. Professor Rankine has pointed out that these limits would result in less width than are usually adopted for piers, the common thickness of which at the springing is from $\frac{1}{6}$th to $\frac{1}{7}$th of the span.

Arches which rest on piers which are built on this principle must, for obvious reasons, be constructed simultaneously. A flood or other destructive cause which carries away one pier will not only wreck the two arches on either side of it, but will cause so great a bending moment on the further piers as to endanger their stability to a serious extent. To localize the dangerous effect of any such accident it is customary in a long series to build at intervals *abutment piers, i.e.*, piers designed with sufficient breadth to resist by themselves the thrust of any one adjacent arch. Abutment piers.

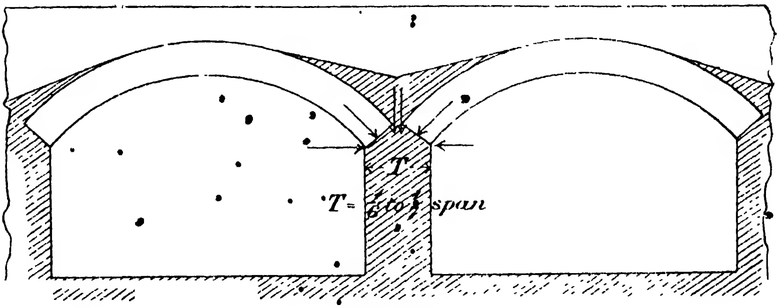


Fig. 199.

While considering the *destruction* of arches it may here be pointed out that in those that are designed (as they usually are) of circular segmental form, the theory in the foregoing pages points to the best position of attack. We have seen that the tendency of arches is to fall in at the crown and to open out at the haunches, and for this reason we load the haunches, and have a minimum load at the crown. To destroy the arch the reverse process should be adopted, *i.e.*, the earth covering on the haunches should be dug up and piled on the centre. A comparatively small explosive under the haunches ought then to have the desired effect (*Fig.* 200).* ...mplest ...thod of ...troying an ...h.

Thickness at the Crown.

Mr. Trautwine's rule for the crown thickness (as given in Part I., p. 323) is Crown thickness.

$$d = \frac{\sqrt{r + \frac{1}{2}s}}{4} + 0{\cdot}2 .$$

* This application of theory to an important part of a military engineer's duty has been pointed out by Capt. and Bt.-Major J. R. L. Macdonald, R.E.

for a first class cut stone arch, increased by $\frac{1}{8}$th for second class work, and by $\frac{1}{3}$rd for brickwork or fair rubble. Here r = radius at crown, and s = span, both in feet. Professor Rankine gives for a single arch—

$$d = \sqrt{0{\cdot}12r},$$

and for a series

$$d = \sqrt{0{\cdot}17r}.$$

The rules followed by Continental Engineers give larger results in short spans, and lower results in large spans.

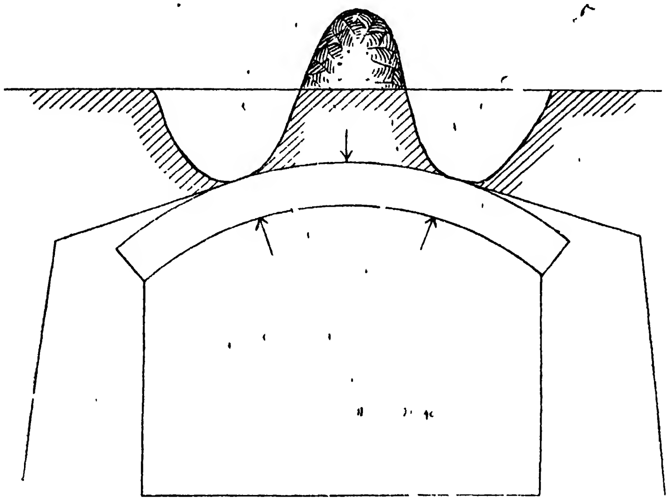


Fig. 200.

As an illustration of the foregoing principles we may take the following:—

Example. EXAMPLE 16.—*It is required to design a masonry arch, 40′ span, 10″ rise, to be built of very hard bricks (of quality equal to second-class stone work), carrying an ordinary road 16′ wide between parapets, which are each 2′ thick.*

Greatest weight on the bridge 80 lbs. per foot super, or a traction-engine with 9 tons on driving wheels, 5 tons on leading wheels, with 10′ 8″ wheel base, and distance of wheel centres 6′ 2″.

The depth of the earth over the extrados at the crown is 3′.

Weight of earth 100 lbs. per foot cube, and of brickwork 112 lbs.

Abutments to be 12′ in height from floor.

The first step is to design the arch by any known rules (say Trautwine's), and then investigate its stability. Depth at crown.

The depth of the ring at the crown, according to Trautwine, would be

$$d = \frac{\sqrt{r + \frac{1}{2}s}}{4} + 0{\cdot}2,$$

and the radius being

$$\frac{\left(\frac{s}{2}\right)^2 + a^2}{2a}$$

(see p. 323, Part I.); a being the rise, or 25 feet, the value of d is 1·877 for cut stone or 2 feet for second quality stone work.

The value of the abutment thickness

$$t = \frac{25}{5} + \frac{10}{10} + 2 = 8'.$$

The tangent to the extrados is drawn from a point G (*Fig.* 201), such that $NG = \frac{1}{2}IT$, and the line GN is found by the steps described on pp. 324–5, Part I. Design of abutment, etc.

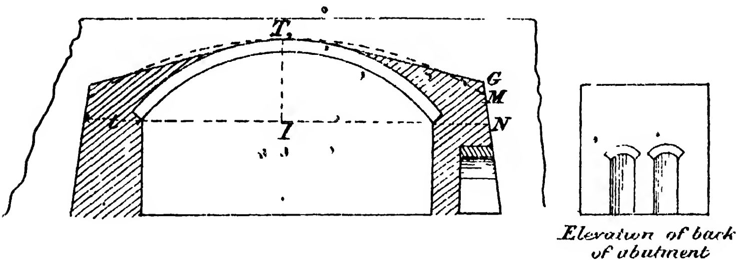


Fig. 201.

It will be observed that this does not give a thickness gradually increasing from the crown to the joint of rupture, which may be taken at about 45°, but it does assist generally in the stability of the structure as a whole by adding weight to the abutment. The curved line shown in *Fig.* 201 would in all respects be better, because it adds to the weight on the arch ring at once after leaving the crown, and the c.g. is nearer the inner edge of the abutment than that of the other form. As, however, most arches are built with the straight tangent, this will be here investigated.

As regards the weights coming on each of the 16 imaginary voussoirs into which the arch ring is divided (see *Plate* XVIII.), it is External loads

assumed that, vertical lines having been drawn from the edges of that voussoir on the extrados, the weight coming on 1′ in length of the voussoir is made up of—(*a*), the moving load W_1 on the roadway: (*b*), the vertical weight W_2 of the earth and metalling between the roadway and the masonry; (*c*), the horizontal thrust W_3 of the earth on the vertical projection of the masonry; (*d*), the weight W_4 of the backing acting at its c.g.; and (*e*), the weight W_5 of the voussoir itself. These five forces have a resultant which is represented in magnitude and direction for each voussoir on *Fig.* 1, *Plate* XVIII., and is shown on the load diagram (*Fig.* 2).

Two conditions of investigation

We may investigate the stability under two conditions:—(1), where the whole arch is covered with people, bringing a weight of 80 lbs. per square foot; and (2), where the traction-engine is on the bridge, and is rolling off, causing an unsymmetrical load.

1. Uniform load

(1). The right half of the ring on *Fig.* 1 in *Plate* XVIII. is shown without the Line of Resistance, as that line would be the same for both halves. It is left blank to show the construction lines.

On the left half of the arch the weights on each voussoir are shown acting through the common centre of gravity. The horizontal component of the thrust is very slight, even in the voussoirs near the springing. The weights are shown on the load line XP.

The horizontal thrust being at the crown, a horizontal line OP is drawn to any point O, and the lines Ox_1, Ox_2, etc., drawn.

From E, the upper point of the central half at the crown, a horizontal line is drawn, meeting the vertical through the c.g. of voussoir 8 at y_8. From this point y_8 a line $y_8\ y_7$ is drawn parallel to Ox_7, meeting the vertical through voussoir 7 at a point y_7, and so by drawing lines parallel to the lines in *Fig.* 2, we get the dotted polygon EQG (*Fig.* 1), where QG is parallel to OX, and EG is horizontal. Then a vertical through G passes through the common c.g. of all the loads.

Join FG where F is the lower limit of the central half of AB, and draw O′X parallel to FG, and join $O'x_1$, $O'x_2$......$O'x_7$. Then starting from E, again draw a new force polygon as before, and mark the points where this polygon cuts the joints.

Position of Line of Resistance

These points give the Line of Resistance, shown in thick red lines on the drawing. This falls close to the limits of the central half, and though it comes near the intrados at the first voussoir, and just outside the central half, it is so near the limit that it is not necessary to take a new value.

To find the maximum pressure on the arch ring we measure the

distance from the edge of the centre of pressure, and find it to be 5″. The width of the joint is 24″.

As the pressure on the joint ($O'x_1$ which scales 21,600 lbs.) comes outside the centre third the maximum pressure will be found from the principles illustrated on *Fig.* 275, p. 304, Part I. Here $d = 5''$, and $P = 21,600$. Hence maximum pressure

$$= \frac{2 \times 21600}{3 \times 5} = 2880 \text{ lbs. on } 12'' \times 1'' \text{ run,}$$

i.e., 240 lbs. per square inch, or

$$\frac{12 \times 2880}{2240} = 15{\cdot}4 \text{ tons per square foot.}$$

From Table VIII., p. 33, Part I., it will be seen that to give a margin of safety it will be necessary to use very hard bricks laid in cement. The strength of the single brick is not the governing factor, but the brickwork in mortar. Staffordshire bricks in cement should be used for this arch. Even then the factor of safety would be small. It would be safer to increase the thickness of the arch ring to 27 inches, or three whole bricks. If of Portland cement concrete, the thickness might be left at 2 feet.

(2). For the rolling load (*Fig.* 1, *Plate* XIX.) it is assumed that the weights on each wheel are distributed over a cone, the apex angle of which is 90°. In *Fig.* 1 the traction-engine is supposed to be rolling off the bridge to the right, and the heavier wheels to be just clear of the arch. This will be the position least favourable to stability (approximately). The lines of resultant pressure through the centres of gravity of the voussoirs and their loads are shown in red, with amounts written against each. 2. Unsymmetrical load.

The reaction of the right support is then easily calculated by taking the algebraic sum of the moments at the opposite support. The amount of that reaction is 21,100 lbs. That is equal to the weights on voussoirs 10—16 inclusive, plus a part of voussoir 9, the amount of which can easily be reckoned by proportion. The weight of the whole of voussoir 9 being 2,652 lbs., and the difference between 21,100 and voussoirs 10—16 being 1,854, we draw the joint JH dividing voussoir 9 in the proportion of

$$798 : 1854, \quad (1854 + 798 = 2652).$$

This is the joint of horizontal thrust.

The line through the c.g. of the weights on the right of this joint is found by the same process as before. Then taking moments about F (the lowest point of the centre half of the joint AB) of the weight W, and the thrust T acting at the highest point of the centre half of the joint HJ—

$$T \times he = W \times dF,$$

where $$he = 11{\cdot}6', \; dF = 7{\cdot}8',$$

and $$W = 21{,}100 \text{ lbs.}$$

Whence $$T = 14{,}380.$$

This value OP is laid off, and the points O*x*, etc., joined as before. A trial Line of Resistance (shown dotted) is drawn with this value, but it comes above the limits at the crown, and below the limits at the springing. A slightly increased value of T should be taken.

Try $$T = 15{,}000.$$

Then $$h'e = \frac{21000 \times 7{\cdot}8}{15000} = 10{\cdot}8.$$

With this value a new Line of Resistance is drawn, which just falls within the limits.

The maximum stress would be at the joint between voussoirs 15 and 16. The centre of pressure is at 6″ from the edge, and the amount of pressure is 23,600 lbs. (O′*x*15). The maximum pressure is

$$\frac{2 \times 23600}{3 \times 6} = 2{,}622 \text{ lbs. on } 12'',$$

i.e., 235 lbs, per square inch, or

$$\frac{12 \times 2622}{2240} = 14 \text{ tons to the square foot.}$$

Hence good hard bricks in cement mortar are required, if the thickness be 2 feet. As stated above, it would be better if made 27 inches.

Stability of Abutments.

Stability of abutment.

In both the above cases the stability of one abutment is shown on the left. The weight of the masonry abutment and the earth above is first found, and a vertical line drawn through its c.g. Its amount is found to be 21,200 lbs., and it is represented by *ac*. The resultant thrust GF on the skewback is produced, and its amount

(24,000 lbs., in *Fig.* 1, *Plate* XVIII.) is laid off at *ab*. The resultant of these two forces is *ad*.

This must be compounded with the resultant earth pressure at the back of the wall *gh*. As will be seen in a subsequent chapter, this thrust acts at $\frac{2}{3}$ of the total depth below the surface, and its horizontal component

$$=\frac{1}{2}wh^2\frac{1-\sin\phi}{1+\sin\phi},$$

where w = weight per unit of the earth (100 lbs.), h = height in feet (=27 feet), and ϕ is the angle of repose of the soil, which we may take at 30°. This gives a value to the horizontal thrust of 25,000 lbs., and to the inclined thrust, acting at an angle = to the angle of repose, of about 28,000 lbs. Compounding this ($a'e$) with $a'd'$ (= ad) we get a total resultant pressure shown in a thick red line 115,000 lbs., in *Fig.* 1, *Plate* XVIII., which intersects the base almost at the centre*, showing that the pressure is nearly uniformly distributed over the 10 feet of base, giving 11,500 lbs., or 5·1 tons per square foot. This can, of course, be spread out over the footings, so as to reduce the pressure per square foot on the soil to any desired amount.

It is sometimes desirable to have an approximate idea of how much the thrust of a uniformly loaded segmental arch amounts to without going into the above lengthy investigation. This may be estimated by considering the Line of Resistance as a parabolic arc (which it is approximately), and considering the skewback at the springing to be at right angles to the tangent to the curve there. This will not be far wrong for arches where the skewback is inclined between 35° and 60° to the horizontal. If θ be the angle of the skewback to the horizontal, w the average weight per foot run on the arch, l the span, and d the average rise (*i.e.*, to centre of ring at the crown) the thrust on the skewbacks is approximately $\frac{wl^2}{8d}$ cosec θ. In this case $\theta=37°$, $d=11'$, $l=41'$ (measuring to inner edges of centre half) and $w=830$ lbs. per foot run, on a strip 1 foot in width. Hence the thrust

Approximate value of abutment thrust.

$$=\frac{830\times 41\times 41}{8\times 11}\times 1\cdot 66=26,320 \text{ lbs.}$$

From *Fig.* 2, *Plate* XVIII., OX = 24,500 lbs., the difference, therefore, being on the safe side.

* Of course, this is approximate. The exact position depends on the angle of repose of the earth backing.

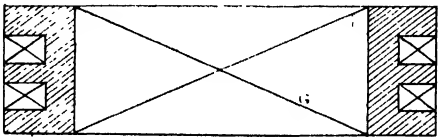
Trautwine's rules in accordance with theory.

From the example above it will be seen that such an arch, designed according to Trautwine's rules, will give good results. Probably, if the backing had been disposed along a curve passing through the crown, and passing a little below the edge of the spandril filling (see M, *Fig.* 201), it would have given still better results.

Where it is necessary to take heavy loads over a bridge it is desirable to spread that load as uniformly as possible by temporary planking, or in some such manner.

Counterforts

When an arch is built, as is frequently the case, with counterforts, so as to save masonry in the abutments, as in *Fig.* 202 (which represents the plan of the arch in *Fig.* 201), the *total* thrust of the arch at springing (*i.e.*, thrust due to 1 foot in width × total width) should be compounded with *total* weight of abutment and counterforts. The resultant should pass within the centre third of abutment and counterforts.



Plan of abutments with counterforts

Fig. 202.

Construction of Arch Ring.

Material of ring.

From the above investigation we also see that the best materials are necessary in important arches. The investigation has been based on the assumption that the arch ring is homogeneous and hence if the ring is built without proper bonding the calculations are of little value. Frequently arches are built of half-brick rings. These may be, and often are, quite sound, but if so their stability largely depends on the strength of the mortar. Numerous instances may be seen of arches built in this way which have shown signs of failure from one ring sliding on another.

In other cases arches are built with "rubber" bricks laid in Flemish bond, the bricks having been rubbed to a voussoir shape by the bricklayer. This is admissible in walls of houses where little real weight comes on the arch, but in important bridge work it is out of the question, as the bricks are far too soft to stand a heavy crushing weight.

Hard bricks with taper joints not recommended.

In other cases hard bricks are used, and the necessary shape is given to the voussoirs by increase of the mortar joint near the extrados. This is not to be recommended, as the strength of the

ring cannot be homogeneous, and at best is measured by the strength of the mortar.

This difficulty has been very successfully overcome by Mr. C. Richardson,* Engineer of the Severn Tunnel. He used the ordinary 9″ × 4½″ × 3″ brick, and two special bricks, voussoir shaped, and each of the same cubic content as the ordinary 9″ × 4½″ × 3″ brick. The lower one of these special bricks was 9″ long, 6″ broad, and tapering from 2″ to 2½″, while the other was 9″ long, 5″ broad, and tapering from 2½″ to 3″. The arrangements in a ring, 27″ wide, such as that for the bridge shown in *Plates* XVIII. and XIX., would be as in *Fig.* 203. It will be seen that by the use of these special bricks bond is given to the whole arch ring, both longitudinally and transversely. Special bricks.

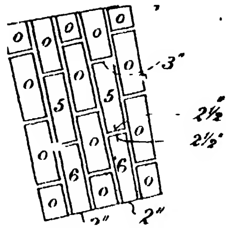


Fig. 203.

Concrete Arches.

With ashlar arches, or concrete arches, where the spandril filling and the arch are constructed simultaneously, it would be quite justifiable to consider the spandril filling as part of the arch ring. The limits of the area within which the Line of Resistance can fall are therefore largely increased. The curved form for the extrados is in this case even more desirable than in the case of brick arches.

There is a diversity of opinion as to how the concrete should be laid. Some engineers consider that, as the consolidation of the concrete is of the utmost importance, it should be laid in horizontal layers all over, as in that position it can best be rammed. Others consider that it should be laid in voussoir-shaped masses from springing to crown, and not rammed. Others again consider that, as far as possible, it should be laid and rammed in concentric rings. Circumstances must influence an engineer in deciding as to which of these expedients he should adopt. Possibly, a combination of all three may be the best for a certain case. Method laying.

Considerable economy has been effected in arch construction by the use of the Monier system, where a network of steel rods is laid Monier system.

* *Engineering*, 13. 1. 88.

in the concrete at those parts where the tension is greatest. This would point to the intrados at the crown, and the extrados at the haunches, as being the best positions in a circular segmental arch.

Austrian tests. The Monier system has been recently tested on a somewhat extensive scale in Austria, and has given very remarkable results. Arches of 23 mètres (75·4′) span, and 4·6 mètres (15·1′) rise, and 2 metres (6·56′) were experimented on, and were built of brickwork, rubble, ordinary concrete, and Monier concrete. The following were the breaking strains:—

Brickwork	67·5 tons
Stone...	74·02 „
Concrete	83·27 „
Monier system	146·12 „

The loads were applied from one abutment up to the crown of the arch in each case, applied very gradually, and the deflections were very carefully measured along the entire arch ring. The load was put on one side only, because it was considered that under such conditions the arch would be distorted to a much greater extent than would be likely to occur in practice.

The arches failed, by cracking, in every case, at the positions indicated by theory.

The brick and stone arches were both built in Portland cement mortar, 1 cement to 2·6 sand. The crown thickness in these arches was 0·6 mètres (1′ 11$\frac{3}{4}$″), and at the haunches 1·14 mètres (3′ 7$\frac{1}{2}$″). The concrete arch was 0·7 mètres (2′ 3$\frac{1}{2}$″) throughout. The Monier arch was 0·35 mètres (1′ 1$\frac{3}{4}$″) thick at the crown, and 0·6 mètres (1′ 11$\frac{5}{8}$″) at the haunches. The longitudinal rods were 0·55″ in diameter, and the transverse rods 0·276″ diameter, the mesh being 2$\frac{1}{2}$″.

The widest spans that have been built in the Monier system as yet have only been about 130 feet, but there seems to be little doubt that in the future much larger spans will be constructed.

Large spans. As a result of the Austrian experiments a stone arch, 213′ span by 58·6′ rise, has been built over the Pruth, at Jaremcze (exceeded only by one arch, the Cabin John, at Washington, 220′ span), and a similar arch has been designed by the same engineer for a span of 120 mètres, or 393′.*

* These data are obtained, partly from a lecture by A. E. Carey, Esq., M.Inst.C.E., delivered at the S.M.E. in January, 1898, and partly from *The Engineering Magazine* for September, 1897.

In England the longest span, arched in masonry, is the Grosvenor bridge, over the Dee, at Chester, which is 200′ clear width, with 42′ rise. The Severn bridge, at Gloucester, is 150′ span, 35′ rise.

In India some remarkably bold arches were built by the late Major-General Sir James Browne, K.C.S.I., R.E., on the Kangra Valley Road, in the Punjab; some of these were constructed of brick, some of concrete; in all cases of material, and with labour, locally obtained. The greatest span was 142′.

With concrete arches Sir James Browne recommended that they should not be segmental but elliptical, so as to enable the concrete to be consolidated better at the springing.

With all concrete arches wooden centering is objectionable, though frequently inevitable, as the spring of the wood prevents proper consolidation by ramming. Where it is possible to do so, it is better to build up the centering of dry rubble plastered over on the surface. This is largely used in India with successful results.

*Groined Arches.**

Groined arches.

In the case of groined arches, the most important point is to investigate the stability of the piers.

The pier ABDC (*Fig.* 204) has to support the thrust of the half arch CQRD, as well as the thrust of the two quarter groins SEQC, TFRD, in the directions EC, FD, the two together producing an additional thrust in the direction of the axis of the pier, while it is strengthened against overturning by the weight of the two half arches SUAC and TVBD, the horizontal thrusts of which neutralize each other.

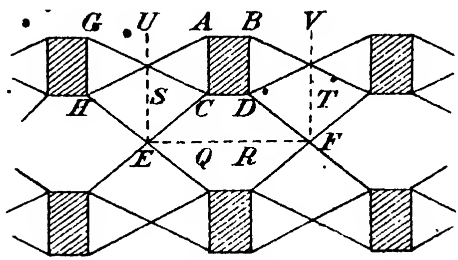


Fig. 204.

The forces acting on the pier are:—(1). The total horizontal thrust at the crown acting along EF. (2). The weight of the arch ring CEFD, acting though its c.g. (3). W, the weight of the pier and the two half arches SUAC, TVBD, resting on it, acting at their common c.g.

The resultant of these forces should pass through a point within the centre third of the base.

Domes.

Domes.

The stability of a dome differs from that of an arch in that there is no thrust at the crown of a dome; it is, indeed, common to have domes open at the top.

* General Wray's *Instruction in Construction*.

A voussoir block* in a dome is kept in position not only by the blocks above and below it, but by those on each side of it. The latter exercise a horizontal radial force, which, without any pressure from above, will, when combined with the weight of the voussoir, give a resultant passing within the limits of the joint below. The sliding of the voussoirs must also be prevented.

In *Fig.* 205 let *ab* represent the weight of the stone HGLK. Draw OB perpendicular to HGO, and make YOB = ϕ the angle of repose of the material (say 33° the angle of repose of stone on stone without mortar). Since *ab* does not cut HG, horizontal thrust must be developed for equilibrium, and such a thrust as will prevent the resultant from forming with the normal to the joint an angle less than the angle of repose. Assuming the horizontal thrust to act through L,† the upper edge of the stone, we lay off from *b*, the intersection of the horizontal line through L with the vertical through the c.g. of the stone, the weight *ab* to scale; then from *b* draw *bc* parallel to OY, and draw through

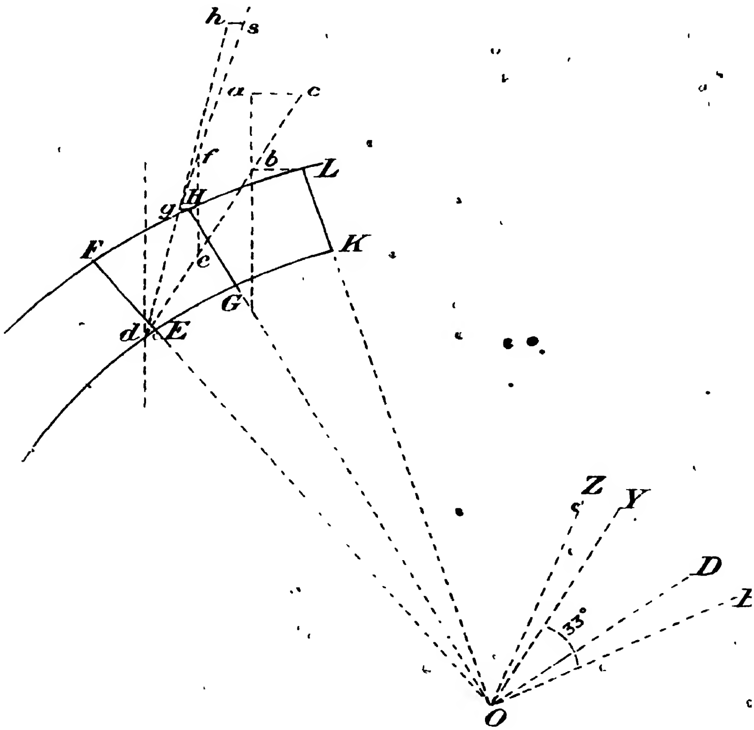


Fig. 205.

* From De Lanza's *Applied Mechanics.*

† This is Dr. Scheffler's method, his reason being that the thrust at the top is a minimum. The true position is probably near the middle.

a ac horizontal to meet bc. Then ac will be the horizontal force that will be furnished by the other stones in the course to produce equilibrium. The pressure upon the joint HG is bc, and it acts at the intersection of bc and HG.

Now prolong bc to meet the vertical drawn through the c.g. of the next stone HGFE at d. Combining this with the weight of the stone by making $de = bc$, and drawing ef (=equal weight of stone) vertical, we get a resultant fd. This makes an angle with the normal to FE greater than ϕ, hence draw OD perpendicular to FE, and draw OZ so that $ZOD = \phi$: then from g, the intersection of fd with the horizontal through H (the highest point of FHGE), produce gf to s, making $gs = df$; through g draw gh parallel to OZ, and through s draw sh horizontal. Then is sh the horizontal thrust which will be furnished at H to keep FEHG in place, and hg is the total pressure upon the joint FE, acting at the point of intersection of FE and hg.

In like manner the pressure on the other joints is ascertained. It will be seen that the horizontal thrust gets less as we go further from the crown (sh being much smaller than ac), and at last the resultant pressure cuts the joint within the limits of the joint itself, showing that there no horizontal thrust is necessary.

Instead of taking the thrusts as acting at the top of each joint we may take the central third, or any other limits of stability.

In ascertaining the pressure on the joint at the springing, we may either construct it graphically, or we may compute it by (a) finding the resultant of all the vertical forces acting on a given strip between two meridional planes; (b), finding the resultant of all the horizontal forces ($e.g.$, ac applied at L and sh applied at H); (c), compounding these two results.

CHAPTER XI.

RETAINING WALLS.

Uncertainty of the Subject.—Practical Considerations.—Professor Rankine's Theories.—Subsequent Modifications.—Wedge Theory.—Graphic Representations.—Examples.—Appendix.

RETAINING WALLS.
Definition of terms.

THE technical terms adopted in the case of retaining walls are generally as follows:—

Retaining Wall, properly speaking, is applicable to a masonry, brick, or concrete revetment built to retain an artificial bank of earth.

Breast Wall is the name applied to the revetment built on the scarp of a cutting in *natural* soil, to prevent disintegration or sliding. Generally breast walls have to sustain much less thrust than retaining walls of the same height and for the same earth. But in cases where there are strata of rock with alternate layers of clay sloping towards the breast wall, its thickness may require to be far greater than that required for an ordinary retaining wall.

Batter is the slope given to the face of the wall, thereby giving certain advantages in point of stability, which will be noticed in due course.

Buttresses are projections in front of a retaining or a breast wall. *Counterforts* are projections at the back of a wall, largely used in days gone by, in fortification, to limit the effect of breaching shot, and to confer stability. It is, however, very doubtful if they are not a source rather of weakness than of strength, and they are now not much used.

Land Ties are iron rods connecting the face of the wall with an

anchorage embedded in the solid earth behind, so as to give further resistance against overturning, etc.

Surcharge.—If a retaining wall supports earth which slopes upwards from its summit at an angle which must necessarily be less than the angle of repose, the height of the embankment at the top above the highest part of the wall is called the surcharge.

The *Angle of Repose** of the earth is the angle at which a bank of that particular earth will stand permanently when the weather has destroyed the cohesion of the particles of earth, leaving the earth to depend for its stability on the friction of the particles alone. The values of this angle for various earths is given in Table VII.

Table showing angle of repose of various materials.

TABLE VII.

Angles of Repose and Coefficients of Friction of Various Building Materials.

Material	Coefficient of friction = tan ϕ.	ϕ.
		Degrees.
Masonry and brickwork dry . .	·6 to ·7	31 to 35
,, with wet mortar	about ·47	about 25½
,, slightly damp mortar.	,, ·74	,, 36½
,, on dry clay . .	,, ·51	,, 27
,, on moist clay .	,, ·33	,, 18¼
Timber on stone .	,, ·4	,, 22
Iron ,, ...	·7 to ·3	35 to 62⅔
Timber on timber dry .	·5 ,, ·25	26½ ,, 14
,, ,, ,, soaked	·2 ,, ·04	11½ ,, 2
,, ,, metal	·6 ,, ·2	31 ,, 11⅓
Metal ,, ,, ...	·25 ,, ·15	14 ,, 8½

* See p. 306, Part I. Professor Rankine says: "The properties of earth with respect to adhesion and friction are so variable that the engineer should never trust to tables . . in designing earthworks, when he has it in his power to obtain the necessary data either by observation of existing earthworks in the same stratum or by experiment."

Angles of Repose of Various Earths.

(Chiefly from Newman's *Slips and Subsidences*).

Earth.	Angle of Repose ϕ.	Weight per cubic foot.	Coefficient of friction.
	Degrees.		
Sand, very fine and dry . . .	33 to 27		
,, wet	26	89 to 118 lbs.	
Soft chalk, impure and argillaceous	32 to 26		
Vegetable earth, dry .	29 ,, 18		
,, ,, (loam)	33 ,, 26	100 to 120 lbs.	
,, ,, very wet. .	17 ,, 14.		The coefficient in all cases $=\tan\phi$.
,, ,, ...			
Clay, dry	29		
,, damp	18	120 to 135 lbs.	
,, sound yellow, well drained	26 to 18		
,, wet .	17 ,, 1		
Gravel, clean and compact	45		
,, with sand	26 to 33	90 to 110 lbs.	
Loose shingle	35 ,, 39		

Three conditions of thrust.

In order to investigate the effect of the thrust of a bank of earth against a wall, it is necessary to know the three conditions, viz.—magnitude, direction, and point of application of that thrust.

Three conditions of stability.

In order to investigate the stability of a given wall to resist this thrust, we need to combine the weight of the wall with the thrust of the earth and see whether the resultant pressure is such that the structure is safe against overturning, crushing and sliding.

There are, however, certain difficulties in the theoretical investigation of the thrust which make the exact determination impossible. The theories on the subject which will hereafter be given are at best approximate. So indeterminate, indeed, is the problem, that some engineers simply follow the empirical rules given in Part I., pp. 321-22, without any modification due to theoretical considerations. This, however, seems to be unjustifiable, because it is clear, at all

events, that the direction of the thrust of the earth, as well as its magnitude, must depend, to an extent perhaps which is not quite known, but none the less certainly, upon the angle of repose of the soil; and that the resistance afforded by the wall must depend upon the weight of the material of which the wall is built. Both these considerations will affect to a very great extent the stability of the structure, and neither of them have been taken into account in the empirical rules referred to. Hence some theoretical investigation is evidently desirable, even though in our present state of knowledge it may be lacking in precision.

Take the case of a mass of earth (*Fig.* 206) supported by a wall Method of failure in a bank of earth.

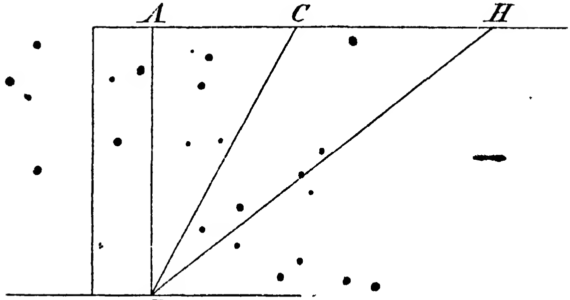


Fig. 206.

AB. If we conceive the wall to be entirely removed, a wedge or prism of earth ABC of a greater or less size, according to the consistency of the earth, will (as a matter of experience) be first detached from the general mass. Let BC be the line along which rupture takes place. Then, after a certain time, the influence of the weather will have the effect of detaching successive particles of BCH, until ultimately the bank will assume a slope BH, the natural slope of the earth, making an angle φ with the horizon, the angle of repose, which will be a constant whatever the height of the bank may be.

Now the prism BAC (*Fig.* 207), *if in one mass*, will be in equilibrium without exerting any pressure against AB, because its tendency to move down BC will be met by the friction of the surface.

But if we consider another prism BAE, where BE is at a greater angle than the angle of repose, it is clear that this will exert against the wall a pressure due to its own weight, and diminished by the

friction of the earth on the slope BE. If this prism be very small, as BAD, close to the wall, it will evidently exert less pressure than BAE. There must, therefore, be some prism or wedge between AB and BC, where the pressure on the back of the wall is a maximum.

Position of line of greatest thrust.

It can be proved (see Appendix I.) that this maximum pressure occurs when the area of the triangle BEF = ABE, EF being the perpendicular on BC. When the top of the bank is horizontal, the angle AB is bisected, in such a case, by BE.

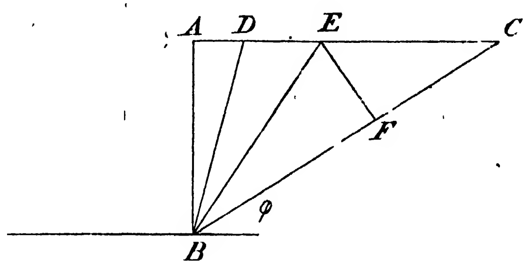


Fig. 207.

We can, on this basis, ascertain the *magnitude* of the greatest thrust on the vertical face AB, which need not necessarily be the back of the wall, but may be taken as any vertical plane at right angles to the section of the embankment under consideration. The *direction* of the reaction of this plane would be normal to its face (if there were no friction), and the *amount* $= \frac{1}{2}wx^2$,* where w = weight of the earth per unit of volume, and x = EF, the perpendicular on B above mentioned.

If (in *Fig.* 208) we produce CA to G meeting the perpendicular from BC at G, and if we draw AH at right angles to CB, and call BG $= c$, AH $= a$, BC $= b$, it can be proved (see Appendix) that when the thrust is a maximum

$$x = b \tan \beta - \sqrt{b \tan \beta (b \tan \beta - a)} = c - \sqrt{c(c-a)}.$$

Magnitude of greatest thrust.

Hence the maximum thrust

$$T = \tfrac{1}{2} w \{c - \sqrt{c(c-a)}\}^2.$$

Now as c and a are known quantities, if we can by any geometric methods obtain a graphic value for the expression within the brackets

* For proof see Appendix.

we have found the *amount* and the *horizontal direction* of the maximum thrust. These geometric methods will presently be detailed.

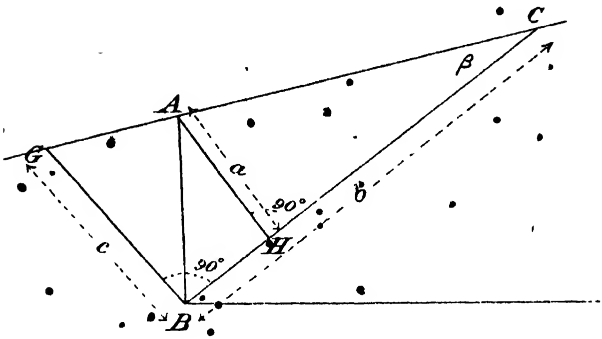


Fig. 208.

The direction, however, will be modified by the friction on the plane AB; the total amount of such friction must therefore be compounded with the amount just found (which we may call T) to give the total value of the thrust T′. As the coefficient of friction $= \tan \phi$, the amount of the thrust T′ due to friction will $= T \times \tan \phi$. Hence if we represent the thrust T by a line drawn to scale PQ (*Fig.* 209), and draw the angle PQR $= \phi$, then RP = component of thrust due to friction, and the resultant pressure T′ at the back of the vertical plane will be represented by RQ.*

The *point of application* of this thrust is, however, still to be ascertained. It is evident that the total pressure on the vertical Point of application of maximum thrust.

* This is Dr. Scheffler's theory, and is at least on the safe side. Its accuracy, however, is open to question. According to it the friction increases the overturning force. This, in the extreme case of a wall having a very great coefficient of friction, would involve large pressure intensity on the material, which seems improbable. Mr. Trautwine points out that if a wall were to overturn round its outer toe H (*Fig.* 214) its back DB would rise, and in so doing would rub against the earth in contact with it. This rubbing must evidently *impede* the overturning. The friction, therefore, must assist the stability, and not assist the overturning thrust. To what extent it would do so is difficult to determine, because the full frictional resistance will not be developed until rotation has actually begun (see Minchin's *Statics*, Vol. I., p. 68), and in that case it is not possible to say what amount of earth thrust will now be developed. All that one can say is that, although Dr. Scheffler's thrust may err as regards its magnitude (T′ being $= T \tan \phi$), it is, at all events, on the safe side, and as it is improbable that T′ will be as little as T $\sin \phi$, the difference in ordinary cases of angles of repose, about 26°, will not be so great as to matter very much, or lead to practical reduction of the size of the wall.

plane varies as some function of the height. In water it varies as the square of the height, *i.e.*, the pressure uniformly increases with the depth. If the soil be homogeneous, it is probable that this is also the case in the pressure on a vertical plane in a bank of earth, and this assumption has been adopted as practically true by Professor Rankine, Dr. Scheffler, and all writers who have treated this subject theoretically.

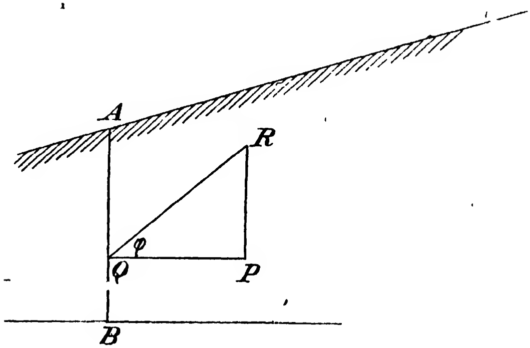


Fig. 209.

Professor Rankine, however, also considers that the pressure acts in every case parallel to the surface. This is tantamount to saying that in a wall with a vertical back, without surcharge, the direction of the pressure is horizontal and is similar to water pressure. As this takes no account whatever of the variations in various classes of earth, it has been discarded by subsequent writers, as there evidently must be a difference in the direction of the pressure afforded by a wet clay, and by a dry sand.

Professor Rankine's equations for the value of the earth thrust are—

Rankine's formulæ. (1). For a wall without any surcharge

$$P = \tfrac{1}{2} w h^2 \frac{1 - \sin\phi}{1 + \sin\phi} = \tfrac{1}{2} w h^2 \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \qquad .(\alpha).$$

(2). For a wall with an indefinite surcharge, the angle of the bank with the horizontal being $= \theta$ —

$$P = \frac{wh^2}{2} \cos\theta \, \frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}} \qquad .(\beta).$$

In the above P = value of the earth thrust in lbs., w = weight of earth per cubic foot in lbs., h = height of the back of the wall, or other vertical plane, in feet, ϕ is the angle of repose of the earth.

In any case, however, we may take it that with earth of a fairly homogeneous nature we have the pressure uniformly increasing from the surface downwards, and hence the centre of pressure will be at a point $\frac{2}{3}$ of the height of the plane from the surface.

We have thus found the magnitude, direction and point of application of the resultant earth pressure at the back of a vertical plane in a mass of earth. Before going on to show how, by means of simple geometry, we may obtain the magnitude of the horizontal component of the thrust graphically (the value of which may be compared as a check with Rankine's equation (a) above), it may be as well to point out the defects of the theory as stated.

*Defects of the Theory.**

I. The assumption that the surface along which the rupture takes place (EB, *Fig.* 207) is a plane. It would be probably correct, or nearly so, in clean, sharp sand, where the soil is devoid of cohesion, and is homogeneous. It would be considerably in error in tough, tenacious soil. Objections to theory.

II. That the direction of the resultant pressure is inclined to the horizontal at an angle = to the angle of repose, is an assumption not by any means proven.

III. That the point of application of the resultant pressure is at $\frac{2}{3}$ the height from the surface. This would really only be true with perfectly homogeneous soils.

While recognizing these defects, it may be said that no theory has as yet been advanced which eliminates the uncertainties which surround the subject, and, in default of better, we can at all events apply the rules which are thus given to us with a very reasonable amount of security, and fair economy.

Graphic Method of Finding Thrust.

We now pass to the consideration of the graphic methods of finding the thrust, which, as we have seen above, is expressed by Graphic methods.

$$\{c - \sqrt{c(c-a)}\}^2 \times \tfrac{1}{2}w.$$

* From Professor Baker's *Masonry Construction*, pp. 340 *et seq.*

There are four cases—

(1). When the surface of the earth is horizontal.

(2). When the surface has its maximum slope $=\phi$, the angle of repose.

(3). When the surface has a slope less than ϕ.

(4). When the surface slopes *up* to the wall.

Cases (2) and (4) are of rare occurrence.

Case 1. Surface horizontal

CASE 1.—*Surface horizontal.*

In *Fig.* 210, produce the surface to meet the perpendicular from B at the point G. From centre G, with radius GA, draw the arc PA; then BP $= \sqrt{c\,(c-a)}$.

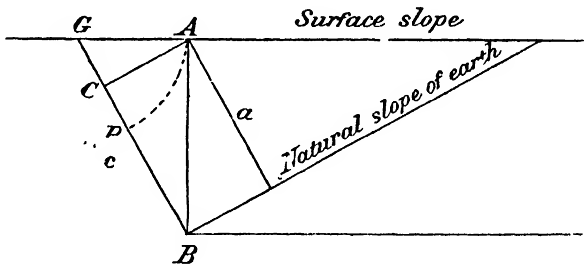


Fig. 210.

For, AC being perpendicular to GB—

$$GP^2 = GA^2 = BG \times GC,$$

but BG $=c$, and GC $=c-a$; $\therefore$ GP $=\sqrt{c\,(c-a)}$, and BP $=c-\sqrt{c\,(c-a)}$. Hence $T = \frac{1}{2}w BP^2$.

Case 2. Bank sloping at an angle $=\phi$.

CASE 2.—*Bank sloping at an angle* $=\phi$.

Here (*Fig.* 211) $c=a$; hence $c-a=\theta$;

$$\therefore\ c-\sqrt{c\,(c-a)} = c = BP;$$

$$\therefore\ T = \tfrac{1}{2}wBP^2.$$

Case 3. Bank sloping at an angle less than ϕ.

CASE 3.—*Bank sloping at an angle less than* ϕ.

Produce the line of the bank to meet the perpendicular from B, as before at G (*Fig.* 212). On BG describe the semi-circle GOB, and

from A draw ACO at right angles to GB, and from centre G, with radius GO, describe the arc OP.

Then $$GP = GO = \sqrt{BG \times GC} = \sqrt{c(c-a)};$$

$$\therefore\ PB = c - \sqrt{c(c-a)}.$$

Hence $$T = \tfrac{1}{2} wBP^2.$$

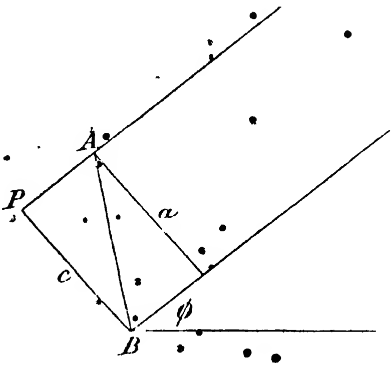


Fig. 211.

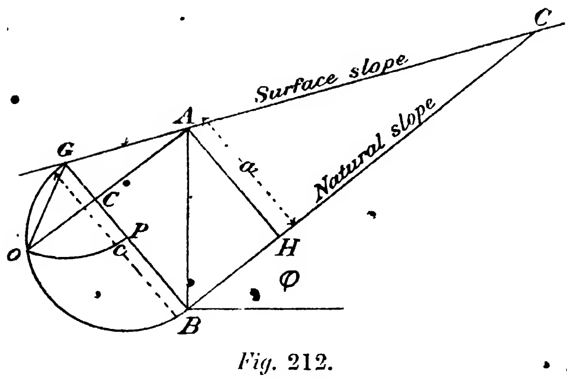


Fig. 212.

CASE 4.—*Bank sloping up to the wall.* CASE 4. Bank sloping up to the wall.

According to Rankine's theory of the thrust being always parallel

to the surface, there would here be an upward thrust, which is evidently impossible.

The geometric construction here (*Fig.* 213) is the same as in the former case.

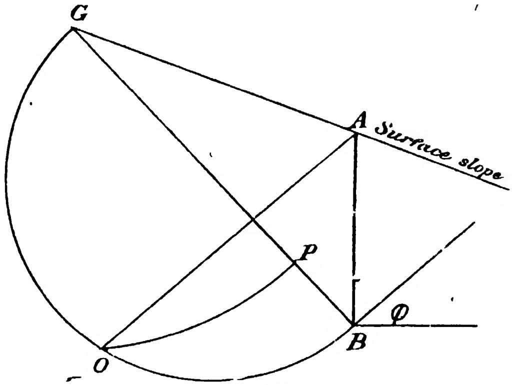


Fig. 213.

Design of the Wall.

Design of cross section.

To ascertain the form of the cross section of the wall we must evidently assume some form, and see whether the three conditions of stability are fulfilled when the thrust at the back of the wall is compounded with the weight of the assumed section.

In the dimensions of the assumed section the top width may be as small as possible, consistent with practical requirements. It will be governed by the consideration of the object for which the wall is intended, *e.g.*, in a dock the width at the top will be greater than in an ordinary revetment for a fort. As there will generally be a coping of some sort, 2′ is about the least width that would be admitted in practice.

Data for trial section.

The width at the ground line may be taken, as a trial, at some ratio of the height, measured normally to the face. With brick walls and no surcharge, $\frac{1}{3}$ of the height will generally be found to give fair results. With heavy stone, such as granite, less width will suffice. But in any case this width will be ultimately dependent on the angle of repose of the earth retained, as we shall see when working out an example, a loose dry sand requiring less width than a soft wet clay.

With these data a trial section can be drawn, as in *Fig.* 214. The

plane against which the overturning thrust should be calculated will be that ascertained by drawing a vertical line to the surface from the inner edge of the base (BA, *Fig.* 214). The forces, then, which tend to maintain stability are the weight of the wall plus the weight of the triangle or wedge of earth ABD.

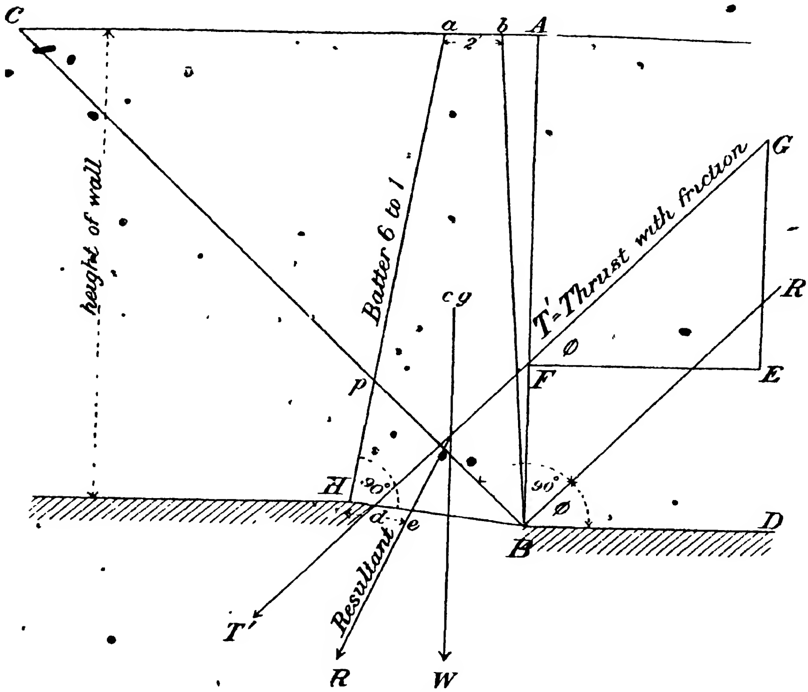


Fig. 214.

To ascertain the stability of the wall *against overturning*, the resultant thrust T' on the plane BA must be compounded with the resultant weight W of the wall and the wedge of earth ABD, acting through the common centre of gravity. The resultant R of T' and W should cut the base within the centre third, so as to obviate the chance of any tension being brought on the joint at B (see p. 299, Part I.). Stability against overturning.

The stability of the wall against *crushing* can then be investigated by Formulas (α) or (γ), p. 302, Part I. The distance d from the Stability against crushing

outer edge H of the centre of pressure e is measured. The value of the normal component N of the resultant pressure R is estimated, or measured from the drawing. The base HB or t being known, we know, in the formula

$$Y = \frac{2N}{t}\left(2 - \frac{3d}{t}\right) \text{ (Part I., p. 302),}$$

all the expressions on the right side of the equation, and may therefore find Y, the maximum pressure in lbs. or tons per square foot, on the outer edge. This can then be compared with the safe resistance of the material of which the wall is built.

Stability against sliding.

The stability of the wall against *sliding* can easily be ascertained by measuring the angle which the direction of the resultant R makes with the normal to the joint at the point where R cuts the base. This angle should not be greater than $\frac{4}{5}$ of the angle of repose of the material of which the wall is built, which is, of course, a very different angle from the angle of repose of the earth (see p. 306, Part I., masonry on damp mortar say 30°–35°.)

Effect of batter.

The batter on the face of the wall has a double effect in assisting the stability; it tends to throw the centre of gravity of the wall further back, and therefore brings the line of R nearer the centre of the joint HB, and since the courses of masonry are built at right angles to the batter, there is less tendency to slide forward than there would be if the courses were horizontal. It has also the advantage in point of apparent stability, for if the wall is originally vertical and gets pushed out of plumb, it *appears* to be unsafe. If it has a batter originally, the difference is not manifest.

Design of the Foundations.

Foundations.

This will vary with the soil on which the wall is built, but in any case it is desirable to distribute the weight equally.

We may assume that the soil is sufficiently good to be capable of bearing a definite weight per square foot. If special treatment is required, the same broad principles as hereafter detailed would be followed.

We may draw the base of the concrete bed XZ (*Fig.* 215) parallel to HB and at some depth below the frost limit (see p. 289, Part I.), say 4′, measured vertically below H. Then produce the back of the wall bB to meet XZ at Z.

Breadth of concrete.

The base XZ will be cut at some point f by the direction of the resultant R. Make $fX = fZ$, which will give an approximate width

to the base with a uniformly distributed pressure all over. If fX results in a very long projection beyond the toe of the wall, it will be desirable to limit it so that XK = HK, the prolongation of the face of the wall.

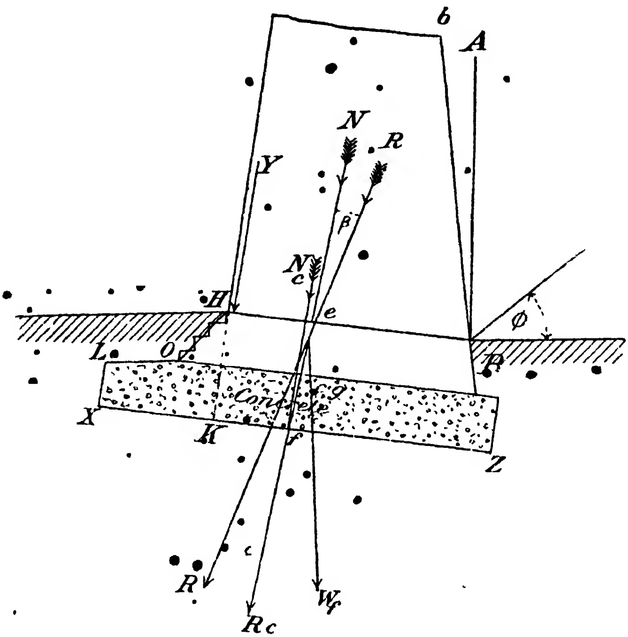


Fig. 215.

A thickness of concrete may be then taken, say 2 feet, and HO may be drawn representing the equalizing line for the offsets of the footings. We can then easily find the pressure on the foundations by (1) obtaining and compounding the centres of gravity of the areas of concrete and footings, calling the weight of the foundation work W_f, acting at a point marked c.g. (*Fig.* 215); (2), by compounding R with W_f we obtain R_c, the total resultant pressure on the foundation bed; (3), resolving normally, as before, we obtain N_c, the normal pressure which should cut the base approximately at the centre. The maximum stress intensity will then be $N_c \div$ base, and should be within the safe limit of the soil. Thickness

The projecting toe of concrete will be an inverted cantilever

uniformly loaded. It should not be too long, or the concrete will tend to crack along the line OO′ at O′.

Thickness of Base for a Wall with a given Surcharge.

Wall with definite surcharge.

In the investigation for earth thrust at the back of walls retaining a bank of earth, the slope of which is inclined to the horizontal, the surcharge has been taken as sufficiently great to admit of a line drawn from the inner base of the vertical wall at the natural slope of the earth meeting the sloping surface (BC, *Fig.* 212). The pressure will not be affected by any increase of surcharge beyond the limits of C, and therefore such a wall may be termed one having an indefinite surcharge. To meet the case, frequently occurring, of a wall having a given surcharge less than that indicated by C*d* (*Fig.* 216), such as C′*d*′, some modification in the value of the thrust is necessary. To find this out by any graphic method would be difficult, but the problem may be solved approximately by adopting a modification of the base thickness, as proposed by Professor Rankine, as follows:—

Let h = the height of the wall.

c = the height of the surcharge.

t = thickness at the base required for the wall when the bank has a horizontal top.

t' = thickness required at the base with an indefinite surcharge, and when the *maximum intensity of compression* is the same as when the top is horizontal.

t'' = the required thickness for the wall with the given definite surcharge.

Then

$$t'' = \frac{ht + 2ct'}{h + 2c}.$$

The application of this practically will be seen in an example.

Miscellaneous Questions relating to Retaining Walls.

Modification of section.

The design of the cross section of a retaining wall must frequently be modified from various circumstances of site, or loads likely to come on the earth above, and other causes.

Where, for instance, the plan of the wall is on a salient angle, the thickness should be increased, as the wall will evidently there be in the position of a wedge with a thrust from the point of the wedge. The tendency to fail will be increased if the wall has to

support a railway, as the centrifugal force of passing loads at high speed will exert an increased outward thrust.

On the contrary, if the plan of the wall is a re-entering angle, the section may be reduced.

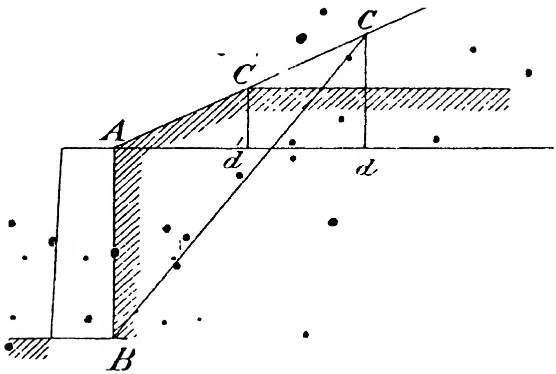


Fig. 216.

Where the earth is loaded with an external load, such as a Building on building (as in *Fig.* 217), the effect is to increase the value of the ground above

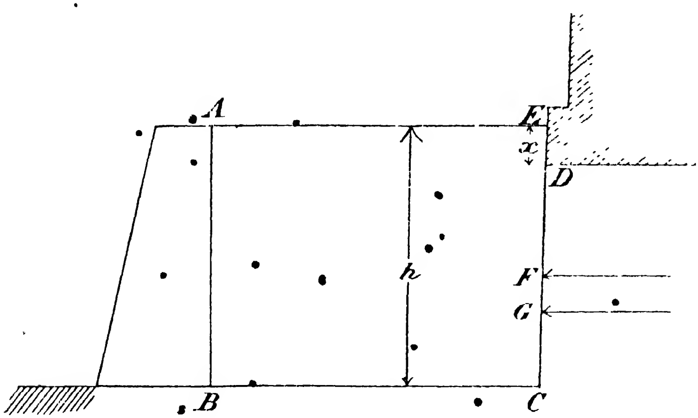


Fig. 217.

horizontal pressure P on the vertical plane at the end of the building beyond that which is due to the earth itself. The intensity of the vertical pressure at D, the edge of the building, is the weight of the building per unit of surface. Calling this unit weight w_1, the intensity of the horizontal pressure due to it

$$= w_1 \frac{1 - \sin \phi}{1 + \sin \phi},$$

which acts at every depth below the building on the vertical plane DC, its total amount being

$$(h - x) \frac{1 - \sin \phi}{1 + \sin \phi},$$

where x = the depth ED from the surface to the bottom of the foundation. In addition to this, there is the horizontal pressure due to the earth itself on plane DC, which (by Professor Rankine's formula) is

$$= w_1 \frac{(h - x)^2}{2} \frac{1 - \sin \phi}{1 + \sin \phi}.$$

This pressure has its centre at G, where $GC = \frac{1}{3}EC$, whereas the other pressure, due to the building alone, has its centre at F, where $FC = \frac{1}{2}DC$.

These two pressures make up the total pressure on the retaining wall when the front of the building is at A. If the building is at some distance behind A, the pressure is diminished to an extent, which is, however, uncertain.

Buttressed horizontal arches.

Buttressed horizontal arches (*Fig.* 218), connected in some cases above by means of arches, or below by inverts, are frequently used

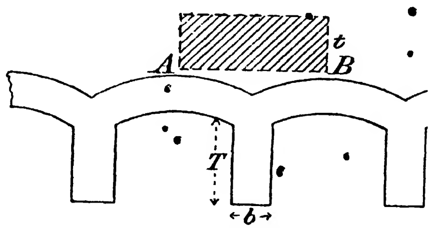


Fig. 218.

as retaining walls. Professor Rankine gives the following rules for the buttresses:—

If T = thickness of a buttress DE.

B = AB, the mass of earth against one buttress.

b = breadth of the buttress.

t = thickness which would be required for a rectangular wall to sustain the same bank of earth.

w = unit weight of both wall and buttress.

h = height of both wall and buttress.

Then, equating the Moments of Resistance of the buttress and the equivalent rectangular wall about their outer edges—

$$whTb\frac{T}{2} = whtB\frac{t}{2},$$

whence $$T = t\sqrt{\frac{B}{b}}, \text{ and } b = B\frac{t^2}{T^2}.$$

This takes no account of the weight of the arches between the buttresses.

As regards the construction details of retaining walls generally, see p. 322, Part I.

The practical application of the foregoing theoretical rules will be apparent from the following:—

EXAMPLE 17. Wall without surcharge.

EXAMPLE 17.—*Design a retaining wall 24' high, batter 1 in 5, material limestone weighing 156 lbs. per cubic foot. The soil to be retained is clay, weighing 120 lbs. per cubic foot, and with an angle of repose of 29°. The foundation is on sound yellow clay capable of bearing a safe load of 5 tons per square foot (p. 288, Part I.). The foundation bed to be of concrete weighing 140 lbs. per cubic foot, composed of 1 cement, 2 sand, 6 broken stone. Compare the value of the earth thrust found graphically with that found by Rankine's formula.*

It may be observed that the material of the earth to be retained in this example is by far the most troublesome of all the soils which an engineer may be called upon to revet. The swelling of clay when wet causes a thrust at the back of retaining walls, which is unquestionably very great, and the nature of which is very imperfectly understood. The most careful draining is, therefore, necessary, the dry rubble packing at the back of the wall, and the ample provision of weep holes, are, in the case of such soils, more than all others, absolutely indispensable. The angle of repose is comparatively small, only 29°, and hence, even with a comparatively heavy material, such as that proposed for this example, it is probable that the width of the base

will require to be more than $\frac{1}{3}$ height, usually taken as an approximation to economical requirements.

Trial value for base.

If we start by taking a base of $\frac{1}{3}$ height $= 8$ feet, and follow the graphic construction in *Fig.* 214, we shall find that the resultant pressure cuts the base outside the centre third.

Increasing the width of the base to 9·15, say 9′ 2″, and making further trials as in *Fig.* 2, *Plate* XX., the resultant pressure now passes just within the centre third. The calculated data necessary for construction are—

$$\text{Earth thrust } T = \tfrac{1}{2} w BP^2.$$

BP measures 15·2 feet, and w = unit weight of clay = 120 lbs. per cubic foot.

$$\therefore \qquad T = \tfrac{1}{2} \times 120 \times 15{\cdot}2^2 = 13862 \text{ lbs.} = 6{\cdot}18 \text{ tons.}$$

The weights per foot run of the masonry and wedge of earth are—

Masonry

$$= 150 \times \{\tfrac{1}{2} \times 2 \times 24 + \tfrac{1}{2} \times 21 \times 8{\cdot}8 + \tfrac{1}{2} 8{\cdot}8 \times 1{\cdot}8\}$$
$$= 20{,}628 \text{ lbs.} = 9{\cdot}21 \text{ tons.}$$

Earth wedge

$$= 120 \times \{\tfrac{1}{2} \times 2{\cdot}2 \times 28{\cdot}8\} = 3105{\cdot}6 \text{ lbs.} = 1{\cdot}52 \text{ tons.}$$

Hence total weight $W = 10{\cdot}73$ tons, acting at the common centre of gravity. (To find this, see p. 12, Part I.)

This weight must be compounded with the thrust T' at the back of the plane AB, acting along the plane of repose. Constructing the triangle of forces at $\frac{1}{3}$ of the height AB, where the base of the triangle $= T = 13862$ lbs., we find $T' = 15680$ lbs. $= 7{\cdot}0$ tons.

Compounding T' and W, we find the direction and value of $R = 34496$ lbs. $= 15{\cdot}4$ tons.

Of this pressure, the component N, normal to the base, is found by resolution to be 33,600 lbs. $= 15{\cdot}0$ tons.

The maximum intensity of pressure Y at the outer edge of the base will then be found from equation (a), p. 302, Part I.

$$Y = \frac{2N}{l}\left(2 - \frac{3d}{l}\right),$$

when Y = maximum pressure in tons per square foot.

N = normal pressure in tons = 15·0.

t = width of base = 9·15 feet.

d = distance of centre of pressure from outer edge, *i.e.*,

$$He = 3{\cdot}1 \text{ feet.}$$

$$\therefore \quad Y = \frac{2 \times 15}{9{\cdot}15}\left(2 - \frac{3 \times 3{\cdot}1}{9{\cdot}15}\right) = 3{\cdot}22 \text{ tons.}$$

As the crushing resistance of limestone is at least 175 tons per square foot (Table III., p. 28, Part I.), this leaves an ample margin of safety.

The angle between N and R is 12° 10′, which is less than $\frac{4}{5}$ of the angle of repose of masonry with wet mortar, *i.e.*, $\frac{4}{5}$ of $25\frac{1}{2}$° = 20° 24′. It is sufficient to consider the mortar damp in practice, *i.e.*, angle = 30°.

The section is therefore safe against overturning, crushing and sliding.

We may now proceed to the design of the foundations. Following the construction as described in the foregoing pages, we obtain loads as follows:—

Masonry in footings—

$$150 \times 2 \times 10{\cdot}45 = 3{,}135 \text{ lbs.} = 1{\cdot}44 \text{ tons.}$$

Concrete

$$= 140 \times 2 \times 16 = 4{,}480 \text{ lbs.} = 2{\cdot}0 \text{ tons.}$$

Total weight of foundations

$$\therefore \quad 3{\cdot}44 \text{ tons.}$$

Find the centres of gravity and lay off the values W_1 and R.

Compounding as before, we obtain the value of R_c, which measures 18·45 tons, and cuts the base at 9 feet from the toe of the concrete.

Two feet of this can be cut off as unnecessary for structural reasons, and so bring the resultant to the centre.

The normal component N_c = 18·34 tons, and the maximum of pressure will = $N_c \div$ base, since the load is equally distributed all over.

Hence $Y = \frac{18{\cdot}34}{14} = 1{\cdot}31$ tons, which is well within the limit (5 tons) which the soil can safely bear.

To compare Rankine's thrust with that found graphically, we

have seen that the latter $(T) = \frac{1}{2}wBP^2 = 13,862$ lbs. By Rankine's formula—

$$P = \frac{1}{2}wh^2 \frac{1 - \sin\phi}{1 + \sin\phi},$$

when w = weight of earth per unit of volume.
h = height of vertical plane under pressure.
ϕ = angle of repose of soil and $\sin\phi = \cdot 48481$.

Whence

$$P = 60 \times 25{\cdot}8^2 \times \frac{1 - \cdot 4848}{1 + \cdot 4848} = 13,858 \text{ lbs}$$

which is practically the same as T found above.

EXAMPLE 18. Surcharged wall.

EXAMPLE 18.—*Design a retaining wall under the same conditions as in the previous example, but the top of the bank sloping at $\frac{1}{4}$, and the height of the surcharge 6 feet.*

This example is given to illustrate the difference in working for walls with surcharge from those with the earth surface at the top horizontal.

The essential point about the design is that a trial section should be obtained, worked out on the principle shown above (Case 3, p. 258), where the maximum intensity of pressure Y should be the same as the value of Y found for a wall with horizontal top, as in the last example.

In this case it will be necessary to increase the base beyond the 9′ 2″ found in the previous example, and it is manifest also that, if the new value of Y is to be equal to the old one, the position of the resultant pressure must be nearer to the centre in this case than in the former one.

Take a base 11·5 feet wide, and work out the graphic construction as described on p. 259. The construction is shown on *Fig.* 1, *Plate* XX.

Then

$$T = \frac{1}{2}wBP^2 = \frac{1}{2}120 \times 17{\cdot}7^2 = 8{\cdot}4 \text{ tons},$$

and by construction—

$$T_1 = 8{\cdot}4 \text{ tons.}$$

Then, working as before—

$$W = 11{\cdot}5 \text{ tons}, R = 20{\cdot}8 \text{ tons, and } N = 20{\cdot}42 \text{ tons.}$$

The resultant cuts the base 4·42 feet from H. Hence Equation (a) becomes

$$Y = \frac{2 \times 20{\cdot}42}{11{\cdot}5}\left(2 - \frac{3 \times 4{\cdot}42}{11{\cdot}5}\right) = 3{\cdot}01 \text{ tons per square foot.}$$

The former value, as found in the previous example, was 3·22 tons. Hence there is a difference of 0·21 tons. The true value of the base, in order to make the pressure equal, will be slightly less than 11·5 feet. Call it 11′ 3″.

The proper value of the base with a surcharge of 6 feet is therefore found from Rankine's formula

$$t'' = \frac{ht + 2ct'}{h + 2c},$$

where t'' = the required thickness.

h = height of wall = 24 feet.

t = thickness without surcharge = 9·15.

t' = thickness with indefinite surcharge = 11·25.

c = surcharge = 6.

Substituting values

$$t'' = \frac{24 \times 9{\cdot}15 + 2 \times 6 \times 11{\cdot}25}{24 + 12} = 9{\cdot}8, \text{ say } 10 \text{ feet.}$$

The foundations would be worked out in exactly the same way as in the former example, and need not be again described.

APPENDIX.

THE method of finding the position of the plane of rupture when the earth pressure is a maximum is as follows:— Position of plane of rupture when the earth thrust is a maximum.

Let *Fig.* 219 represent a retaining wall with the earth above sloping at any angle less than the angle of repose. Let ϕ be the angle of repose, and let θ be the unknown angle which the plane of rupture makes with the natural slope of the earth BD.

Then, considering the wedge of prism BAC, the forces acting on it are

(1). The weight W of the prism acting at its c.g.

(2). The reaction R of the plane AB acting at ⅔ AB from the surface A. Here the friction along the plane AB is neglected, but if we can ascertain the value of the thrust, ignoring friction, we can easily find out subsequently the value of the actual thrust when friction is taken account of.

(3). R′, the minimum reaction of the plane BC, when the earth is on the point of sliding, which, owing to friction, makes an angle with and above the normal N = ϕ.

Draw FE through the c.g. of the triangle ABC, and equal to W on any

scale of weights. Draw FH parallel to R and EH parallel to R′. Then the three forces acting on the prism ABC are as the sides of the triangle ABC. It is required to find the position of the plane CB so that HF shall be a maximum.

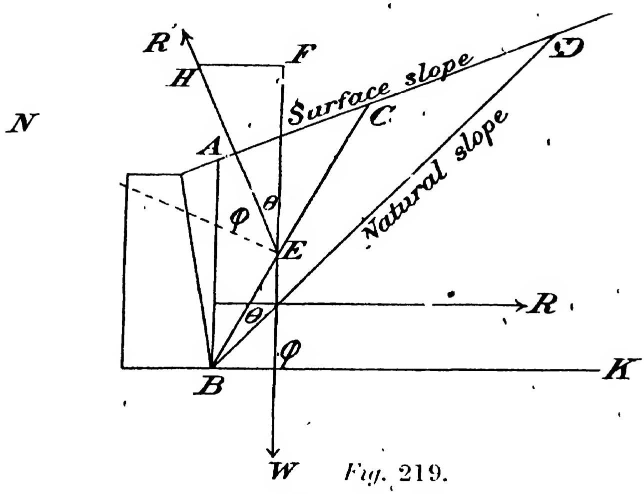


Fig. 219.

We see that in the triangle HEF the angle HEF = CBD = θ, because NEC = 90° − EBK + BEW. But FEC = BEW, therefore FEN = EBK = $\theta + \phi$, and as NEH = ϕ, HEF = θ. Hence HF = FE tan θ = W tan θ, where W is the weight of the prism ABC one unit thick. Hence the maximum horizontal thrust = area of triangle ABC × tan θ.

Now in *Fig.* 220 draw the triangles ABC, ABD, as in *Fig.* 208, and from A

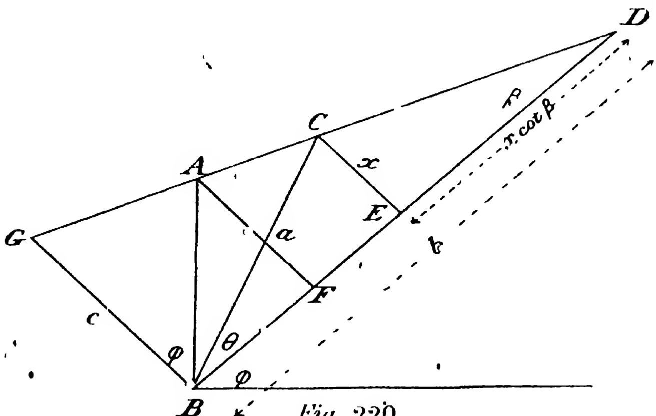


Fig. 220.

and C let fall AF, CE perpendicular to BD; also draw GB perpendicular to BD, meeting at G the surface slope DA produced.

Let GB $= c$, BD $= b$, AF $= a$ and CE $= x$, the last varying with the angle θ. Let the angle CDB $= \beta$.

Then the horizontal thrust on AB $= w \times$ area ABC $\times \tan\theta$, which

$$= w\left(\tfrac{1}{2}ab - \tfrac{1}{2}xb\right)\tan\theta = \tfrac{1}{2}wb \times \frac{ax - x^2}{b - x\cot\beta},$$

where w = weight of earth per unit of volume.

Differentiating for a maximum—

$$(b - x\cot\beta)(a - 2x) + (ax - x^2)\cot\beta = 0,$$

or

$$\left.\begin{array}{l} ab - ax\cot\beta - 2bx + 2x^2\cot\beta \\ \quad + ax\cot\beta \qquad - x^2\cot\beta \end{array}\right\} = 0.$$

$$x^2\cot\beta - 2bx = ab \qquad \ldots\ldots(\alpha),$$

which is an ordinary quadratic equation for finding x. It may be written

$$ab - bx = x(b - x\cot\beta) = x \times \text{BE}.$$

But $ab - bx$ is the triangle ABC, and $x \times$ BE is the triangle CEB. Hence the earth thrust is greater when the area ABC = CEB.

Hence $\qquad$ HF $= w \times$ area ABC $\tan\theta$

$$= w \times \text{area CBE} \tan\theta$$

$$= w \times \frac{x \times \text{BE}}{2} \times \frac{x}{\text{BE}} = \tfrac{1}{2}wx^2 \qquad (\beta).$$

From Equation (α) above, solving for x—

$$x = b\tan\beta - \sqrt{b\tan\beta\,(b\tan\beta - a)},$$

and substituting values of $\tan\beta$ from *Fig.* 214—

$$x = c - \sqrt{c(c - a)} \qquad (\gamma).$$

maximum horizontal thrust

$$= \tfrac{1}{2}w\{c - \sqrt{c(c - a)}\}^2,$$

for which general expression graphic values are given in the text.

When the surface is horizontal, the angle ABD is bisected by CB, and HF $=$ EF $\tan\theta =$ weight of prism ABC $\times \tan\theta = \tfrac{1}{2}wh^2\tan\theta = \tfrac{1}{2}wh^2\tan^2\tfrac{1}{2}(90 - \phi)$, since ABD ($= 2\phi$) is ($90 - \phi$). This is the same as Rankine's formula, p. 256. Hence, as in Example 14 in the text, the horizontal thrust found by the graphic method should agree with that calculated by Rankine's method.

CHAPTER XII.

RESERVOIR DAMS.

Practical Points in Construction.—History of the Theory of Design.—Summary of the Main Principles of Design.—The Pressure of Water at Rest.—General Wray's Method.—Sir Guilford Molesworth's Formulæ.—Messrs. Tudsbery Turner and Brightmore's Theories.—Arched Dams.—Abutment Walls of Service Reservoirs.—Appendix.

IT might be supposed that the design of a masonry dam presented less difficulty than that of a retaining wall for earth, inasmuch as the elements of uncertainty, which, as we saw in the previous chapter, enter into the accurate design of the latter, are no longer present when we deal with water.

Difference in height between retaining wall and dam

On the other hand, the height of retaining walls is seldom more than 10 feet, whereas reservoir dams of four times that height have been built. The Periyar Dam in Travancore, 173 feet high, built by Colonel Pennycuick, C.S.I., R.E., is the highest as yet built, the Chartrain Dam in France being nearly as high; but there are some of a greater height now under construction in America, notably the Quaker Bridge Dam for New York, which is 240 feet high.

However well, practically, approximate calculations may do in the case of comparatively low retaining walls, the greater height and weight of dams require more exact treatment. The disaster which attends the failure of these structures necessitates the most careful consideration of the conditions of their stability.

Practical points in construction necessary in order to justify the

In these pages the assumption usually made in treatises on this subject will be adhered to, viz., that the masonry is a rigid homogeneous mass resting on an elastic foundation. It follows, therefore, that in actual construction the utmost care should be taken to ensure homogeneity throughout; the masonry should be of uncoursed rubble or concrete, with no horizontal joints; there should be no voids or hollows left in any part, every stone should be firmly bedded, and a thorough incorporation of the courses (if concrete is used) must be arranged. The mortar should be of hydraulic lime or cement, and

Masonry

Concrete

Mortar

scrupulous care should be taken to have it free from all dirty particles, hence the water used must be perfectly clean. Hydraulic lime rather than Portland cement is now advocated by some eminent engineers, because it sets more slowly, and is, therefore, less liable to be weakened by the inevitable settlement which must take place during the progress of the work. In the Periyar Dam the mortar was hydraulic lime, locally procured, and *surkhi* (crushed brick); concrete made of this gave most excellent results in point of strength. At the Neuadd Reservoir in Wales now (1897) under construction, the mortar used is Aberthaw lime and crushed rock. This lime is very carefully ground. The engineer of this dam is Mr. G. F. Deacon, M.Inst.C.E., whose experiments on hydraulic lime have been most exhaustive.*

The foundation of a dam must be taken to a solid rock bed, and care should be taken that no water gets below the foundation, either by leakage from the reservoir, or by springs from below the dam. If water thus gets below the structure, it will seriously affect its stability by causing "uplift." At the Vyrnwy Dam in N. Wales there are elaborate arrangements for draining off water from springs below the actual foundation bed, which are well worth studying (see *Minutes of the Proceedings of the Institution of Civil Engineers*, Vol. CXXVI., pp. 29, 30). Foundation springs

History of the Theory of Design

Before proceeding to discuss the principles of design involved in masonry dams, it may be of use to give a brief narrative sketch of the development of the theories on the subject.

Up to the middle of the present century, masonry dams of which there were a good many in existence, especially in Spain, had chiefly been built with little pretensions to scientific adaptation of means to end. Most of these were huge blocks of masonry, much of which was wholly unnecessary, and some of it was in danger of being crushed by the mass above. In 1853, however, French engineers (to whose labours about that period, in connection with a different branch of hydraulics, viz., the flow of water in pipes, the world at large is so much indebted) began to investigate the true principles of masonry dams, and to demonstrate that in regard to these structures certain definite principles should be followed. Those Earliest French writers

* *Vide* Appendix IV., *The Water Supply of Barracks and Cantonments*, by the author.

Principles adopted by French engineers.

principles were :—(1). That the pressures sustained by the masonry or its foundations must not exceed a safe limit. (2). That there must be no possibility of any part of the masonry sliding on that below, or on the foundation. It was also laid down that these conditions should be fulfilled when the reservoir was full, and when it was empty. As it was considered that there would be little likelihood of failure by sliding, the earliest writers on the subject, MM. Sazilly and Delocre, were of opinion that investigation might be limited to the consideration of the maxima pressures at various horizontal joints when the reservoir was full. On these

Furens dam.

lines several dams were designed in France, notably one at Furens* in the valley of the Loire, 50 mètres high, which fairly startled the engineering world by its boldness.

Professor Rankine's views.

The next writer on the subject was Professor Rankine, who, while accepting the conclusions of the French engineers, pointed out that some difference should be assigned to the maxima permissible stresses, as calculated by ordinary methods, in front and in rear of the section of a dam. The former writers had treated the subject in much the same way as enclosure walls, buttresses, etc., are treated in Part I. of this treatise, where the wall is assumed to be subject to the same rules as a beam, and therefore the component of pressure, which is normal to the pressed surface, varies uniformly. This assumption, according to Professor Rankine, is "probably very near the truth in walls of uniform, or nearly uniform, thickness; whether, or to what extent, it deviates from exactness in walls of varying thickness is uncertain in the present state of our experimental knowledge." He considered that the limit adopted for the permissible intensity of pressure should be lower at the outer than at the inner face of the dam, because the direction of the pressure exerted among the particles of a joint in the masonry close to either face is necessarily a tangent to that face, and unless the face is vertical, the pressure found by means of the ordinary formulæ is not the total pressure, but only its normal component.

No tension at any part.

Professor Rankine introduced another and most important element of security, viz., that it was most desirable that there should be *no tension at any joint* at any section, or in other words, that the line of resistance should everywhere, and under all circumstances lie within the centre third of the joint. He proposed a form of dam of which

* The details of this dam, and the calculations connected with it, are given in Spon's *Dictionary of Engineering*, article, "Damming."

the inner and outer faces were logarithmic curves. He fixed the limit of pressure intensity at 20,000 lbs. per square foot of horizontal surface.

In 1872 Colonel (now Major-General) Wray, C.M.G., R.E., following Professor Rankine's principles, devoted attention to the subject in his *Some Applications of Theory to the Practice of Construction.* He considered that Professor Rankine's limit should be modified at the outer face to 20,000 lbs. $\times \frac{1}{2}$ $(1 + \cos\theta)$, θ being the angle which the outer face makes with the vertical. If T (*Fig.* 221) be the pressure tangential to the face, and V = AD = vertical component, he proposed that T should be taken between CA and AE, *i.e.*, V sec θ and V cos θ. Working out examples of reservoir walls 200 feet high, under certain conditions of weight of masonry, he demonstrated that Professor Rankine's typical curved section demanded modification to suit various classes of masonry. General Wray's investigation

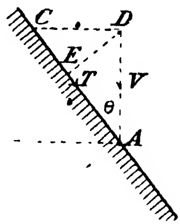


Fig. 221.

The fact which Professor Rankine had been the first to notice, viz., that the maxima stresses permissible should vary for the up-stream and down-stream faces, had meantime attracted attention in France, and in 1875 M. Bouvier, in the *Annales des Ponts et Chaussées*, published a paper on the subject embodying researches which he had made. He pointed out that if ABCD represent (*Fig.* 222) the section of a dam, the reservoir being full, and *mn* be an imaginary horizontal joint, the real expression of the pressure is not found by taking the normal component of the resultant pressure acting at *o* (the centre of pressure), but by considering a joint *mn'*, M. Bouvier's theory.

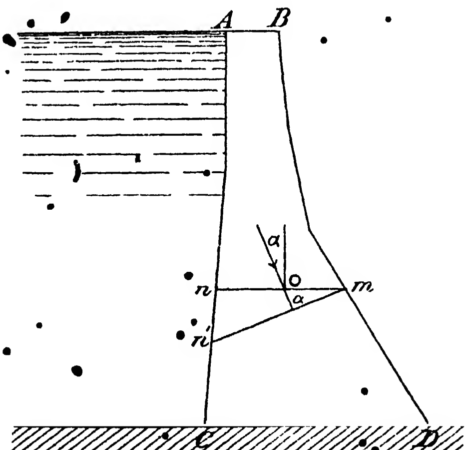


Fig. 222.

which forms with mn an angle, α = the angle between the resultant R and its normal component. Then the maximum intensity of compression becomes

$$Y = \frac{2R}{t \cos \alpha}\left(2 - \frac{3d}{t}\right)$$

for all cases where d is greater than $\frac{t}{3}$.

Where d is less than $\frac{t}{3}$.

$$Y = \frac{}{3d \cos \alpha}.$$

These pressures will be considerably greater (in the proportion of 1 to $\sin^2 \alpha$) than those calculated on the hypothesis formerly held, where (as shown on p. 302, Part I.)

$$Y = \frac{2N}{}\left(\quad \frac{3d}{t}\right)$$

N being the normal component of R.

Conditions adopted by French engineers generally.

In general it may be stated that the French school of writers on this subject devoted their attention to the form of dam in which the compressive stresses should nowhere exceed certain limits, whether the reservoir were full or empty.

Sir G. Molesworth's formulæ

In 1874 Sir Guilford Molesworth framed some empirical formulæ for connecting the pressure, the depth below the surface of the water, and ordinates from a vertical line to the up-stream face and down-stream face respectively. These formulæ were modified by him after the publication of M. Bouvier's researches, and were published in a paper printed at Roorkee in 1883. The formulæ are as follows :—

If x = depth below the surface of a horizontal plane in feet.

y = the ordinate from a vertical line on that plane to the outer edge of the down-stream face.

z = the ordinate from the same vertical line to the edge of the up-stream face in feet.

P = limit of pressure permissible in tons per square foot.

Then $$y = \sqrt{\frac{\cdot 05x^2}{P + \cdot 03x}} = 0\cdot 6x \text{ as a maximum,}$$

$$\left(\frac{\cdot 09x}{P}\right)^4$$

In these formula, variations for the specific gravity of the masonry are not included. Sir G. Molesworth points out that this does not materially affect the profile of the dam, because, though heavier masonry generally involves increased pressure on the base of the dam, yet the resultant pressure in such a case cuts any joint at a point nearer to the centre than would be the case with a light masonry; hence the maximum intensity of pressure is not increased in proportion to the heavier weight of masonry. In the paper above alluded to, examples are given, worked out in detail, for a dam 180 feet high, with masonry weighing 145·6 lbs. per foot cube (see *Fig.* 3, *Plate* XXI.), and there are also several other sections of existing dams, with the section that would have resulted, had these formulæ been used. Specific gravity of masonry does not affect formulæ.

Before the publication, however, of the modified formulæ just mentioned, Major (now Colonel) Pennycuick, R.E., in a report on the Periyar Project, published at Bangalore in 1882, had brought forward a new view of the case. He maintained that in order to obtain the maximum pressure at a given point such as E (*Fig.* 223) on the outer face of a dam it is not sound to consider a *horizontal* joint through that point, as advocated by the earlier writers on the subject; nor did he accept M. Bouvier's theory, maintaining that there seemed to be no reason for transferring the effect on any horizontal joint to some other *imaginary* joint. He considered that the greatest intensity per square unit at a given point such as E Colonel Pennycuick's investigations.

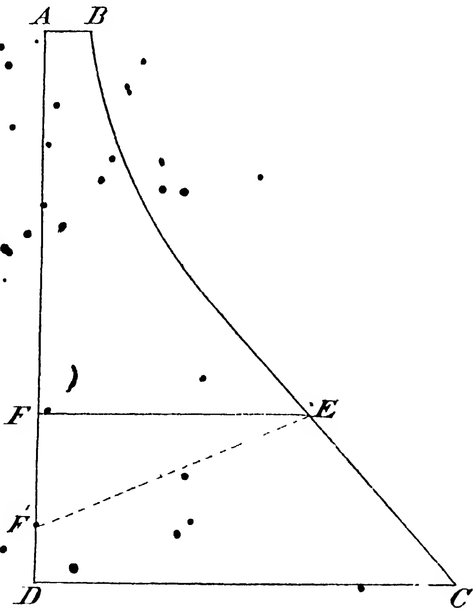


Fig. 223.

would not be found by considering a horizontal joint as EF, but some *inclined* joint, such as EF′, the portion ABEF′ being subject to greater stress, obviously than ABEF. The position of FF′, which gives the maximum pressure at E, is approximately $\frac{EF^2}{h}$, where h is the depth of water above EF. "Neither the old method nor M. Bouvier's take any account of the portion of the dam below the line EF, and the pressure at E is the same, whether the total height be AF or whether it be infinite." "If the depth FD be less than $\frac{EF^2}{h}$, the maximum pressure which would occur in a dam of infinite height can never be reached, and the greatest pressure at E can only be that due to the portion of the dam ABED."

Colonel Pennycuick's formula.

Following out this principle, Major Pennycuick evolved a formula, which had been found to give results so nearly accurate that it might be adopted without risk of error, where x and y having the same values as in Sir G. Molesworth's—

$$y = 7{\cdot}67\sqrt{\frac{x\{x^2 + 7(x - d)\}}{p}},$$

where p = limit of pressure in lbs. per square foot.

General Mullins' formula

Lieut.-General Mullins, R.E., has proposed* another general formula—

$$y = b + \sqrt{\frac{0{\cdot}05\,(x - b)^3}{P + 0{\cdot}05\,(x - b)}},$$

where b = breadth in feet of top of the dam, the remaining symbols being the same as in Sir G. Molesworth's formula. For dams whose top is less than 12 feet wide the formula is somewhat modified, and becomes

$$y = b + (12 - b) + \sqrt{\frac{0{\cdot}05\,(x - 12)^3}{P + 0{\cdot}05\,(x - 12)}}.$$

Mr. Wegmann.

In 1893 Mr. Wegmann, in America, published a monograph on masonry dams, in which he gives suggestions for modifications of Rankine's profiles, and proposes various "practical profiles" for various cases.

Messrs. Turner and Brightmore.

In the same year Messrs. Tudsbery Turner and Brightmore published their admirable treatise on *The Principles of Waterworks*

* *Minutes of Proceedings, Institution of Civil Engineers*, Vol. CXV., p. 173.

Engineering. Taking into account all that had been previously published on the subject (except Sir G. Molesworth's revised formula, and Major Pennycuick's views on the matter), they elaborated a new method of determining, by strictly rational processes, an economical section that was in strict accordance with the conditions of strength and stability. This process will be described later.

In 1893, also, Professor Kreuter, of Munich, read a paper at the Institution of Civil Engineers in which he proposed a section where the water pressure on the up-stream face is, after the dam has reached a certain height, utilized to assist the stability, and thereby to introduce economy into the section. This is done by increasing the ordinates of that face to form a considerable curve up-stream (see *Fig.* 224). This section, however ingenious from a theoretical Professor Kreuter.

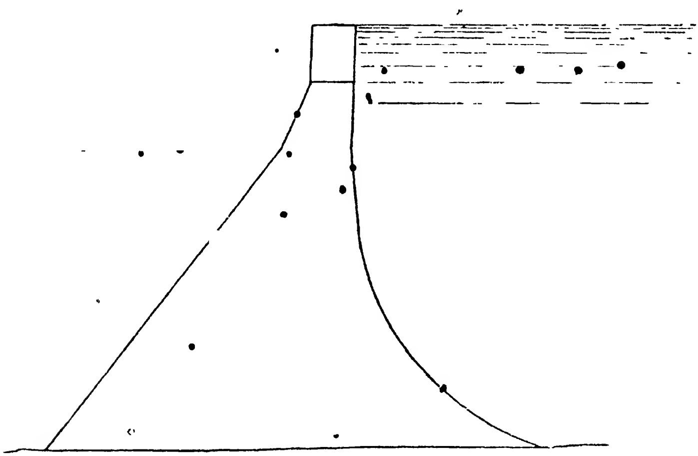


Fig. 224.

point of view, did not meet with approval from practical men, because it not only neglects the effects of bending moments, but assumes that the masonry is, and always will remain, water-tight, a condition of things impossible to realize in practice.

In April, 1895, the dam at Bouzey, near Epinal, France, burst, affording an object lesson to engineers of all countries, which they were not slow to take advantage of. The fact that this dam had Failure of dam at Bouzey.

been designed of such a section that at certain parts there was a definite amount of tension at the up-stream side is very noteworthy. This was one of the points overlooked by the engineers of the French school, though insisted on by Rankine.

Professor Unwin's views.

As a natural consequence of this failure much attention has recently been paid to the subject, and many articles, etc., have been written about it. Of these, the most important are the *Report of the Special Commission of the "Ponts et Chaussées,"* and an article by Professor Unwin in *Cassier's Magazine* for November, 1896.* In the latter a fresh danger attending the existence of tension on the up-stream side of a dam has been pointed out. If the resultant pressure, as in *Fig.* 225, lies outside the centre third, the stress at *b* is tensile. Now we know that the adhesion of mortar to stone is not great, even though the mortar itself be of good quality; hence a slight amount of tensile stress is likely to produce cracks. When once a crack is formed, as at *g* (*Fig.* 226), the water pressure inside

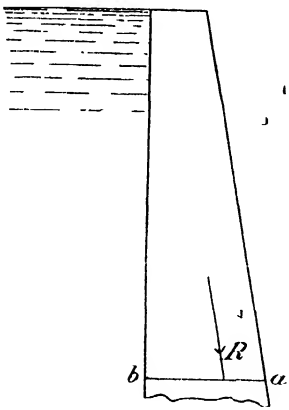


Fig. 225.

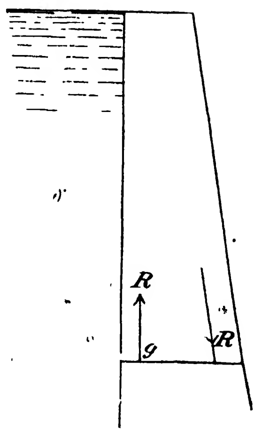


Fig. 226.

the fissure, acting upwards, reduces the weight of the rock above, causes the resultant to move nearer the down-stream edge, and consequently increases the tension at *g*. Thus the fissure increases,

* And in *Minutes of Proceedings, Institution of Civil Engineers,* Vol. CCXXVI., p. 94.

and the process goes on until the dam is overturned. The effect of the tension is, therefore, progressive, and this may account for the failure of a dam such as that at Bouzey, which had stood sufficiently well for years. There is, therefore, this peculiar danger of tension in a masonry dam, in that, if a fissure does open, a new and *continually increasing* straining force comes into play at the weakest point.

There is yet another danger from water percolating into a dam. As long as the masonry is water-tight, the resistance of the dam to sliding will be far greater than the shearing force produced by the water pressure: but when a thin film of water intervenes, the friction of the mass above is very materially reduced. (This is the well-known principle of the *chemin de fer glissant*). It follows, therefore, that if there be a longitudinal fissure extending for some distance into the masonry the effective area of resistance to shearing will be much reduced, and thus a force, which under water-tight conditions is not worth considering, becomes, under the altered conditions, of serious importance. When the masonry is built of horizontal courses the tendency to shear will become all the greater. Danger of sliding through percolation.

In 1896 the Periyar Dam was finished, and an account of its construction read in a paper by Colonel Pennycuick at the Institution of C.E. in January, 1897. The dam had been severely tested by floods shortly after completion, and had shown no signs of weakness. Designed on the principles above indicated in the Report of 1882 (though slightly modified), its section is shown on *Plate* XXI., where, for purposes of comparison, sections to the same scale of a dam worked out on Messrs. Turner and Brightmore's principle, and another worked out by Sir G. Molesworth's formulæ, are shown on either side. In the discussion which followed Colonel Pennycuick's paper, it was described by one of the members (Mr. Farren) as "the only rational dam in the whole of the British Empire." It is interesting to note that the cost is extremely moderate; it can impound 6,800 million cubic feet of water at a cost of less than £14 per million cubic feet, or about £2¼ per million gallons. The cost of the Quaker Bridge Dam is estimated at £26 per million gallons. Periyar Dam

Principles of Design.

The following are, therefore, the main principles in the design of masonry dams:— Principles of design of gravity section in a masonry dam.

1. The compressive stress on any unit shall not exceed in intensity the safe limit of pressure which the masonry is capable of bearing.

2. There shall be no tensile stress at any joint, least of all on the up-stream face.

3. The resistance to shearing at any horizontal plane shall be greater than the horizontal thrust of the water at the level of that plane.

4. The thickness at the top must be sufficient to withstand the impact of waves or the thrust of ice.

5. The foundation must be so secured, and the superstructure so built, that in no part of the structure is there danger of percolation.

The last condition is never perfectly fulfilled, but by careful use of materials, and careful supervision in construction, any sensible amount of percolation may be prevented. It is *by far* the most important of all considerations in the matter.

The problem of the design of a masonry dam to fulfil all the above conditions, with the most economical disposition of the material, is difficult and complex.

Before considering the solution of the problem, it may be as well to recapitulate a few axioms, probably already well known to all readers of this work, regarding

The Pressure of Water at Rest.

This pressure is—

(1). Directly proportional to the area of the plane or surface on which it presses, and normal to that surface.

(2). Directly proportional to the depth of that surface below the surface of the water.

(3). Equal in intensity in all directions.

Intensity of pressure of water.

Let ABCD be a small tank with one vertical and one sloping side (*Fig.* 227), and let EFGH represent any layer of unit breadth at a

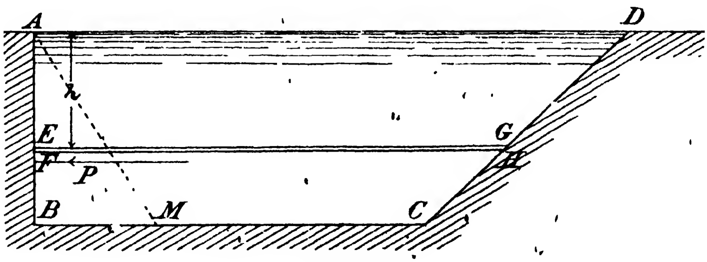


Fig. 227.

mean depth $= h$, then the pressure at EF and at GH will be the same, being equal to the weight of a prism of water of mean height h, and of unit area. The *direction* of this pressure will be normal to the surface in both cases. The *magnitude* of this pressure will be unit area × unit weight of water × depth $h = wh$.

Units of measure and weight of water.

In this country the units usually taken are:—For depth, feet; for weight, lbs., the weight of water being taken at 62·5 lbs. per foot cube.

Graphic representation of pressures.

As the pressure is directly proportional to the depth, we may represent graphically the pressure on the vertical wall AB by a triangle ABM where BM represents the pressure at $B = w \times h$, and the total pressure on the wall $= wh \times \frac{1}{2}h = \frac{1}{2}wh^2$, which acts at the c.g. of the triangle, *i.e.*, at $\frac{2}{3}$ the distance from the surface (*Fig.* 227). *Fig.* 228 shows the graphic representation of the pressure on the sloping wall.

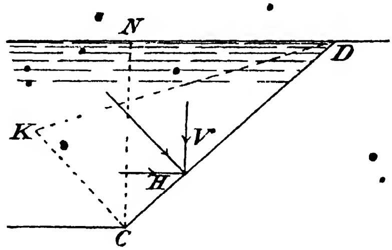


Fig. 228.

Vertical and horizontal components of pressure.

If we consider the vertical and horizontal components of the latter, the horizontal component will be the same as the pressure on AB, and the vertical component will be the weight of a prism of water of unit length, and of cross section = the triangle CDN acting through its centre of gravity.

Assumptions necessary in theoretical investigation.

There is, therefore, no difficulty in finding the pressure produced by water on a face, vertical or sloping, of a masonry dam, assumed to be impervious. The main difficulty of the problem lies in the arrangement of the material in the dam so as to meet the conditions of stability mentioned above. The hypotheses assumed, viz.:—That (1) the dam is rigid and homogeneous, and (2) the construction such that the stresses transmitted are either uniform in intensity, or are uniformly transmitted, are only partially true, but, since in our present state of knowledge we cannot exactly say how far they are erroneous, we assume for our purposes that they are wholly true.

General Wray's Method.

General Wray's method.

The following method of ascertaining the section, so as to give stability at various levels, is given by General Wray:—

Some thickness at top necessary.

The actual section adopted in any given case depends to some extent on the thickness required for the top. It would be possible

(if we neglect the thickness required to resist the impact of waves, etc.), to give a feather edge at the top, but such a section would practically be objectionable, even in the case where there is no chance of any ice to induce a thrust. In most cases the dam is made wide enough to admit either of the surplus water of the reservoir flowing very gently over it, or else it is wide enough to admit of the full width of a road crossing the valley. Then, taking this definite width, and working downwards from the top, there is (1) a certain depth at which, by maintaining the same thickness of wall as at the top, the conditions of stability for reservoir full is just fulfilled, and for reservoir empty is more than fulfilled (*Fig.* 229).

Sections of stability at various levels.

First part, rectar

(2). Below this section there is another (shown in *Fig.* 230), where, by giving a batter to the down-stream face, and leaving the up-stream face vertical, the lines of resistance always lie within the centre third of the joint.

Second part, outer batter.

(3) Below this section there is a third, where it is necessary to give a slight batter to the up-stream face, so as to prevent the line of resistance falling too near the edge on the up stream side when the reservoir is empty (*Fig.* 231).

Third part batter on both faces.

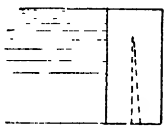


Fig. 229.

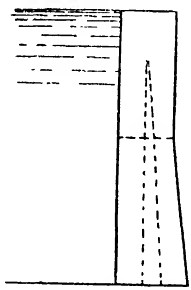


Fig. 230.

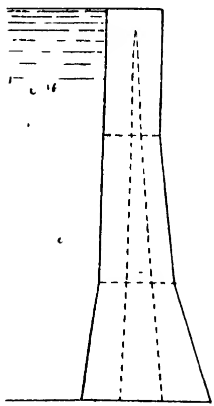


Fig. 231.

(4). Below this it is necessary to spread out the width (*Fig.* 232), not in order to keep the lines of resistance within the centre third, but to keep the pressure intensity within the prescribed limits for the material.

Fourth part, breadth increased.

The design of the first three parts can be obtained by algebraic expressions :- Calculation of First part.

First Part.—

Let w = weight of water per cubic foot = 62·5 lbs.

w' = weight of masonry per cubic foot. If p be the specific gravity then $w' = pw$.

a = depth from top of wall to surface of water.

b = uniform breadth of the wall.

h = required height to satisfy conditions of resultant falling within centre third when the reservoir is full (*Fig.* 233).

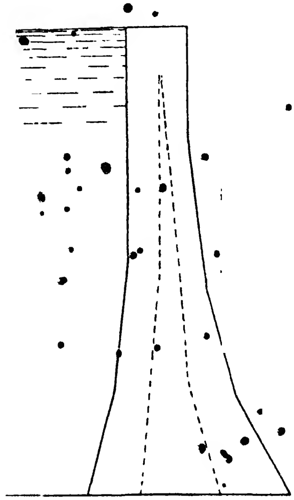


Fig. 232.

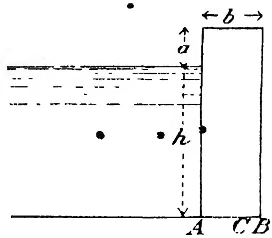


Fig. 233.

Then taking moments about C, the outer edge of the centre third, moment of water pressure

$$= \frac{wh^3}{6} = w'(h+a)\,b\,\frac{b}{6} \qquad (1),$$

whence h is obtained.

If $a = 0$, and we call $w' = pw$, then Formula for height.

$$\frac{wh^3}{6} = p\,\frac{w'hb^2}{6},$$

whence

$$h = b\sqrt{p},$$

a very easily applied expression.

Second Part (*Fig.* 234).—Take any layer m feet high of width x. Then taking moments about the outer point of the centre third of DE, the moment of water pressure = rectangle KL × distance of c.g. from the centre of Second part with outer batter

pressure, + rectangle DK × distance of its c.g. from the centre of pressure + triangular portion × distance of its centre of gravity from the same point, *i.e.*—

Formula width.

$$\frac{w(h+m)^3}{6} = W\left(\tfrac{2}{3}x - c\right) + w'\{bm\left(\tfrac{2}{3}x - \tfrac{1}{2}b\right) + (x-b)\tfrac{1}{2}m\left[\tfrac{2}{3}x - b - \tfrac{1}{3}(x-b)\right]\}$$

$$= W\left(\tfrac{2}{3}x - c\right) + \frac{w'm\,(x^2 + bx - b^2)}{6} \quad \ldots\ldots\ldots(2),$$

from which the value of x at the depth $a+h+m$ is ascertainable. In this W = portion already designed in the first part, KH.

To find the limit beyond which m must not be taken, the c.g. of the wall above the lowest extremity of m must be obtained, and a vertical line drawn through it. If this line falls outside the centre third, a fresh value of m must be taken.

This method is therefore one of trial and error. After ascertaining the greatest permissible value of m, the maximum intensity of pressure must be determined by M. Bouvier's rules. If this pressure is more than the material can safely stand the base must be increased and the operation repeated.

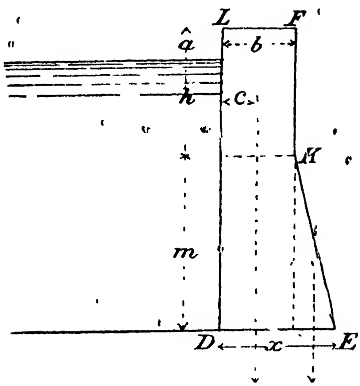


Fig. 234.

Third and Fourth sections.

Third and Fourth Sections.—In these sections the values of the breadth at successive depths must be found somewhat on the same principles as the foregoing. Trial and error must be applied, and it is considered that by a few trials the proper dimensions can be arrived at. The vertical components of the water pressure on the internal batter may be taken into account in ascertaining the stability when the reservoir is full.

All the dimensions which have been calculated may then be plotted, and the stability of the wall checked by graphic statics, as in the Example of a retaining wall in the previous Chapter.

Sir Guilford Molesworth's Method.

Example of Sir G. Molesworth's formula applied to a dam 180 feet high.

Applying the formulæ arrived at by Sir G. Molesworth, given above, to the case of a dam 180 feet high (divided into imaginary planes, four in number, as shown on *Fig.* 3, *Plate* XXI.), built of masonry weighing 145·6 lbs. per square foot, and with a permissible limit of stress = 20,000 lbs. per square foot, the following are the values of the various symbols, the width at the top being = 0·4y, at a = 10·8 feet:—

Plane.	A	B	C	D
x (depth of plane below surface)	45	90	135	180
y (ordinate from vertical line to down-stream face) $= \sqrt{\dfrac{\cdot 05x^3}{P + \cdot 03x}}$	21·10	56	97·4	142·6
y as minimum $= 0{\cdot}6x$	27	54	81	108
y as maximum	27	56	97·4	142·6
z (ordinate from vertical to up-stream face) $= \left(\dfrac{\cdot 09x}{P}\right)^4$	0	0	3·44	10·8

This gives the approximate dimensions of the dam, and the next step is to ascertain whether the intensity is at all points within the prescribed limits. Taking the section of the dam drawn to scale, we can easily find the c.g. of each portion, and, in the case of the lowest portion, the common c.g. of the weight of the masonry between the planes c and d, and the vertical water pressure on the sloping side, up-stream, of the dam between these planes. The methods of finding these centres of gravity have already been illustrated in the previous Chapter on retaining walls, as well as in Chapter II., Part I. (Sir G. Molesworth uses polar diagrams and funicular polygons for ascertaining these common centres of gravity. These are useful means to the end in view, but, as they do not necessarily form part of the solution of the problem, are here omitted).

We then draw verticals through these centres of gravity and find the points where the line of resultant water pressure above each plane cuts each of these verticals. These points are marked on the section (*Fig.* 3, *Plate* XXI.) by small circles. The line of action of the resultant force is then easily obtained by laying off vertically to any scale the value of the weight above the plane plus any vertical water pressure on the inclined face, and to the same scale, horizontally, the value of the horizontal water pressure. Completing the triangle by joining the extremities of the lines thus drawn we have the resultant of the weight of masonry and of the water pressure, and where this line intersects the plane will be the centre of pressure of the plane in question. Line of resistance, reservoir full.

Line of resistance, reservoir empty.

When the reservoir is empty the line of pressure is found by drawing vertical lines through the centres of gravity of the portions above each plane in succession.

Messrs. Tudsbery Turner and Brightmore's Method.

Messrs. Turner and Brightmore's calculations.

It has been mentioned above, in describing General Wray's method of investigation, that at a certain plane (fourth section) it is necessary to make the width of the dam greater than that which is required to bring the centres of pressure within the centre third, because of the great weight of masonry above, combined with water pressure, which tend to cause a maximum intensity of pressure greater than the material can safely stand.

"Low" dams and "high" dams.

Dams whose height is less than the limit thus obtained are denoted "low" dams, while those where the maximum intensity of pressure occurs in the base, and which are designed accordingly, are called "high" dams. "The relation of the breadth of the dam to its height is in the latter case dependent not only upon the position of the resultant stress, but also upon its amount, and the determination of this latter quantity strictly involves a knowledge of the very relation that is sought. In fact, the general form and the mass of the dam are indispensable data for the calculation of its base."* General Wray proposes, as we have seen, trial and error, a method which has disadvantages. In the method proposed by Messrs. Tudsbery Turner and Brightmore the problem is solved directly.

If Y = maximum stress intensity on any plane section (using the same notation as we have always done in this treatise).

t = breadth of the section.

h = height below the water level.

d = the distance of the centre of pressure from the extremity of that section.

W = total weight of masonry, plus vertical component of water pressure on the inner face, above the given plane section, the dam being considered of unit thickness.

P = corresponding horizontal thrust of the water against the inner face.

w = the weight of water per unit.

Formula for breadth at any depth.

It may be proved (see Appendix to this Chapter) that

$$t = \sqrt{\frac{wh^2}{Y}\left(1 + \frac{w^2h^4}{4W^2}\right)} \;\ldots\ldots\ldots\ldots\ldots (a).$$

* *Principles of Waterworks Engineering;* p. 217. Quoted by the kind permission of the authors.

From this equation the breadth is found with a high degree of accuracy, if W is only approximately known. It might appear at first sight as if the presence of W in this equation made the whole useless; but as it only forms a small fraction of the whole the result is comparatively affected by a slight error in the computation.

Having found t, the next step is to ascertain how much of it lies under the inner crest of the dam, *i.e.*, within the vertical through the crest, in order to bring the resultant weight of the whole superincumbent mass of masonry and water to the distance one-third t from the inner toe.

This is done by equating the moments of the vertical forces about the point in question to zero. Thus (*Fig.* 235), considering a lamina of unit thickness and of depth l, the upper and lower breadths being t_0 and t_1 situated at depths h_0 and h_1 below the surface, and x_1 being the increment of the breadth under the inner face of the dam, it may be proved (see Appendix to this Chapter) that

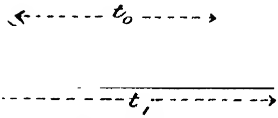


Fig. 235.

$$\frac{\rho w l}{24}\{3t_0^2 - t_1^2 + 6x_1(t_0+t_1) + 2t_0t_1\}$$

$$-\frac{wx_1}{12}(h_0+h_1)(2t_1 - 3x_1) \quad W_1 \quad t_1 - t_0 \quad = 0 \ldots\ldots (\beta).$$

This is an ordinary quadratic equation for finding x, and as t_1 has been already determined from (α), the precise value of W_1* may be fixed. The process of calculation may be repeated and applied to lower laminae, so as to determine other values of t. Formula for ascertaining necessary batter.

Fig. 1, *Plate* XXI., shows a section calculated for a dam 195 feet high, with $\rho = 2{\cdot}25$, and $s = 10$ tons per square foot. The triangular portion at the top is given for resisting wave action or to carry a roadway. It does not affect the conditions of the problem down to the point A, where the vertical through the c.g. of the triangular lump cuts the inner third of the breadth. Below that point it is necessary to give a slight batter to the inner or up-stream face, so as to bring the resultant of the masonry (dam plus triangular piece) within the centre third limit. At the point B the weight of the triangular part becomes comparatively insignificant, and the inner Example.

* W_0 and W_1 are the weights of the superincumbent masses of masonry above t_0 and t_1 respectively.

face may again be vertical down to C, where the limit of the "low dam" is reached.

Arched Dams.

In some dams the plan is curved, so as to present a convex surface up stream. In some few cases the cross section of the dam would be insufficient in itself to secure stability. Usually the curvature is given so as to furnish an extra factor of strength from the form of the horizontal arch.

Advantage of arched dams.

Major-General Sir J. C. Ardagh, K.C.I.E., C.B., has pointed out that "curvature on plan in long dams, though the increased power of resistance to water pressure afforded by the horizontal arch may be insignificant, is of very great importance as an automatic compensation for changes of length due to temperature, and this point has not received the attention it deserves."

Suitability in a deep gorge.

The Aqueduct Commissioners to the Quaker Bridge Dam at New York are of opinion that, "in designing a dam to enclose a deep narrow gorge, it is safe to give a curved form in plan, and to rely upon arch action for its stability; if the radius is short, *i.e.*, under about 300 feet, the cross section may be reduced below what is termed the gravity section. A gravity dam built in plan on a curve of long radius derives no appreciable aid from arch action so long as the masonry remains intact, but in case of the yielding of the masonry the curved form might prove of advantage . . . . The curved form better adapts itself to changes of form due to changes of temperature."

Pressure parallel to the chord of the

As the pressure of the water is everywhere normal to the tangent of the curve, it may be resolved into components at any point parallel to, and at right angles to, the chord of the arc (*Fig.* 236). The former series of resolved compressions, PP, tends to produce compression in the up-stream face, and that compression would tend to counteract any tendency to crack, *c*, caused by expansion or contraction of the mass, *which is inevitable.* This compression would be entirely independent of the arch action of the dam, since that would take place if the pressure were everywhere perpendicular to the chord

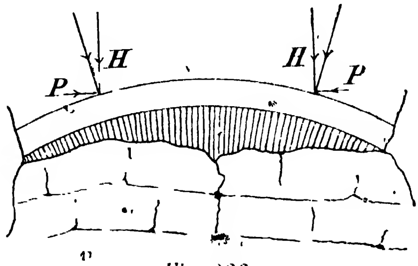


Fig. 236.

of the arc, as HH (*Fig.* 236). It further tends to consolidate the masonry transversely, a most desirable effect.*

The increase in length, due to the curved form, is comparatively slight.

Hence the introduction of the arched plan is generally an advisable precaution where additional security is required beyond that which is afforded by the stability of the cross section, calculated on the principles discussed above.

Dams of the purely arched form are rare; there are only three in existence, one in France, two in the United States. They are undoubtedly cheap, *e.g.*, that of the Bear Valley in California impounds water at the rate of £1 8s. 3d. per million gallons. Instances of arched dams.

Messrs. Tudsbery Turner and Brightmore point out with reference to purely arched dams that, provided the same maximum stress intensity is permissible in arched dams as in those of the gravity section, there will only be a saving of breadth when the radius of curvature is less than 250 feet; further, that the breadth should theoretically increase with the depth. Hence the use of such dams must be very limited, unless a higher stress intensity be allowed in them than that ordinarily permitted in gravity dams. In the Bear Valley Dam the stress intensity is calculated at 53 tons per square foot. Evidently such a structure is in a condition of internal strain far in excess of that usually considered as safe. Limits of employment of arched dams.

Abutment Walls of Service Reservoirs.

Service reservoirs are usually covered over, and this covering is frequently arched. Where the reservoir is above ground, and the abutment walls have no earth backing, they are subject when full to three distinct forces (*Fig.* 237), viz., the thrust of the arch Q, the weight of the abutment W, and the thrust of the water P. Of these, Q and P can be found, the former approximately by the principles explained in Chapter X., and the latter accurately. The third weight W is unknown, and must be found by trial and error. Forces acting on abutment walls of service reservoirs. No earth backing.

Where there is earth backing a fourth force, viz., the earth pressure T, acts at the other side of the wall from the water pressure, and at a point $\frac{1}{3}$ of the height of the earth above the base of the abutment, and making an angle with the horizontal equal to the angle of repose of the earth (*Fig.* 238). Forces when there is earth backing.

The resultant stress in either case must cut the base within the

* Professor Baker's *Masonry Construction*, p. 332.

centre third. No tension should be possible, especially on the side where the water presses. (See also *The Water Supply of Barracks and Cantonments*, p. 200).

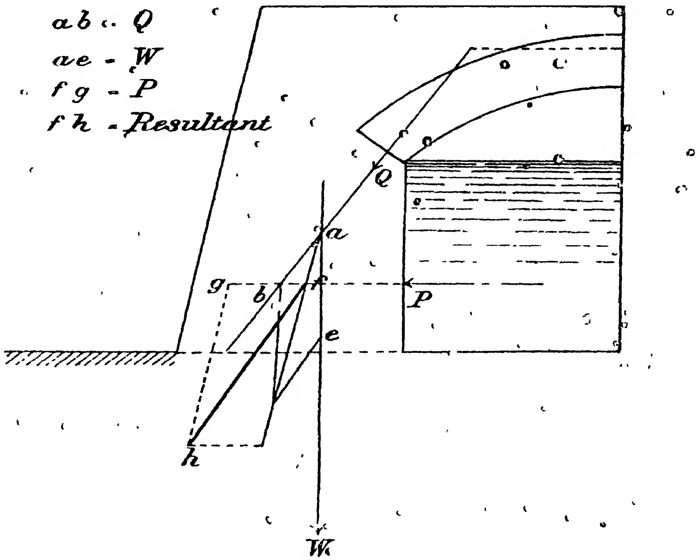


Fig. 237.

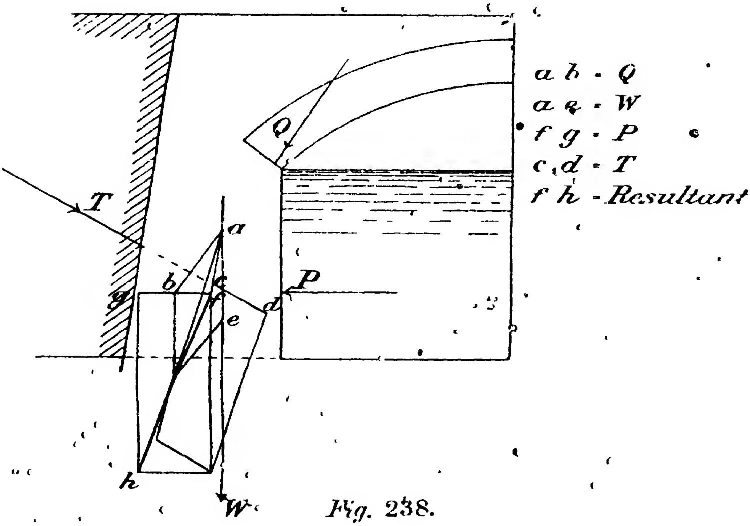


Fig. 238.

APPENDIX.

Proof of Messrs. Tudsbery Turner and Brightmore's Formulæ.

Let W (*Fig.* 239) = total weight above a horizontal plane AB of the masonry, and the vertical component of water pressure over the inner face of a dam (a vertical section of unit length being considered), and let P = the corresponding horizontal thrust of the water against the inner face, and let R = the resultant pressure on the plane section AB. Then

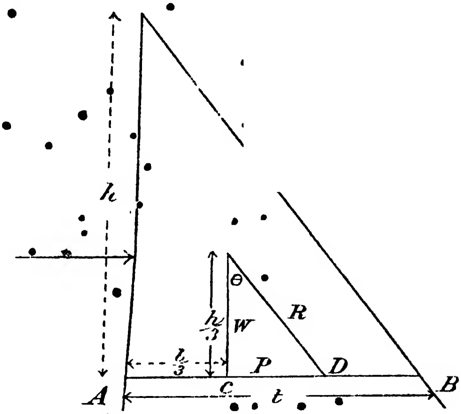


Fig. 239.

$$P = \tfrac{1}{2}\,wh^2,$$

and

$$R = \sqrt{W^2 + P^2}.$$

The maximum stress intensity on a horizontal section is, according to M. Bouvier's principle—

$$= Y\cos\theta = Y\frac{W}{\sqrt{W^2+P^2}},$$

and

$$n = \frac{\sqrt{W^2+P^2}}{t}.$$

Here Y, n and t have the same values as on p. 301, Part I., *i.e.*, t = breadth of the base AB, Y = maximum intensity of pressure, AC, and n = the mean intensity of pressure, EF (*Fig.* 240).

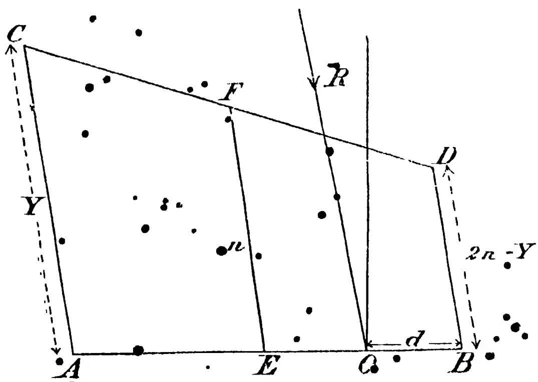


Fig. 240.

But from Equation (a), p. 302, Part I., if d = distance of the centre of pressure from the edge—

$$l=\frac{t}{3}\left(2-\frac{Y}{2n}\right)$$

which holds good, whether the thrust be normal or oblique. Hence (*Fig.* 240)

$$t = AB = AC + CD + DB - (\text{when } d \text{ is one-third of } t),$$

and substituting the value of n as found above), whence

$$\frac{Ph}{Wt}=\frac{YtW}{2\sqrt{W^2+P^2}},$$

$$t^2=\frac{2Ph\sqrt{W^2+P^2}}{YW^2},$$

$$\sqrt{\frac{wh^3}{Y}\left(1+\frac{w^2h^4}{4W^2}\right)},$$

as on page 290.

To find how far t must lie within the vertical through the crest, in order to bring the incidence of the resultant weight of the whole superincumbent mass of masonry and water to the distance $\frac{t}{3}$ from the inner toe, let O mark that distance, and equate the moments of the vertical forces about O to zero (*Fig.* 241).

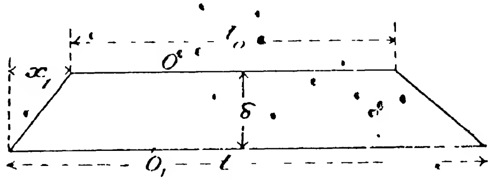


Fig. 241.

Considering a lamina of unit thickness and of depth δ, the upper and lower breadths of which are respectively t_0 and t_1, situated at depths h_0 and h_1 below the crest of the dam, let W_0 = total superincumbent weight on the upper surface of the lamina, acting at O_0, $\frac{t}{3}$ from the inner face, and let x_1 = the required increment of breadth under the inner face of the dam due to the addition of the lamina. Then the weight of the lamina is

$$\frac{t_0+t_1}{2}\times\rho w\,\delta,$$

and the distance of its c.g. from the vertical axis (which is assumed to be the middle of the average depth, which of course requires δ to be very small) is

$$\frac{x_1}{2}+\frac{t_0+t_1}{4}-\frac{t_1}{3}=\frac{3t_0+6x_1-t_1}{12}.$$

The weight of the water over the inner face of the lamina is

$$\frac{h_0 + h_1}{2} \times w x_1,$$

and the distance of its c.g. from O is $\frac{t}{3} - \frac{1}{2}x_1$, and the moment of W_0 ab

$$W_0\left(\frac{t_1 - t_0}{3} - x_1\right).$$

The condition of equilibrium of the entire mass above t_1 and about O_1 is therefore expressed by the equation

$$\frac{\rho w c}{24}\{3t_0^2 - t_1^2 + 6x_1(t_0 + t_1) - 2t_0 t_1\} - \frac{w x_1}{12}(h_0 + h_1)(2t_1 - 3x_1) - W_0\left(\frac{t_1 - t_0}{3} - x_1\right) = 0,$$

as on page 291.

CHAPTER XIII.

SEA DEFENCES.

Materials.—Action of Sea-Water on Concrete.—Timber.—Cast Iron.—Wrought Iron.—Wave Action.—Classes of Waves.—Forces produced by Waves.—Internal Destructive Forces in Structures.—Design of Piers in Plan.—Section of Piers.—Various Methods of Construction.

Importance of subject to military engineers.

THE subject of the design of structures to resist the shock of waves is of considerable importance for English military engineers, one of whose most important duties is the construction of coast fortifications, as well as of piers, sea-walls, and other similar accessory works in connection with W.D. lands on the seashore.

Peculiar nature forces.

The design of such structures differs so far from others that they are exposed to external forces of great violence and peculiar application.

Before discussing, however, the nature of these external forces, it may be well to say a little about the materials which can be used.

Materials.

One of the chief requisites of a material which is intended to resist the action of the sea is strength and stolidity, and hence it has frequently been the case that massive stonework has been adopted where in a similar structure not exposed to the violence of the waves a lighter material would have been quite sufficient.

Stone.

The heavier classes of stone, such as granite, basalt and gneiss, particularly granite, are well adapted for marine structures; but,

Concrete.

owing to the expense of such stone in certain localities, concrete is frequently substituted. It may be either formed in large blocks and deposited on horizontal beds, or these blocks may be arranged with sloping beds, as shown in *Fig.* 242. In either case it is deposited so as to have some form of key between one block and another.

Another method of construction is to have two walls of blocks with loose concrete filled in between. Another method is to have the concrete deposited in large bags, which serve the purpose of protecting it from the wash of the waves until the cement has set. Another method is to have the concrete *en bloc*.

In all these cases the question arises how far the action of the sea has a deteriorating effect upon the cement which is used for the matrix of the concrete, and what proportion of cement to sand in the matrix gives the best resistance. To these important questions the best answer perhaps is furnished by experiments which have been made in various parts of the world extending over a period of several years, and under conditions adapted closely to practice. Such experiments have been made in the harbour of La Rochelle* for some 10 years by French engineers, and their conclusions may be summarized as follows : – Action of sea-water on cement.

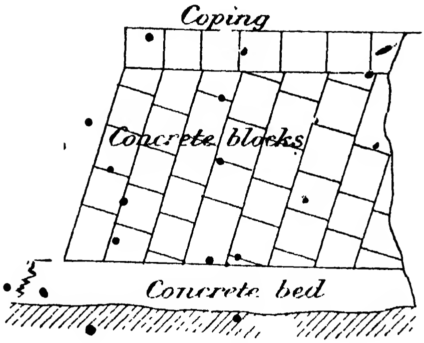


Fig. 242.

(1). Neat cements are destroyed more rapidly than mortars.

(2). Mortars made of 1 cement to 1 sand, or 1 cement to 2 sand, are those which offer the greatest resistance to sea-water.

(3). Mortars of hydraulic lime mixed in any proportion in most cases commence to disintegrate after one or two years' immersion in sea-water. They crumble into pulp after periods varying in length, but usually not exceeding 15 years. Conclusions of French engineers.

(4). Concrete resists sea-water better than masonry, owing to the greater density imparted to it by ramming. Of course this remark applies to concrete carefully and properly made, not carelessly mixed and consolidated, as is too frequently the case.

(5). A rapid setting cement may commence to disintegrate after six or eight years, but it has been observed that it *may* last longer than 38 years without crumbling.

(6). Free lime is most dangerous when contained in compounds burnt at high temperature. Any cement containing free lime is unfit for use with sea-water.

(7). It has been found that in lime mortars the quality of the sand had an important effect upon the resistance of the resulting concrete. Thus coarse sand gave much better results than fine sand.

* From the *Engineering Record*, December, 1897.

Probably crushed quartz and similar rock would give the best results of all (see page 35, Part I.). The main point to bear in mind seems to be that the voids in the mortar and concrete should be as completely filled up as possible, thereby increasing the density of the mass and reducing its porosity.

(8). It was found that blocks of neat cement did not stand nearly so well as a mixture of cement and sand, and some authorities consider that the best mixture would be obtained by mixing a certain percentage of puzzuolana with Portland cement. The puzzuolana will combine with the free lime, will contribute towards hardening, and will prevent the lime from forming dangerous compounds under the influence of the sulphate and magnesia contained in sea-water.

Washing away of the cement

The great difficulty in connection with concrete work under water has been that the action of tides and currents washes away the cement and renders the mass of no value. This difficulty has been overcome, in some cases, by depositing the concrete in very large masses, where, as the cement has a very much higher specific gravity than the water, there is comparatively little danger of the whole being permeated. At the breakwater of La Guaira, in South America, where this method was adopted, there were special arrangements for depositing the concrete in masses of about 100 tons at a time. Such a method as this involves, of course, special plant for deposition. Such a system does not entirely do away with the disintegration of the concrete, and it generally requires the occasional assistance of some divers.

Some engineers have recommended that the concrete should be allowed to set partially before it is deposited; others (engineers) have recommended a mixture of Medina cement with Portland cement It is, however, doubtful whether this quick-setting cement would last under the action of sea-water.

Grouting

Another system which deserves careful consideration is by "grouting," described in the *R.E. Professional Papers*, 1889, by Mr. Kinipple. This consists in laying first under water the aggregate of the concrete, then letting down a pipe with one end penetrating the loose material, and the other end furnished with a funnel through which the mortar may be poured. It has been found that the cement permeates the whole of the loose mass of aggregate, displaces the water in the interstices, and cements the whole together. Mr. Kinipple recommends that "the cement should be mixed to the consistency of a stiff paste, then put in a tub and slightly thinned down by adding water in very small quantities and stirred until the paste is reduced

to a thick grout, or just soft enough to leave the bucket from which it is to be poured as rapidly as possible into the funnel at the head of the pipe. The finer the cement, and the quicker it is poured down the pipe to keep up a continuous flow, the better."

Comparing masonry and concrete, the latter has many advantages:—

1. All squaring and dressing is dispensed with.
2. Small, hard, local stone can be used.
3. It seldom requires powerful machinery to move.
4. It adapts itself to the irregularities of the foundation.

To pass now from concrete to other materials.

Timber, although largely used for piers and jetties, cannot be regarded as *permanently* a good material, because it is liable to the attacks of boring worms. These creatures are prevalent all the world over, and although it has been alleged that certain processes, such as vulcanising, have the effect of rendering the timber unpalatable to them, yet the information on this subject has not as yet passed the early experimental stage, and it is impossible as yet to place any reliance upon it. Of all timbers, greenheart is that which is most durable for marine work. Generally it may be said to be proof against worms, and when it will not hold out against them, no other timber will. Some of the Australian timbers are said to be very good. Timber.

Cast iron is largely used for the standards of piers and jetties exposed to the action of the waves. There is no doubt that the sea water has some deleterious effect upon it. General Sir Chas. Pasley, R.E., in reporting on the metals found in the *Royal George* and *Edgar*, stated that cast iron had generally become quite soft, and in some cases resembled plumbago, whereas wrought iron was not so much injured, except when in contact with copper, or brass, or gun-metal. Cast iron intended to resist sea-water should be of close-grained, hard, white metal.* In such the small quantity of contained carbon is chemically combined with the metal, but in the dark or mottled iron it is mechanically mixed, and such iron soon becomes soft when exposed to sea water. Generally it may be said that cast-iron piles made from hard, white, or light grey castings are the best for sea work, and may remain secure a long time. Cast Iron.

With regard to *wrought iron*, which we know corrodes more rapidly Wrought Iron.

* Trautwine.

in fresh water than cast iron, there appears to be comparatively less deterioration when exposed to sea-water. At the same time, there is no doubt that considerable corrosion does take place, and particularly in those parts between tides which are exposed alternately to water and air.

How far a screw-pile pier will be effective in breaking the force of waves is uncertain. After some time, continual vibrations tend to loosen the joints, and deterioration will be produced in them, by repeated shocks and by rust.

Resistance afforded by structure.

Whatever the materials used, the resistance which they will afford may be either :—

(*a*). Dependent solely on the mass or weight, or

(*b*). Dependent on the strength of the various members and their connections with each other and the bottom.

Of these two classes of resistance the former is unquestionably preferable to the latter, if circumstances admit of its practical application.

External Forces.

We now consider the peculiar action of the forces upon any engineering structure exposed to the sea.

Wave action. Exposure.

Wave Action.—This is by far the most important of all the considerations under review. The exposure on any given coast may to a certain extent be estimated by an experienced person from the appearance of that coast. In general, however, the safest guide to the exposure is the level below the surface of low water at which mud reposes on the bottom of the sea.* On the coast of Holland mud is found at from 12 to 16 fathoms; at the mouth of the Elbe, at 8 fathoms; at Wick, from 60 to 70 fathoms; at the Moray Firth, about 35 fathoms; off the Firth of Forth, about 29 fathoms; and on the Coast of Norway, from 40 to 50 fathoms. Thus, taking the North Sea alone, these data would indicate that the Southern and South-Eastern coasts are the least exposed, and the North of Scotland the most, a conclusion which is amply justified by facts. Similarly, in Ireland, on the West, the mud level is 40—60 fathoms; near Dublin it is about 20 fathoms, and at Belfast Lough 5 fathoms, below low water.

Waves are caused by wind-pressure acting on the water of the ocean. They appear to roll onwards, though really the form and

* Stevenson, p. 19.

energy of the wave only, and not the mass or volume of the water, is transmitted through the undulations.

Apparently the volume of water is moving forward, but really this is not the case, as we can see if we fix our attention on some floating object, which, it will be seen, does not move forward, but oscillates to and fro within certain limits.

According to Professor Rankine,* the path described by each particle of water is elliptical, following the direction of the curved arrows in *Fig.* 243, in which the straight arrow denotes the direction of propagation.

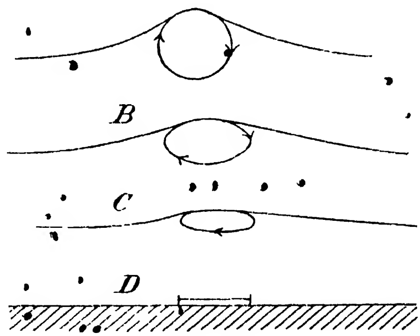


Fig. 243.

Near the surface the particles describe the largest orbits, the extent of the motion, both horizontal and vertical, diminishing as the depth below the surface increases, "but that of vertical motion more rapidly than that of the horizontal motion, so that the deeper a particle is situated the more flattened is its orbit, as indicated at A, B and C, a particle in contact with the bottom moves backwards and forwards in a horizontal straight line as at D."

The *length* of a wave is the distance between two successive crests; the *depth* is the vertical height from summit of crest to the lowest point of the trough; the *period* of a wave is the time occupied in traversing a distance equal to its length, or the time which a particle takes to make one complete revolution. Length, depth, and period of waves.

The form of waves is approximately cycloidal, and usually the orbit of the surface particles approaches a circle.

Where a series of waves advances into water becoming gradually shallower the periods remain the same, but the orbits of the particles become distorted, as in *Fig.* 244. The velocity of the lower portions is checked by the friction of the shore, until at length the front of

* *Civil Engineering*, p. 753.

the wave passes the vertical, its crest falls, and the wave breaks into surf, with a forward motion due to the velocity of the wave.

When waves roll againt a vertical wall, or even one with considerable batter, they are reflected, and the particles of water for a certain distance in front of the wall have motions compounded of those due to the direct and the reflected waves.

Long low waves more destructive than short high ones.

Since the velocity of waves is the chief determining factor in their impact, and since the period of waves is constant at a given time and place, it follows that long low waves, traversing a greater space in the same time as short high waves, must have the greater velocity, and therefore greater destructive effect, *cæteris paribus*, than waves which may appear from their height to be more formidable, but which may be only due to surface agitation.

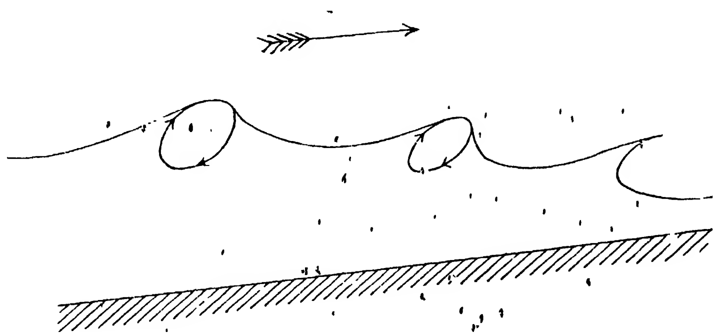


Fig. 214.

The magnitude of the force in the case of long low waves is only manifested when the continuity of transmission is broken by a solid obstacle, such as a cliff or a light-house. The spray dashed up on cliffs after a heavy storm, when the surface of the sea may be unbroken, is evidence of this.

There are two classes of waves :—

Surface waves and rollers.

1. Oscillating or surface waves, where the length is small compared with the depth of the water.

2. Waves of translation or rollers.

Of the two the latter are by far most dangerous to structures. They constitute, in fact, a vast mass of water transmitting energy in one direction with considerable momentum.

Fetch.

It has been found by experience that waves are largest in the open sea, hence there must be some ratio between the height of the

waves and the distance from which they have travelled. This distance is called "fetch," or "reach" of the open sea. It may be measured from a map. Its effect is modified in any given case by the intervention of a headland in the vicinity of the site in question.

The following empirical formulæ have been used, and are fairly accurate :— Formulæ for height of waves.

If h = height of waves in feet, and

d = distance from the windward shore in miles,

a = co-efficient varying with strength of wind.

Then $$h = a\sqrt{d} = 1{\cdot}5\sqrt{d} \text{ as a rule.}$$

For short distances the formula may be taken as follows :—

$$h = 1{\cdot}5\sqrt{d} + 2{\cdot}5 - \sqrt[4]{d}.$$

In oceans the height amounts at times to 30 or 40 feet. During a gale the forms of waves are very irregular, and it is not possible to trace any individual undulation for a long distance.

The limit of depth below which one can reckon that wave-action ceases is, approximately, 20 feet below low water. Immediately below low water level a sudden reduction of force takes place. Limit of wave action.

A few instances of recorded wave force may be mentioned here. At Wick, in December, 1872, a block of concrete weighing 1,350 tons was bodily moved. In 1877 a mass weighing 2,600 tons was removed. In both these cases the superstructure of the wall alone was affected, and the foundations were left intact. Mr. Stevenson quotes several most remarkable instances of large rocks having been *quarried* from positions above high-water level and moved a considerable distance. These cases occurred on the exposed coasts of Shetland. Wave force instances.

The actual force of waves has been measured by the blows delivered on a dynamometer placed so as to be immersed at ¾ tide. From experiments made by Mr. Stevenson in the Atlantic Ocean, near the Hebrides, and on the North Sea, at Dunbar, and in Banffshire, it was found that the maximum wave force was 3 to 3½ tons per square foot,* being greater in winter than summer.

The direct horizontal force of the waves, however, is not the only one which we have to consider in their effect upon a structure. There are indeed four distinct forces :— Four distinct forces produced by waves.

* There can be no doubt that higher results may be obtained (Stevenson).

(1). Direct horizontal force.

(2). Vertical upward force acting against any projection.

(3). Vertical downward force, as in *Fig.* 245, from spray or waves falling.

(4). Back draught. This is the force produced by a receding wave, which tends to move the bed of the foundation and to undermine the lower courses.

Horizontal force.

(1). *Horizontal Force.*—We have already seen that the maximum horizontal force measured by dynamometer was 3 to $3\frac{1}{2}$ tons per square foot. This maximum force probably acts at about the level of high water. But it has been found that the most dangerous waves happen, not at high water itself, but at $\frac{3}{4}$ tide, both on the ebb and flow.

Upward vertical force.

(2). As regards *upward vertical force*, experiments made by Mr. Stevenson have shown that the vertical force tending to raise the projecting coping of a sea-wall 23 feet above water level was at least 84 times greater than the horizontal force tending to thrust it inwards.

Downward vertical force.

(3). *Vertical Downward Force.*—This is due to the vertical descent of spray dashing against an obstacle.

It has been found that spray rises seven times higher than the height of the waves which produce it on a hollow or curved wall, and $6\frac{1}{2}$ times higher on a vertical wall. The actual amount of this force per square foot can be calculated from the consideration of the momentum produced by a cubic foot of water falling freely from such a height.

Back draught.

(4). *Back Draught.* From experiments made by Mr. Stevenson, back draught was found to be three times the direct force.

We thus judge that as regards the vertical section of a pier or structure the weak points are the top and toe. Hence the importance of the *contour of the wall*, the *quality of the masonry*, and the *form of the parapet*. No projecting coping or string course can be admitted. The surface of the roadway should be paved with such heavy stones that they cannot readily be dislodged by falling spray.

Su- an-

Wave action dynamical.

It must be remembered that the action of waves on a structure is dynamical. It has been referred to in Part I, p. 5, as an instance of live load, but this is hardly correct, as it cannot be regarded in any way as statical load. Water flowing through an ordinary supply pipe, and shut off suddenly, will exert, as is well known, an extra pressure throughout the pipe over and above the pressure due to the head of water in the pipe. This extra pressure is caused by the sudden arresting of the velocity of the moving water.

Fet

In the case of wave action on marine structures we have a moving mass of water whose velocity is suddenly arrested, and whose mass is far in excess of any quantity ever passed into supply pipes by human skill. The dynamic force, therefore, in almost every case must be great, and in times of storms it becomes gigantic.

The relations between velocity and head or hydrostatic pressure (referred to on p. 329, Part I) hold good in all cases; hence if we can ascertain the velocity of the waves, we would easily calculate the corresponding hydrostatic (but not hydrodynamic) pressure that would be produced. Mr. Shields, in his book on *Harbour Construction*, notes that in a storm at Peterhead, in 1888, he calculated from observations that the velocities of waves on a particular occasion were 41 feet per second. From the equation $v = \sqrt{2gh}$, this would correspond to a head of 26 feet, and this head would be equivalent to a pressure of 1622 lbs. per square foot. But the dynamical force of the waves has been registered at as much as 6,083 lbs. per square foot, horizontally.

When we come to consider the vertical force produced by falling masses of waves, we see that the dynamic force will be that due to the mass falling through the height to which the water has been projected. That height has often been observed to be at least 100 feet,* which would correspond to a velocity of 80 feet per second.

Thus we see that the vertical downward force is much greater than the direct horizontal force.

Destructive Forces Acting Internally in a Masonry Structure.

In addition to the external forces above mentioned, there are internal forces produced in the interior of a masonry pier or breakwater exposed to wave action:

(1). By vibration from the shock of waves through the whole structure.

(2). By the direct communication of the impulse of the wave action through fluid in the joints or interstices of the loose hearting.

(3). By sudden condensations and expansions of the air in the hearting, tending to loosen the face stones.

* Sir John Jackson, contractor at the Alderney Breakwater, has stated that he has seen water projected to a height of 200 feet in a gale (*Minutes of Proceedings, Institution of Civil Engineers*, Vol. XXXVII, p. 90). This corresponds to a velocity of 113 feet per second, and each cubic foot of water would have a momentum of $62 \cdot 5 \times 113^2 = 798,062$ foot-lbs.

(4). By the hydrostatic pressure of the water which is in the interstices, and which, when under any pressure, will act upon all surfaces with that pressure like a Bramah press.

For instance, in the pier shown on *Fig.* 245, built of two face walls, loose hearting and a roadway, if the interstices of the hearting are all filled with water before they can be protected by the roadway, any pressure produced subsequently by the vertical downward force will be transmitted laterally and tend to burst the work asunder. This has frequently been a cause of ruin in sea walls.

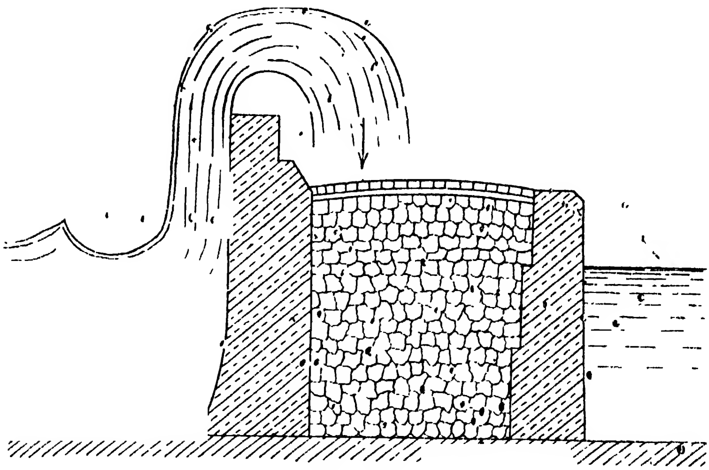


Fig. 245.

If the roadway which protects the hearting be made air-tight, and if the joints of the face be left open, the direct horizontal force of the waves tend to injure the work by the imprisoned air inside the hearting finding no means of escape, and forcing up the roadway.

Generally, loose rubble hearting in a sea wall of this type is a source of weakness.

Effect of back draught.

The effect of "back draught" is, among other things, to cause a certain amount of rarefaction in the surrounding air, which may tend to push stones outwards from the face of a sea wall, causing projections which ultimately may lead to serious consequences. The stones, therefore, on the face of a sea wall may be subjected to pressure due to simultaneous expansions of condensed air behind and rarefaction in front.

Before dismissing the subject of wave action, we have to consider the nature of wave force in plan. The following general principles have been laid down by experienced engineers:—

H = Height of waves which produce maximum effect due to the line of maximum exposure.

X = Greatest force brought to bear upon a pier.

Sin α = sine of the angle formed between direction of piers and line of maximum exposure.

Then resolving normally to the piers, X varies as $\sin^2 \alpha$, and along the piers as $\sin^3 \alpha$.

It has been recommended by experts that piers should be laid out so as to form a horizontal angle of not more than 25° with the heaviest billows. Generally, we conclude that the most exposed coasts are those where the waves generated in the line of maximum fetch strike the shore at right angles. Angle of Piers.

Waves are most destructive when their destruction coincides with the principal tide currents. "Wherever there is a tide current, the waves will more or less partake of the properties of waves of translation." (Stevenson). Effect of currents.

Into the complicated question of tide currents and the modifications produced by the shore configuration it is not proposed to here enter.

Design of Piers and Sea Walls.

Having discussed briefly the forces which assail works on the sea-shore, the next subject for consideration is the arrangement of the design of a pier or sea wall. It will be sufficient to confine attention to piers or sea walls, and not to breakwaters, as the latter are not likely to be constructed by R.E. officers. Design.

(1). *Preliminaries in Settling the Plan.*—A good chart of the site is necessary, and the actual position of the proposed work should be surveyed with soundings shown as accurately as possible. Sections of the sea bed may require to be taken parallel to the coast, as well as along the line of the proposed work. Preliminaries.

A thorough examination of the nature of the bottom on which the structure is to be founded is absolutely essential. The neglect of this has been shown by failures. Structures have been founded, for instance, on a layer of rock overlying a substratum of clay, where serious damage has subsequently resulted by slipping of the clay.

It must be remembered that sufficient depth will be required at

the lowest tide for vessels to come alongside the pier. With this proviso, it is well to build on shoal ground as far as possible.

Plan of piers.

In plan the pier should be, if possible, convex in the direction of maximum exposure, but in any case should not be either concave or present abrupt re-entering angles in that direction.

Plan of sea wall.

In the alignment of a sea wall the sinuosities of the coast should not be too rigidly followed along high-water mark. If this is done, not only a longer line will result, but also concave curves will be presented to the action of the waves, and will be a source of weakness. If the line be carried across minor indentions, the foundations of the wall will no doubt be deeper, but the ultimate result will be better than if the coast line had been adhered to throughout.

Advantage should be taken of any natural features of the site, such as ridges of rocks, etc., and the design should be adapted to suit them.

Design of pier-head

As regards the design of the pier-head, the following points may be noted: (*a*). As the pier-head is situated in the deepest water it will be the most exposed part of the structure. (*b*). It has only support from other parts of the structure on the landward side. (*c*). It is subject to rough usage from vessels coming alongside. Hence of all parts of the structure especial care should be devoted to it, and additional strength given in some way or other according to circumstances.

There is frequently a lighthouse at the end, generally there is a parapet wall (with masonry piers) to protect the roadway, and there are bollards, rings, and pawls for mooring vessels (these should be hollow, to act as air-valves for confined air in the hearting), cranes or capstans for lifting stores, and landing steps in sheltered positions. *Figs.* 246, 247 and 248 show some plans of pier ends. *Plate* XXII. shows a pier designed for landing stores, which has been in use for a good many years. The tramway line was designed to carry a movable crane.

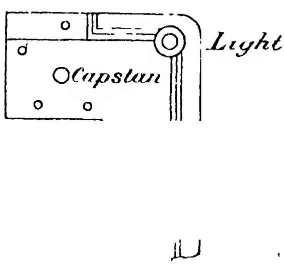


Fig. 246.

This pier is obviously not suited for an exposed position.

Section of pier.

(2). *Profile of Vertical Section.*—Where the *plan* of a wall is either on a a convex curve, or with straight sides and salient angles, special

care must be taken to strengthen the foundations at the salients, because of the increased scour at those points.

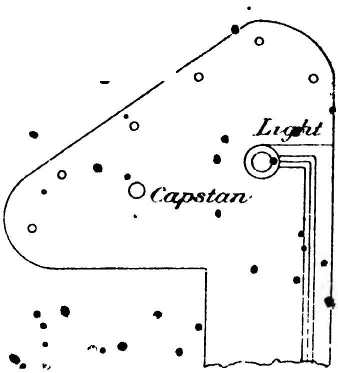


Fig 247.

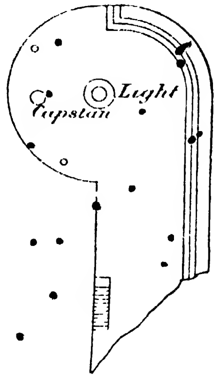


Fig. 248.

A vertical wall is generally unsuitable for a sandy beach on account of the scour at the toe. Mr. Stevenson recommends a cycloidal section, as shown in *Fig.* 249, which may practically be a

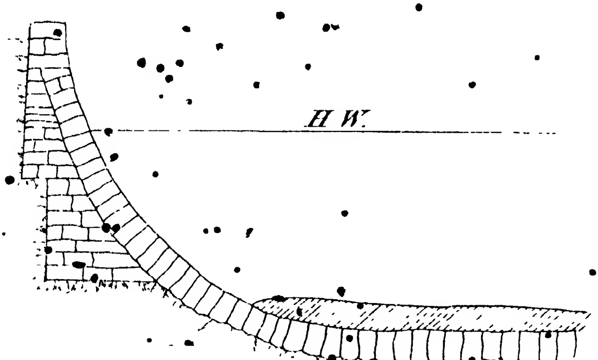


Fig. 249.

vertical wall connected with a horizontal apron by a quadrant of a circle of sufficient radius. The disadvantage of this form consists of the wedge-shaped stones, since the impact of the waves tends to drive the wedges inwards, in the same way as a blow delivered at

the intrados of an arch. Therefore, in such cases, the backing is of great importance. It is evident, also, that a curved form which may be very suitable at one state of the tide may not be equally good at another.

Sections of Breakwaters. The cross-sections of breakwaters are divided, by writers on the subject, into two main types:—(1), mound type; and (2), vertical type.

Mound type. 1. Of these, the former will very rarely apply to piers. It may be either a simple mound of rubble or concrete blocks, thrown at random into the sea, or a foundation of such random blocks with a superstructure carefully built above. Plymouth breakwater is an example of the former, and Portland of the latter (*vide Molesworth's Pocket Book*, p. 330, Edition 1893).

Vertical type. 2. The vertical type is that with which we are here more especially concerned. It has the following sub-divisions:—

1. Vertical wall, with or without slight batter (*Fig.* 250).
2. Curved or talus wall (*Fig.* 251).

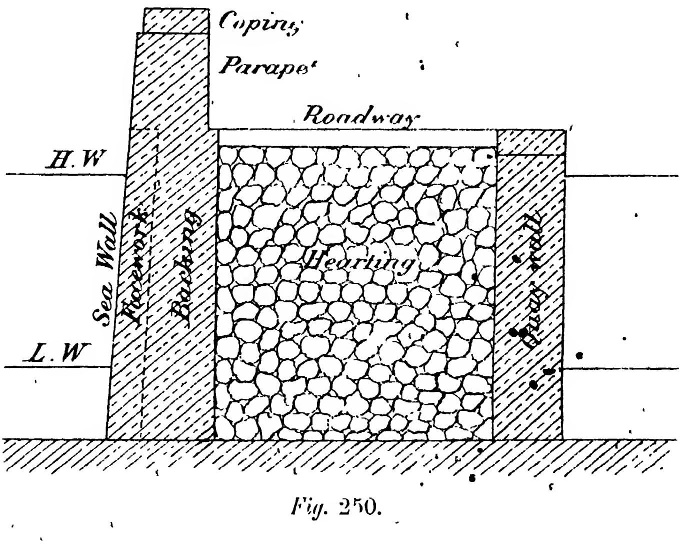


Fig. 250.

Of these, the vertical wall is best when (*a*) foundations are good and hard, *e.g.*, a rocky bottom; (*b*), when good materials are scarce; (*c*), when the stone is easily worked, *e.g.*, stratified rocks; (*d*), hearting stone scarce.

The talus wall would be suitable when (*a*) the foundations are soft or sandy; (*b*), when the stone for the face work though abundant is not easily worked; (*c*), when the hearting stone is plentiful.

In any case loose hearting must be avoided.

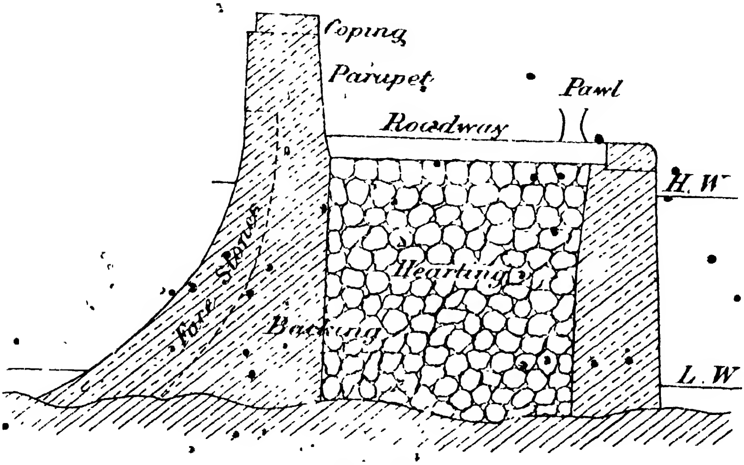


Fig. 251.

The whole may be built in a variety of ways, however, either (i.) with an outer framework of timber, or iron rails, filled with rubble or concrete; or (ii.) with mass concrete wholly or in part; or (iii.) with masonry or concrete outer walls, filled with dry rubble hearting; or (iv.) wholly of concrete blocks. Various methods of construction.

Of these, (i.) is a cheap and useful form where the water is shallow and the seas not heavy. This type is often used for works of a somewhat temporary character. A typical section of such a work is shown on *Fig.* 252.

Fig. 253 shows a pier formed of a mass of concrete below low water, then blocks up to above high-water level, then mass concrete above. The foundation is aggregate grouted.

Fig. 254 shows a section of (iii.) (Kilrush Pier, built by Lieut.-Colonel Jones, R.E.).

The special advantages of (iv.) need not here be enumerated. It is obvious that the larger the blocks the better, and in rough weather the workmen may be employed in the construction of blocks on shore, to be ready for deposition when weather permits.

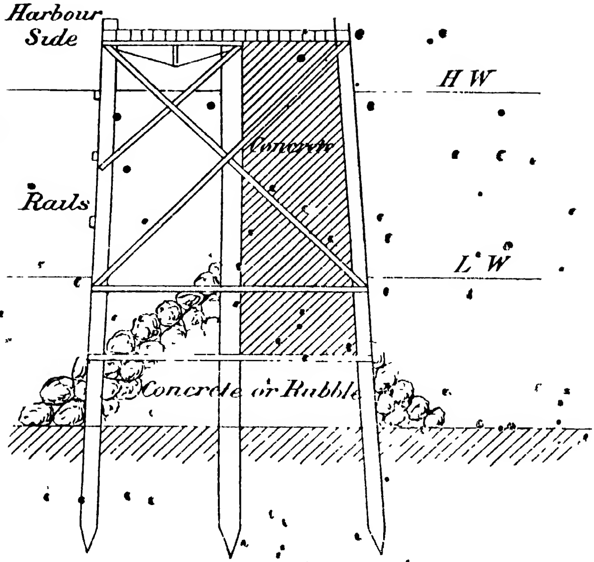


Fig. 252

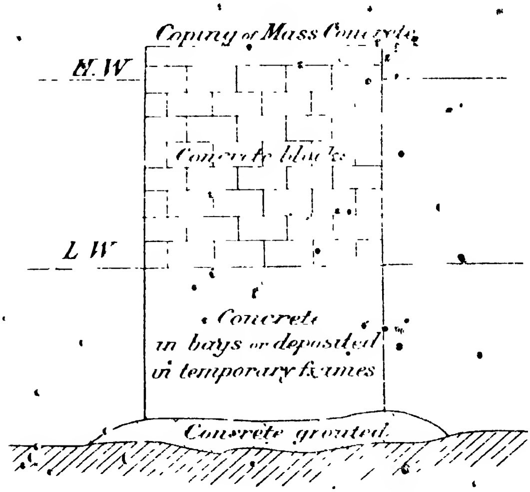


Fig. 253.

The following practical points should be attended to in construction:—

The facing stones should be uniform in size, truly bedded and jointed, all interstices being well filled with mortar, and carefully bonded into the backing as the work proceeds. Practical points to be observed.

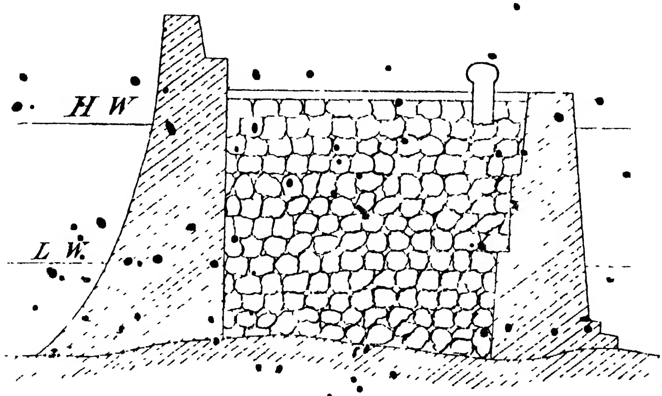


Fig. 251.

The hearting should be free from earthy matter; it should never be tipped indiscriminately. Long stones should be laid lengthwise, and large boulders should be rejected.

It has been found that fine workmanship on the face of the masonry is a positive disadvantage. There is nothing to be gained, therefore, by spending money on extra dressing. Professor Rankine recommends "hammer-dressed ashlar, or a block in-course face, backed with coursed rubble, or with strong concrete, the whole built in strong hydraulic mortar, and the outer edges of the joints laid in cement."

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